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A Model of Endogenous Union Density and Membership

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Abstract

We develop a theoretical model of endogenously determined union density and union membership. A union is formed, continued, or dissolved by majority voting. Given the profitability, production technology, and labor and product market conditions, the union determines the reservation wage that is acceptable to the firm. Based on this reservation wage and other subjective factors, workers vote for or against the union. If the union is formed, the firm determines the employment level at the union wage.

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1. Introduction

While U. S. public sector unions have continued to expand over the past few decades, the private sector experienced a severe contraction, declining from more than 38 percent of the private sector work force in 1950 to 14 percent in 1986 (Freeman, 1988). Though the experience in other industrialized countries has been mixed, a general decline in union membership in OECD countries of about 5 million workers occurred in the 1980's.

Different explanations have been offered, supported by empirical evidence, to rationalize these trends in union membership and densities (the percentage of workers unionized) (Chaison and Rose, 1991; Fiorito and Greer, 1982; Freeman, 1988; Lazear, 1988; Moore and Newman, 1988; Reder, 1988). Few theoretical models of unions, however, can explain these trends. Pencavel (1991, p. vii) indicates that most theoretical research on labor unions has developed independently and that the research does not explain the empirical findings. Bennett and Delaney (1993) note that the insufficient theoretical foundation of trade unions has sparked surprisingly little theoretical work. One explanation for the lack of analysis suggests that, unlike the firm or consumer, no unique theory defines the behavior of a union (Johnson, 1975).¹ Booth (1995) provides a valuable synthesis of the various theoretical models and empirical studies of trade-union research.

We develop a model of endogenous union density and membership. The model examines the conditions under which unions occur and explains what makes certain firms/industries traditionally unionized and others traditionally non-unionized. As such, the existence and size of unions emerges through endogenous events. Specifically, we study the conditions that lead to the formation, continuation, and dissolution of unions and the factors that determine union size. We then use the model to examine the recent trends in union membership and union densities in the United States.

The theoretical union literature can be broadly classified into game-theoretic and non-game-theoretic models (Booth, 1995). Non-game-theoretic models explore the static outcomes of

the bargaining process while game-theoretic models examine the dynamics of the bargaining process. Our modeling strategy falls into the non-game-theoretic camp.

Non-game-theoretic models further divide into monopoly, bargaining, and median-voter models. First, monopoly models argue that unions maximize a utility function subject to the firms' demand for labor (Oswald, 1982; Pencavel, 1984). In these models, the union sets the wage and the firm determines employment. Second, several different bargaining models exist. One, right-to-manage models (Booth, 1995, Ch. 5) continue to have the firm set the level of employment, once bargaining over wages is concluded. The monopoly model, therefore, emerges as a special case of right-to-manage models where the firm's bargaining power is zero. Two, efficient-bargaining models (Calvo, 1978; McDonald and Solow, 1981) assume that unions maximize a utility function subject to a firm's given profit level. In these models, the union and the employer negotiate wages and employment simultaneously. Unlike monopoly models, the outcomes here are Pareto optimal. Third, in voting models, the median voter determines the outcome (Farber, 1978; Blair and Crawford, 1984; Grossman, 1983). The union provides a public good; and, given the different demands for union services, the union satisfies the median voter's demand for union services (Booth 1995, Ch. 4). These models, however, are not mutually exclusive. For example, Booth and Chatterjee (1995) develop a model that combines the efficient bargaining model with the median voter model. Similarly, Booth (1995, p. 113) outlines a hybrid of monopoly and median voter models assuming that the union sets the wage by maximizing the expected utility of the median member.

Monopoly and bargaining models differ from voting models in several important respects. The first two models possess homogenous members, the median voter model introduces heterogeneity. The former models generally do not distinguish between the objectives of the union and its members, assuming them to be identical. For example, maximization of an expected utility function implies that all members are identical and, as such, preferences of individual workers represent those of the union. Similarly, when unions maximize the wage bill, economic rent, or the utility function, these objectives reflect the members' views. Voting models, on the other hand,

stipulate that the union's objectives may differ from those of individual workers. Finally, union size emerges exogenously in the former models and endogenously in some of the voting models (Grossman, 1983).

Few theoretical models can explain the recent decline in union density and union membership. Specifically, they cannot address the following two interrelated questions. Why do unions exist in certain firms/industries and not in others?² And given that unions exist in certain firms/industries, what factors determine the number of members in unions? While some papers examine one question, they do not address the other. For example, Oswald (1982) identifies those factors that affect union density in different firms/industries, but his model does not discuss the determinates of union size. Median voter models, on the other hand, analyze the conditions that affect union size, but do not consider the differences in union density across different firms/industries.

Our paper develops a model of an industrial union that addresses both questions simultaneously. Our model incorporates ideas from the bargaining (right-to-manage) and median-voter models.³ The wage paid reflects the bargaining between the union and the firm. At the same time, workers are heterogeneous as is characteristic of median-voter models.

Though our model adopts majority rule to establish, continue, or dissolve a union, it differs from median-voter models in several important respects. Contrary to these models, we do not assume that unions have monopoly power and can set any wage. For example, in median-voter models, the union sets the wage that maximizes the expected utility of the median member. Instead, we examine the conditions that influence the union's ability to bargain for a higher wage. Since the firm has a reservation wage, which if exceeded, is unacceptable, we assume that the union always offers a wage that is acceptable to the firm. The introduction of union bargaining power enables the union wage to be determined endogenously. Bargaining power derives from an expression of the relative costs of strikes. We also consider the costs of organizing unions, which most studies ignore. Operating costs and membership fees are important factors that determine union density and union size. While we consider the factors that determine the union bargaining power and how

voting among members leads to the formation/dissolution of unions, the bargaining process and the union's political role are not considered explicitly.

The next section outlines the model and describes the behavior of the union and workers. Section 3 enumerates the steps involved in union formation/dissolution. Since we motivate the model by the institutional framework in the United States, the model is used to explain the recent trends in union density and union size in the United States. The paper ends with a conclusion.

2. Model

Assume that the firm employs N^c workers at the competitive wage w without a union. A union is formed, kept, or dissolved by a majority-voting rule. The union wage is set at W^u for the contract period T , which is assumed to be one. Three decision-makers exist: the union (represented by a manager or a leader), workers (employees), and the firm (employer).⁴ All agents possess information on the different variables affecting wage and employment levels. The union maximizes the welfare of its members. Though the union's existence may lead to, among other things, better working conditions, employment security, possible employer retaliation against the members, and so on, the union primarily bargains for higher wages and derives economic rents for its members. The union leader proposes the formation of a union by declaring the expected wage rate and the union membership fee. Individual (risk-neutral) workers vote for the union, if the expected benefits exceed the expected costs. Once the union's wage offer is known, the firm determines employment based on its demand curve $N^d = N(W, P)$, where W and P are the nominal wage and price of the product. The behavior of the union and its members is discussed next.

Union: The union sets the wage and the membership fee that covers the union's operating cost. When setting the wage, the union considers two conflicting factors. On the one hand, unions prefer higher wages to capture more economic rent (and attract more workers as members). On the other hand, a higher wage demand increases the probability that the firm rejects it. The probability that the firm accepts a higher wage demand depends on the relative costs of resisting the higher wage demand and that of agreeing to it (Hicks, 1963). If the cost of the latter outweighs the former,

then the firm does not accept the union's demand. Thus, the union's ability to raise the wage depends on these relative costs.

These relative costs are defined as bargaining power (Chamberlain, 1951; Mulvey, 1978) as follows:

$$(1) \quad BP = [C_d / C_a],$$

where BP is bargaining power, C_d is the cost to the firm of rejecting the union's offer, and C_a is the cost to the firm of accepting it. The cost of rejecting the union's demand is a strike and the loss of profit over the strike's duration t , which is exogenous in the model.⁵ The cost of accepting the union's demand, on the other hand, is the higher wage paid over the contract period T (assumed to be unity), and the corresponding reduction in net-income or profit.

Employers may also oppose unions on ideological grounds (e.g., belief in the market mechanism), where they view, for example, unions "solely as impediments to flexibility" (Chaison and Rose, 1991, pp. 22). This subjective factor is represented by an index $\alpha \in (0,1)$; $\alpha=1$ when the employers do not have any ideological beliefs affecting decisions, and $\alpha=0$ when employers opposition to unions on ideological grounds is total. Assuming that both parties (union and firm) have knowledge of the discounted values of the costs and the index α , the bargaining power of the union can be written as follows:

$$(2) \quad BP = \alpha[(1+ti)^{-1} t P^c] / [(1+i)^{-1} P_w],$$

where i is the discount rate, $P^c = p^c Q^c - wN^c$ is the net income of the firm when it uses labor at the competitive wage w , and P_w is the loss in profit that results from paying the higher union wage during the contract period T . That is, $P_w = dP$ is the total derivative of profit due a change in the wage (dW). Note that dW is the difference between the union wage (W^u) and the competitive wage (w), or the rent r that workers get after unionization.

In words, the contract period T is normalized to one. The cost of the union contract in terms of profit relative to the competitive case is dP . That loss of profit is discounted to the present to give $(1+i)^{-1} P_w$. The cost of the strike reflects the net income from the competitive outcome P^c times the length of the strike, which is t . Since the contract length is normalized to one, it is

probable that the strike length is less than one. The loss of net income because of the strike $t P^c$ is discounted to the present to give $(1+ti)^{-1} t P^c$. Finally, the ratio of costs is multiplied by the subjective factor reflecting ideological beliefs.

The loss of profit due to the higher wage equals the following (see Appendix A for the derivation):

$$(3) \quad dP = [1/(e_{yp}^s + e_{yp}^d)][PY_w(e_{yp}^d - 1) - (N/s_w)[e_{yp}^d e_{nw}(W, P) + e_{yw} e_{np}]]dW - \\ [[(e_{yp}^d R K s)/[W(e_{yp}^s + e_{yp}^d)]] + N]dW$$

where $Y_w = dY/dw < 0$, $e_{yp}^d = -d \ln Y^d / d \ln P > 0$, $e_{yp}^s = d \ln Y^s / d \ln P > 0$, $e_{nw} = d \ln N / d \ln W > 0$, $s_w = WN/TC > 0$, $s = \tilde{K} - \tilde{N} / \tilde{W} > 0$, Y_w is the derivative of output supply with respect to the wage, e_{yp}^d is the price elasticity of demand, e_{yp}^s is price elasticity of supply, e_{nw} is the elasticity of labor demand, s_w is the share of labor cost to total cost, and σ is the *two-factor-one-price* elasticity of substitution (TOES) (Mundlak, 1968; Chambers, 1988).

Substituting the value for $P_w = dP$ into equation (2) gives the bargaining power as follows:

$$(4) \quad BP = [a(1+i)tP^c/(1+ti)dP], \text{ or}$$

$$(4a) \quad BP = p(a, P^c, r, t, e_{yp}^d, e_{yp}^s, s, s_w, e_{nw}).$$

Thus, the bargaining power of the union increases in the strike period (t), the price elasticity of supply (e_{yp}^s), and the profitability of the firm (P^c), and decreases in the index for employer opposition on ideological grounds (a), the rent (r), the price elasticity of demand (e_{yp}^d), the elasticity of substitution (σ), the share of labor in total cost (s_w), and the elasticity of labor demand (e_{nw}) (see Appendix A for the comparative static results).

The union's bargaining position is strong if $BP > 1$; weak, if $BP < 1$. Bargaining power decreases with wage demand. That is, the cost of accepting a relatively small increase in the wage is smaller to the firm and, as such, increases the probability of accepting the union's demand, implying a larger bargaining power. But, if the union demands a relatively high wage, the probability that the firm accepts it is small, thereby decreasing the union's bargaining power. Given the production technology and the characteristics of the factor markets, let $\bar{r} = \bar{W}^u - w$ represent the rent that makes the union's bargaining power equal to one (set $BP = 1$, and solve for

$\bar{r}$). That is, the union can ask for at most a wage equal to $\bar{W}^u$ (and get rent $\bar{r}$) and still expect the firm to accept it. Any wage demand higher than $\bar{W}^u$ is not accepted by the firm (see Figure 1).

The other union decision sets the fee for its members. This fee equals the total cost of operating the union divided by its membership. The cost of establishing a union involves some fixed cost (c_0) and member-dependent variable cost. The cost (C) of union activities increases with membership, but due to economies of scale, at a decreasing rate. The larger the spread of the workers (s), the more the cost of organizing.⁶ The cost of a union, thus, relates positively to the number of members (N^m) and to the spread (s). That is,

$$(5) \quad C = c_0 + c(N^m, s), \quad c_i > 0, c_{N^m N^m} < 0, \quad i = N^m, s.$$

The union fee is given as follows:

$$(6) \quad f = [c_0 + c(N^m, s)] / N^m = C / N^m$$

This fee (f) is an decreasing function of N^m , and a increasing function of the spread of workers.

That is,

$$(7) \quad df/dN^m = (1/N^m)[c_{N^m} - (C/N^m)] < 0; \text{ and } df/ds = c_s/N^m > 0.$$

Note that the cost function (equation 5) implies that the average cost (C/N^m) always exceeds the marginal cost (c_{N^m}), implying that df/dN^m is negative.

Figure 2 illustrates the negative relation between the fee and the number of workers for a given spread. Note that the union does not minimize cost in the classical sense by equating the marginal revenue (here the membership fee, f , which is also the average revenue) to the marginal cost. Instead the marginal (average) revenue is equated to the average cost. The number of workers that vote for a union is determined by the workers (discussed next). The union takes this as given and sets the membership fee to cover operating costs.

Workers: Assume that all workers are risk neutral.⁷ Individual workers vote in favor of the union if the expected net benefit exceeds or equals the cost (fee). That is,

$$(8) \quad E(B) \geq f.$$

Workers expect the union to provide a higher wage, employment security, and better working conditions. The (subjective) costs include possible employer retaliation and subsequent

job loss, and a certain amount of inconvenience (Ashenfelter and Pencavel, 1969, pp. 130). Furthermore, workers possess different attitudes toward subjective factors such as comradeship, altruism, and political beliefs. These subjective factors respond to the legal, political, and social environment surrounding the workers. The public policy of labor relations, public opinion, attitudes towards unions, and other factors, such as laws, institutions, and the political party in power, all affect these subjective factors.

Let h_i equal the net collective value of these other factors (subjective benefits minus costs) as seen by worker i . Assume that the distribution of h_i is uniform across some range of values. Finally, h_i responds positively to the organizing efforts of the union and negatively to the efforts of the firm to prevent unionization.

The expected benefit of joining a union is the difference between the expected total benefit associated from joining the union ($W^u + h_i$), and the competitive wage (w). That is,

$$(9) \quad E(B) = [q(W^u + h_i) + (1 - q)w] - w = q(W^u + h_i - w),$$

where q is the probability of receiving the higher union wage and equals N^u / N^c , where N^u is the employment determined by the firm after the union sets the wage and N^c is the employment level at the competitive wage.⁸

Equation (9) can be rewritten as

$$(10) \quad E(B) = q(r + h_i),$$

where $r = W^u - w$. Given that workers have different h_i s, an array of different $E(B)$ values emerges. Arranging these values in descending order and assuming continuity gives an expected benefit $E(B)$ curve that relates negatively to N , the number of workers. This curve shows the expected benefit that different members get by voting for the union. It is a positive function of the rent (r) and the probability of being employed after unionization (q). That is,

$$(11) \quad E(B) = b(N, r, q), \quad b_N < 0; b_r > 0; \text{ and } b_q > 0.$$

Similarly, the cost (fee) of joining the union is f . As equation (6) shows, the fee (f) varies inversely with the number of members in the union and directly with the spread s . An individual worker favors unionization as long as the expected benefits exceeds the costs. That is,

$$(12) \quad E(B) > f \text{ or } q(r+h_i) > f.$$

The number of workers that vote for the union (N^m) is determined by the intersection of the expected benefit $E(B)$ curve and the fee (f) curve (i.e., equations 11 and 6, respectively). Figure 2 illustrates this result while the formal statement is given by the following:

$$(13) \quad b(N, r, q) - f(N, s) = 0.$$

Stability of equilibrium requires that $b_N < f_N < 0$. Workers having diversified attitudes towards the subjective factors make the $E(B)$ curve steeper, while small economies of scale of operating the union makes the f curve flatter. Equation (13) determines the number of workers that vote for the union (N^m) as a positive function of the rent (r) and the probability (q), and a negative function of the spread of workers (s). That is,

$$(14) \quad N^m = N(r, q, s)$$

$$dN^m/dr = -b_r/(b_n - f_n) > 0; \quad dN^m/dq = -b_q/(b_n - f_n) > 0; \quad \text{and} \quad dN^m/ds = -f_s/(b_n - f_n) < 0;$$

where $(b_n - f_n) < 0$ for stability. Figure 1b illustrates the positive relationship between N^m and r , which comes from Figure 2. That is, a rise in r from r_0 to r_1 in Figure 2 shifts the b curve upward, causing the equilibrium N^m to rise from N^m_o to N^m_1 .

3. Union Density and Union Size: Determinants and Recent Trends

The formation, continuation, or dissolution of a union proceeds as follows:

1. Given employer opposition to unions on ideological grounds, and given the profitability, the production technology, and the structure of the labor and product markets (i.e., a , P^f , s , s_w , e^d_{yp} , e^s_{yp} , e_{nw}), the union sets a wage ($\bar{W}^u$) such that $BP=1$. This reservation wage is the maximum that the firm accepts (See Figure 1a).

2. When the expected rent ($\bar{r} = \bar{W}^u - w$) and the employment and probability corresponding to this wage (N^u and q) are known to workers, they vote on the formation, continuation, or dissolution of the union. Workers having a positive net expected benefit vote in favor of the union. Let that number be $\bar{N}^m$ (see Figure 1b).

3(a). When no union exists, and the workers are voting to form one, then a union emerges if the majority of the workers vote for the union (i.e., if $\bar{N}^m > 1/2N^c$, where N^c is the number of

workers employed at the competitive wage). If for rent $\bar{r}$, $\bar{N}^m < 1/2N^c$, then the union does not emerge.

(b). When an union already exists, it continues if the majority vote for it (i.e., if $N^m > N^u - N^m$, where $N^u - N^m$ are workers who do not vote for the union). The union dissolves if $N^m < N^u - N^m$.

4. If workers vote to form an union (or keep an existing one), then at wage $\bar{W}^u$, the firm determines the employment N^u from its labor demand curve. If $N^m < N^u$, then the firm employs $N^u - N^m$ nonunion workers at the union wage. With our assumption that other subjective factors have a positive effect on workers who vote for the union, they choose to stay in the union as the expected net benefit they derive is greater than zero. If these subjective factors add no additional benefit so that a higher wage is the only benefit the workers get from unionization, then the free-rider problem arises. If $N^m > N^u$, all workers employed by the firm are union members.

The model shows that given the employer's subjective political belief, and given the profitability, the production technology, and the characteristics of the product and labor markets, the union sets the wage ($\bar{W}^u$) equal to the reservation wage of the firm. This wage also helps to determine how the workers vote for a union. For some firms, profitability, production, and market characteristics result in a wage rate $\bar{W}^u$ so that the number of workers voting in favor of the union represents a majority (i.e., $N^m > 1/2N^c$ or $N^m > N^u - N^m$). These firms constitute the traditionally unionized sectors. On the other hand, firms (given their production and market characteristics) that have wage rate $\bar{W}^u$ set at a level such that the number of workers voting in favor of the union are in the minority (i.e., $N^m < 1/2N^c$ or $N^m < N^u - N^m$) constitute the traditionally non-unionized sectors.

Specifically, $\bar{W}^u$ and the likelihood of unionization decrease in firms that operate under competitive product markets, have relatively larger elasticities of labor demand and substitution, and are more labor using. For example, these firms have a smaller bargaining power, and in Figure 3a, are represented by the lower BP curve, BP_s . BP_h represents those firms with higher bargaining power. For the same subjective factors (represented by the N^{m0} curve in Figure 3b) and initial competitive employment N_c , a majority of workers (N^{m0}_h) vote for a union in firms that have higher bargaining power (BP_h), but not in firms (N^{m0}_s) with smaller bargaining power (BP_s).

Moore and Newman (1988) identify four major hypotheses that explain the downturn of unionization ratios: *the structural, the management-opposition, the changes-in-public-policy, and the union-organization hypotheses*. Other empirical studies identify business cycles and saturation effects to explain the variations in union densities (Chaison and Rose, 1991; Dickens and Leonard, 1985; Farber, 1987; Freeman, 1988; Stepina and Fiorito, 1986; Reder, 1988). Using our model, we discuss how these factors affect union densities and membership.

The structural-shift hypothesis states that over time production and employment shift from the traditionally unionized sectors to traditionally non-unionized sectors, causing a decline in overall union density.⁹ In our model, such structural shifts imply that employment increases in firms that are traditionally non-unionized, which have BP curve that are to the left of those firms that are traditionally unionized. The effect of this is shown in Figure 3, where a lower BP_s curve gives a lower rent r_s and smaller union membership N_s .

Empirical studies, however, suggest that structural shifts can explain at most a quarter of the total decline in union densities (Farber, 1985; Freeman, 1985; Doyle, 1985). Furthermore, as Chaison and Rose (1991, p. 14) note, the structural-shift hypothesis cannot explain the decline in unions and membership in all private sector industry categories in the United States. They assert that most of the decline is explained by public policy and employer opposition to unions. In our model, increases in employer opposition to unions lowers α and this shifts the BP_h curve to BP_s in Figure 3a. This results in lower rent and a smaller number of union members, as seen in Figure 3b by the movement from N^{m0}_h to N^{m0}_s . Unfavorable public policy affects η negatively [i.e., the b curves shift down (not shown) in Figure 2], shifting the N^{m0} curve to N^{m1} (as shown in Figure 3b). This decreases union membership among union firms and the probability of forming new unions among nonunion firms for a given $\bar{r}$. In each case, a decrease in N^m in marginal cases may lead to $N^m < N^u - N^m$, resulting in dissolution of an existing union.

Freeman (1985) asserts that reduced union organizing efforts, another important factor, leads to a decline in union membership, possibly explaining 20 percent of this decline. A decrease in union organizing efforts also decreases h_i causing a decline in union membership and density.

The business cycle explanation of union density argues that union membership increases during economic expansions and decreases during contractions. In our model, business cycles change the profitability of the firm (i.e., Π^c) and this shifts the BP curve accordingly. This is seen in Figure 3 where a recession decreases the profitability of the firm, shifting the BP curve to the left (from BP_h to BP_s). This leads to a lower rent ($\bar{r}_s$) and a corresponding decline in union membership from N_h^m to N_s^m .

Ashenfelter and Pencavel (1969) find that "as membership increases there is a diminishing response to the recruiting efforts of unions" (pp. 447-48). This saturation effect is also found in other studies (Stepina and Fiorito, 1986). Our model gives a simple explanation of the saturation effect. When $N^m < 1/2N^c$, the union aggressively campaigns to increase its membership. When $N^m > 1/2N^c$, the membership drive becomes passive.

4. Conclusion

In our model, unions and their membership emerge endogenously. Given the ideological beliefs of employers, the profitability and production technology of firms, and the characteristics of the product and labor markets, the union determines the reservation wage acceptable to the firm. Based on this wage and other subjective factors, workers vote for or against the union. If the union is formed, the firm determines the employment level at the union wage. According to the model, the likelihood of unionization decreases in firms that operate under competitive product markets, have relatively larger elasticities of labor demand and substitution, and are more labor using. These factors explain why certain firms/industries are traditionally unionized, while others are not. Furthermore, the model shows that membership in unions is affected when the subjective factors change. These factors are institutional (e.g., laws and the political party in power), social (such as social acceptability of unions), and personal beliefs (such as altruism and comradeship). Using the model, we provide insight into the recent trends in union densities and membership. Empirical studies show that business cycles, structural shifts, employer opposition to unions, reduced union organizing efforts, socio-political factors, and saturation affect union membership and densities.

The model demonstrates how these factors affect union densities and membership in the United States.

Appendix: Profit Function and Comparative Statics

The profit function of the firm is given by

$$(A1) \quad P(P, W, R) = P(W, R) \cdot Y^s(P, W, R) - W \cdot N(P, W, R) - R \cdot K(P, W, R),$$

where R is the exogenously given user price of capital. The following dual relationships are used in the derivation (see Chambers 1988, pp.131-34):

$$(A2) \quad -dY(W, P)/dW = dN(W, P)/dP = [dN(W, Y^*)/dY][dY(W, P)/dP],$$

$$(A3) \quad dN(W, P)/dW = [dN(W, Y^*)/dW] + [dN(W, Y^*)/dY][dY(W, P)/dW], \text{ and}$$

$$(A4) \quad dK(W, P)/dW = [dK(W, Y^*)/dW] + [dK(W, Y^*)/dY][dY(W, P)/dW],$$

where $Y^*=Y(P, W)$ is the least cost way to produce an output level. Note that the variables also depend on R , the user price of capital, but are not shown explicitly because it is held constant.

From equations (A2)-(A4), we get the following elasticity relationships (Chambers 1988, pp.135):

$$(A5) \quad e_{nw}(W, P) = e_{nw}(W, Y^*) + (e_{yw}e_{np}/e_{yo}^s), \text{ and}$$

$$(A6) \quad e_{kw}(W, P) = e_{kw}(W, Y^*) + (e_{yw}e_{kp})/e_{yp}^s.$$

where $e_{nw} = d \ln i / d \ln j$ is the elasticity of variable i with respect to variable j . A superscript s on e refers to the supply function.

The *two-factor-one-price* elasticity of substitution (TOES) (see Mundlak, 1968; Chambers, 1988) is given as follows:

$$(A7) \quad \sigma = (\tilde{K} - \tilde{L}) / \tilde{W},$$

where the $\tilde{K} = dK / K$ represents the rate of change. But, given that $e_{nw} = \tilde{N} / \tilde{W}$ and $e_{kw} = \tilde{K} / \tilde{W}$, TOES can be written as,

$$(A8) \quad s = e_{kw}(W, P) - e_{nw}(W, P).$$

Furthermore, from the product market equilibrium [$Y^d(P, M) = Y^s(P, W)$], we get the following:

$$(A9) \quad dP(W, M)/dW = -[PY_w^s]/[Y(e_{yp}^s + e_{yp}^d)] > 0,$$

where $Y_w^s = dY^s/dW$, and $e_{yp}^d = d \ln Y^d / d \ln P$ is the price elasticity of demand for the product sold by the firm. e_{yp}^s is the corresponding price elasticity of supply.

Total differentiation of equation (A1) with respect to W gives the following:

$$(A10) \quad dP = [Y(dP/dW) + P[dY^s/dW] + (dY^s/dP)(dP/dW)]dW \\ - [N + W[dN/dW + dN/dP](dP/dW)] + R[dK/dW + (dK/dP)(dP/dW)]dW$$

Assuming a homothetic production function, using equations (A1-A9), and examining the two coefficients of dW in equation (A10) independently produces the following results:

$$(i) \quad Y(dP/dW) + P[dY^s/dW] + (dY^s/dP)(dP/dW) = [PY_w^s(e_{yp}^d - 1)] / [e_{yp}^s + e_{yp}^d],$$

$$(ii) \quad W[dN/dW + dN/dP](dP/dW) + R[dK/dW + (dK/dP)(dP/dW)] = \\ [N/s_w[e_{wn}(W, P)e_{yp}^d + e_{yw}e_{np}] + RKs e_{yp}^d / W] / (e_{yp}^s + e_{yp}^d),$$

where $Y_w^s = dY^s/dW < 0$, $e_{yp}^d = d \ln Y^d / d \ln P > 0$, $e_{yp}^s = d \ln Y^s / d \ln P > 0$, $e_{nw} = d \ln N / d \ln W < 0$, $s_w = WN/TC > 0$, $TC = WN + RK$, and $s = (\tilde{K} - \tilde{L}) / \tilde{W} > 0$.

Using the values from equations (i) and (ii) in equation (A10) and rearranging gives the following:

$$(A11) \quad dP = [[PY_w^s(e_{yp}^d - 1) - N/s_w[e_{yp}^d e_{wn}(W, P) + e_{yw}e_{np}]] / (e_{yp}^s + e_{yp}^d)]dW \\ - [[RKs e_{yp}^d / [W(e_{yp}^s + e_{yp}^d)]] + N]dW.$$

Comparative Statics

Using equation (A11) in the BP equation gives the following comparative static results:

$$(A12) \quad dBP/de_{yp}^d = [a(1+i)tP^f][PY_w^s + P_w - Ne_{wn}/s_w - RKs/W] / [(1+ti)P_w^2(e_{yp}^s + e_{yp}^d)] < 0,$$

$$\text{if } PY_w^s - RKs/W > P_w - Ne_{wn}/s_w;$$

$$(A13) \quad dBP/de_{yp}^s = [a(1+i)tP^f] / [(1+ti)P_w(e_{yp}^s + e_{yp}^d)] > 0;$$

$$(A14) \quad dBP/ds = -[a(1+i)tP^f e_{yp}^d RKs] / [(1+ti)P_w^2 W(e_{yp}^s + e_{yp}^d)] < 0;$$

$$(A15) \quad dBP/ds_w = [a(1+i)tP^f N(e_{wn}e_{yp}^d + e_{yw}e_{np})] / [(1+ti)P_w^2 s_w(e_{yp}^s + e_{yp}^d)] < 0;$$

$$(A16) \quad dBP/de_{wn} = [a(1+i)tP^f Ne_{yp}^d] / [(1+ti)P_w^2 s_w(e_{yp}^s + e_{yp}^d)] < 0;$$

$$(A17) \quad dBP/dP^f = a(1+i)t / (1+ti) P_{ww} > 0;$$

$$(A18) \quad dBP/da = (1+i)tP^f / (1+ti) P_w > 0;$$

$$(A19) \quad dBP/dr = -a(1+i)tP^f / (1+ti) P_w < 0; \text{ and}$$

$$(A20) \quad dBP/dt = a(1+i)P^f / (1+ti)^2 P_w > 0.$$

NOTES

1. No agreement exists regarding the appropriate objective function for a union and, as such, no clear idea exists as to what unions maximize. For example, Akerlof (1969), Rees (1977), Mulvey (1978), and Penvacel (1984) assume that unions maximize a quasi-concave utility function with wages and employment as arguments, while Dreze and Modigliani (1981), Oswald (1982), and Sampson (1983) assume that unions maximize an expected utility function. On the other hand, Heiser (1970) and Johnston (1972) postulate that unions maximize the wage bill, while Rosen (1970) and de Menil (1971) suggest that unions maximize their rents.
2. Gray, Kandil, and Spencer (1992, pp. 1043) note that certain industries such as manufacturing, mining, construction, transportation, and communication constitute the *contract* sector having union densities of 44 to 92 percent. The *non-contract* sector, on the other hand, contains wholesale trade, retail trade, FIRE, agriculture, forestry, and fisheries, with union densities ranging from 1 to 14 percent.
3. Booth (1995, Ch. 5) notes that the right-to-manage model is inefficient in that the union, the firm, or both can be made better off by bargaining over wages and employment. Observed behavior, however, accords well with the right-to-manage model where the firm sets employment and only bargains with the union over wages. Booth (1995, Ch. 5) suggests that one or more of the assumptions of the efficient bargaining model must not hold in practice (e.g., certainty about future product demand).
4. As in voting models, we assume that the objectives of the workers and the union differ. The union has the objective of increasing the welfare of its members, has altruistic preferences, and is cooperative in nature. Workers, as well as union officials, have some elements of altruism and comradeship, but on the other hand, they are individualistic with actions driven, first and foremost, by self-interest.
5. Hayes (1984) shows that strikes arise due to asymmetric information. Under perfect information as in our model, however, no strike occurs, in fact. Models of strike length include Hicks (1963)

and Hart (1989). Other costs of a strike such as permanently losing clients are not considered here.

6. If the work area is concentrated spatially (e.g., under one roof), it is less costly in terms of time and money to meet and organize workers. This cost increases with the spread of the work area and workers.

7. The assumption of risk neutrality keeps the analysis simple. Risk aversion does not change the basic results, but introduces unnecessary complications.

8. The probability q assumes that voting ex ante for the union conveys no perceived special place in the job queue ex post should the union win the election. That is, the probability of employment if the union wins the election is just the ratio of the employment with the union to the employment without the union. In addition, if seniority is introduced, and if senior employees are less likely to be laid off after unionization, then the probability q becomes an increasing function of seniority. This does not change the qualitative conclusions of the model.

9. In recent decades, the industrialized countries have experienced structural changes that have made the share of the traditionally unionized industrial sector's share in the total GDP smaller, while that of the traditionally non-unionized service sector larger.

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