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The Location of the Poor in a Metropolitan Area: Positive and Normative Analysis

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Abstract

We seek to explain the stylized fact that poor households form the majority in the inner city of most American metropolitan areas. Using numerical simulations, we show that (1) typically there exist two equilibria: one in which the poor form the majority in the inner city and the other in which the rich form the majority; (2) when the metropolitan population is small, rich households 'jump to the suburb' to obtain their desired public service, and this causes the growth path to select the equilibrium in which poor households are the inner city's majority; (3) whether having the poor form the inner city's majority is desirable depends on the metropolitan population and on whether efficiency or equity is the normative criterion.

Journal of Economic Literature Classification: H73, R12, R14

Keywords: urban, equilibria, poor, welfare

1. INTRODUCTION

In Tiebout's (1956) seminal model of fiscal competition, the purpose of a community is to provide public services to its residents. A community's boundary is flexible. A household shops over communities, choosing the community which provides his preferred public service level. If the public service is a normal good, households with different incomes demand different public service levels. In consequence, households with different incomes chose different communities, or all households within each community have the same income (McGuire (1974), Berglas (1976) and Wooders (1978)). There is thus sorting by income between communities. Elickson (1971), Yinger (1982), Epple, Filimon and Romer (1984, 1993) and Epple and Romer (1991) extend the model replacing the flexible boundaries by fixed boundaries. Public service differences are capitalized into land prices, but there continues to be sorting by income between communities.¹

Tiebout's model is non-spatial. Communities are viewed as pieces of land in a featureless plain so that there is no a-priori reason as to which community or which piece of land is inhabited by the poor households. If there are two communities, an inner city and a suburb, then there are two equilibria: one where the inner city contains the poor households, and another where the suburb contains the poor households. Tiebout's model therefore provides no prediction as to whether the poor households congregate in the inner city or the suburb. In contrast to this theoretical indeterminacy, Glaeser et al (2000) describe "the well-documented fact that within U.S. metropolitan areas, the poor generally live in the central cities and middle-income individuals generally live in suburbs."

In contrast to Tiebout's model, Alonso (1964) Mills (1967) and Muth (1969) downplay

the fiscal difference between communities and instead stress spatial aspects. They consider a metropolitan area to be an “as if” single monocentric city to whose center households commute. Land prices decrease as the location moves away from the center, reflecting the increase in the commuting cost. Income sorting still occurs, but it now arises from the interaction of commuting costs and land demand. If land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city’s center is greater for the rich households and compensates them for the associated increase in commuting cost.² In this case rich households live on the outside of the metropolitan area and poor households live in the inner city. Conversely, if land demand is unresponsive to income changes and commuting costs increase with income, the greater commuting benefit to rich households of living closer to center causes them to outbid poor households for the locations closer to the city’s center. Summarizing, whether rich households sort themselves on the outside or on the inside of the monocentric city depends on whether the income elasticity of land demand is greater or smaller than the income elasticity of the cost of commuting.

Wheaton (1977) determines that the income elasticity of land demand is statistically indistinguishable from the income elasticity of the cost of commuting, so that the Alonso-Mills-Muth model is unable to predict whether it is the poor or the rich who live in the inner city. Glaeser, Kahn and Rappaport (2000) find evidence that the income elasticity of land demand is quite small, so that the monocentric city model predicts that it is the rich who live in the inner city. Empirically, therefore, this type of sorting cannot play a major role in explaining the centralization of the poor. Instead, Glaeser et al. suggest that the better access to public transportation - used by the poor to commute to the city’s center - is the primary reason why it is

the poor who congregate in the inner cities.

Although neither the Tiebout model nor the Alonso-Mills-Muth model on their own can explain the concentration of the poor in the inner cities, we find that a model with fiscal differences and commuting costs can explain it. In our model, a circular inner city has an exogenous boundary and is surrounded by a suburb. The public service in each community is determined by voting and all households must commute to the central business district which is located at the center of the inner city. The model has two income-classes. Rich households have a higher demand for the public service so that *ceteris paribus* different income groups prefer to live in different communities. Rich households have higher commuting costs per mile than poor households and land demand is relatively income inelastic. *Ceteris paribus*, therefore, rich households outbid poor households for land nearer the city's center. In the spirit of the indeterminacy of Tiebout's model, we find multiple equilibria over a range of metropolitan populations. In one equilibrium, it is the poor households who form the majority in the inner city, voting low public services in that city; in the second equilibrium, it is the rich households who form the majority in the inner city, voting high public services there.

To determine which equilibria is likely to be selected, we consider the city's growth to be approximated by the comparative statics of an increase in the metropolitan population in the presence of a fixed inner-city boundary. When the population is small, all households live in the inner city; there is a majority of poor households and the public service is low. As the population increases, city rents increase and the edge of urban development moves towards the inner city's boundary. While there is still some undeveloped land in the inner city, some rich households "jump to the suburb" to form a new community with a high public service. This establishes rich

households as the majority in the suburb. As the metropolitan area's population further expands, the configuration of households is maintained: poor households congregate in the inner city and rich households congregate in the suburb. At very high population sizes, high suburban rents plus commuting costs induce rich households to start to move back to the city - gentrification - and at sufficiently large metropolitan populations the inner city changes again to have a majority of rich households.

What matters in our model is the difference in the public service level desired by poor and rich households. For expositional simplicity, we have considered a single public service which is a normal good; an example is public education. However, the qualitative description is unchanged if the public service is changed to be an inferior good; an example is public transportation. When the metropolitan area has a small population, all households live in the inner city and the poor households, forming the majority, vote high public transportation expenditures. As the metropolitan population expands, rich households "jump to the suburb" to escape the high taxes associated with costly public transportation. Further growth leaves this configuration in place - poor households in the inner city with high access to public transportation and rich households in the suburb with low access to public transportation. Glaeser et al., building on the model of LeRoy and Sonstelie (1983), offer evidence that improvements in mass transit attracts the poor into the inner city but this paper suggests that the causality could run in both directions: the siting of the public transportation infrastructure in the inner city also arises as a result of the growth path of the city.

The positive implication of our model is that the poor are concentrated in the inner cities of American metropolitan areas. What are the normative implications of this prediction? In

particular: does the selected equilibrium match the preferred equilibrium? We find that the answer to this question depends on the normative criterion - efficiency or equity - and on the metropolitan population. In metropolitan areas with small populations, the equilibrium in which poor households form the inner city's majority enables poor households to obtain their desired service level and low commuting costs: this implies that the equilibrium is favored for equity. However, it is inefficient because of the high commuting costs paid by rich households living in the suburb. In metropolitan areas with large populations, the ranking is reversed. The equilibrium in which poor households form the inner city's majority is inequitable because it involves the poor paying high rents. However it is efficient because it gives better matching of rich households with their desired public service level.

Because we are comparing different equilibria, it is difficult to use a calculus-based methodology. We therefore use a computable general equilibrium model. We also use a very simple utility function so that the intuition is highlighted. The paper is structured as follows: Section 2 presents the theoretical model and the possible equilibria. Section 3 presents the simulation structure. Section 4 discusses the equilibrium which is selected by the growth path. Section 5 is the normative analysis. Section 6 concludes.

2. THE MODEL

A household lives in a community and obtains utility U from consuming a privately-provided good c and from a public service g provided by the community: $U(c, g)$. The privately-provided good is the numeraire good. The household's demand for lot size is assumed to be exogenous³ and the non-land components of housing are included as part of the private good: therefore housing *per se* does not enter the utility function.

For ease of presentation, we consider a utility function in which the level of the public service desired by a household does not depend on his housing expenditure. This highlights the intuition and makes the welfare comparisons particularly straightforward. In particular, we restrict attention to a utility function of consumer surplus form:

$$U = c + \beta V(g).$$

Because we want the public service to appear normal or to be more valued by households of higher income, we make β to be a function of endowed income M :

$$\beta \equiv \beta(M) ; \quad \beta'(M) > 0 .$$

In this description, households differ in the tastes for the public service, and their tastes vary systematically with endowed income.

All households commute to the central business district which is located at the metropolitan center and, for analytical convenience, is assumed to have no area. A household has a fixed time endowment which he can use either for working or for commuting. If he lives at the metropolitan center, he spends no time commuting and his income is M . If he lives at distance d

from the metropolitan center, his income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to d and the opportunity cost of a unit of his time is proportional to M , so that his commuting cost is tMd . The household's exogenous demand for land area is a and the price of a unit of land at d is $r(d)$. The community provides the public service g . The public service shows constant returns to community size, and the cost of providing a unit of the public service to each resident is one unit of numeraire per resident⁴; the public service is financed by a residency tax g . Therefore the consumption of the private good by the household if he locates distance d from the city center is

$$c = M - tMd - ar(d) - g ;$$

the utility of the household is

$$M - tMd - ar(d) - g + \beta(M)V(g).$$

At equilibrium, a household of income M achieves utility $W(M)$. Denote the bid of a household of income M for a location at radius d from the metropolitan center and in a community providing public service g as $R(d, M; g)$:

$$M - tMd - aR(d, M; g) - g + \beta(M)V(g) = W(M).$$

Hence within the community:

$$tMd + aR(d, M; g) = \text{constant}.$$

The bid decreases as distance from the metropolitan center increases, reflecting the increase in the commuting cost. Differentiate to find the slope of the household's bid-rent curve within the

community:

$$\frac{\partial R(d, M; g)}{\partial d} = - \frac{tM}{a} . \quad (1)$$

The willingness-to-pay of the household to move marginally closer to the metropolitan center is the benefit of the decreased commuting cost per unit of land area purchased.

There are two income classes. Poor households have income M_1 and rich households have income M_2 : $M_1 < M_2$. Associated with each income level, M_1 and M_2 , is a lot size a_1 and a_2 respectively: as noted earlier, a_1 and a_2 are exogenous and $a_1 < a_2$. We assume

$$\frac{tM_2}{a_2} > \frac{tM_1}{a_1} .$$

Using Equation (1), this implies that the bid-rent curve of a rich household is steeper than the bid-rent curve of a poor household⁵ or, within a community, rich households outbid the poor households for the locations closer to the metropolitan center. In consequence, there is income sorting within each community, with rich households living on the inside (closer to the metropolitan center) and poor households living on the outside (further from the metropolitan center).⁶

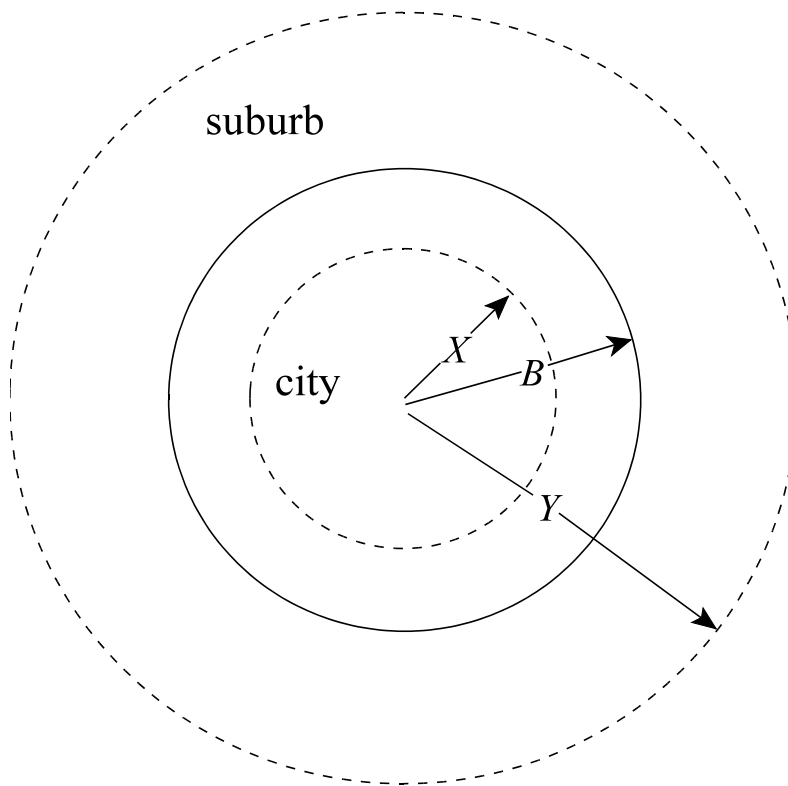


Figure 1: the metropolitan area

The metropolitan area is comprised of a circular inner city (henceforth termed “the city”) surrounded by a suburb. The jurisdictional boundary between the city and the suburb has radius B . There is the possibility of undeveloped land at the fringe of the city if neither poor nor rich households are willing to pay more than the reservation rent to live in the city near the jurisdictional boundary. The limit of development in the city has radius X . If

$X < B$: there is undeveloped land at the fringe of the city;

$X = B$: there is no undeveloped land in the city.

Remembering that rich households live on the inside of the city and poor households live on the

outside, denote the boundary between rich and poor households in the city as occurring at distance x from the center. If

$x = 0$: only poor households live in the city;

$0 < x < X$: rich and poor households live in the city;

$x = X$: only rich households live in the city.

In the suburb, the limit of development is distance Y from the city center and we assume that the outer jurisdictional boundary is sufficiently distant from the metropolitan center that all households live in the city or in the suburb. The boundary between the rich and poor households in the suburb occurs at distance y from the center. If

$y = B$: only poor households live in the suburb;

$B < y < Y$: rich and poor households live in the suburb;

$y = Y$: only rich households live in the suburb.

There are N households of which a fraction θ are poor. Equating land demand and land supply requires:

$$\pi X^2 + \pi (Y^2 - B^2) = \theta N a_1 + (1 - \theta) N a_2 ; \quad (2)$$

$$\pi x^2 + \pi (y^2 - B^2) = (1 - \theta) N a_2 . \quad (3)$$

The model is now summarized descriptively; the formal algebraic formulation is presented in the Appendix.

1. Rent continuity: rent is continuous in a community. If it were discontinuous, a

- household living on the side of relatively high rent could increase his utility by moving across the discontinuity to the side of low rent: his rent would change discontinuously but his commuting cost would change only marginally.⁷
2. No migration: no household can achieve higher utility by moving to another location. This implies that, if an income class resides in both communities, the rents are such that a household in that income class is indifferent between the communities. If an income class does not reside in a community, rents are such that a household in that income class cannot increase his utility by moving into the community.
 3. Reservation land price: the reservation price of land is r_0 . If a community contains no undeveloped land, the rent at the limit of development is at least r_0 . If a community contains undeveloped land, the rent at the limit of development is r_0 .
 4. Determination of the public service level. The public service level in each community is determined by majority voting; households vote myopically, taking the rent schedules as given.⁸
 5. Model closure. We assume that rent is paid to absentee landlords.⁹
 6. The population in each community is considered to be a continuous variable.

There are many possible equilibrium configurations corresponding to which income class forms the majority in the city and whether the city includes one or both income classes, which income class forms the majority in the suburb and whether the suburb includes one or both income classes, and whether there is undeveloped land in the city.¹⁰ In our simulations, we have set the numbers of poor and rich households to be equal (technically there is one additional poor household to ensure that the metropolitan area has an overall majority of poor households). With

a sufficiently small metropolitan population, there may be only one equilibria in which all households live in the city: the potential cost of commuting from the suburb deters households from living in the suburb. At a larger metropolitan population, both jurisdictions are occupied. We now describe these latter equilibria. With the assumed equal numbers of poor and rich households, there are two configurations: Case 1 in which the city has a majority of poor households and the suburb has a majority of rich households, and Case 2 in which the majorities are reversed. We restrict attention to the equilibria found by our simulations, and consider Case 1 first.

Case 1: poor households are majority in city, rich households are majority in suburb.

Our simulations found three equilibrium configurations in which the city has a majority of poor households and the suburb has a majority of rich households. These majorities ensure that the public service is low in the city and high in the suburb. Typical rent profiles of the configurations are shown in Figure 2. Equation (1) shows that, in a region occupied by poor (rich) households, the rent rises at rate tM_1/a_1 (tM_2/a_2) as the location moves closer to the metropolitan center. Therefore, in interpreting this and later figures, we can use the slope of the rent schedule to infer the income of a household living at a location: poor (rich) households live in the region where the rent schedule is relatively flat (steep).

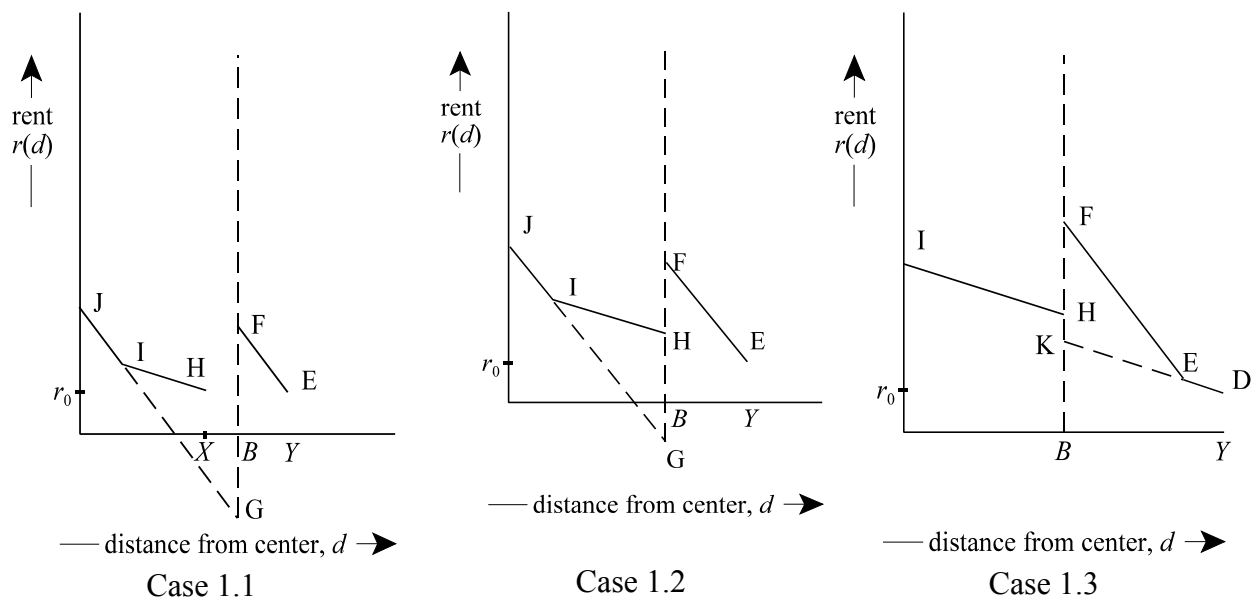


Figure 2: rent schedules for Case 1.

We refer to Figure 2. As the metropolitan population expands, the form of the equilibrium configuration shifts from Case 1.1 to Case 1.2 to Case 1.3. Considering Case 1.1: the metropolitan population is small and there is undeveloped land in the city. All poor households live in the city. Poor households do not live in the suburb: if a household at the edge of development in the city moved to the edge of development in the suburb, he would pay the same rent, get a less desirable public service and incur a higher commuting cost. Rich households are found in both jurisdictions: rents are such that a rich household is indifferent between living in the city (low commuting cost) and living in the suburb (high public service).

The rent at the limit of suburban development is r_0 : this rent anchors the suburban rent schedule. Continuing with Case 1.1: in the suburb, the rent schedule is EF and, as the location moves inward, the rent rises at rate tM_2/a_2 reflecting the commuting advantage to the rich household of locating closer to the city center. As the location moves across the jurisdictional

boundary, the public service changes from the level voted by rich households to the level voted by poor households. This causes the bid of a rich household to fall by FG (and become negative in this example). GIJ is the bid-rent curve of a rich household in the city. In the city, the actual rent schedule is anchored by the reservation rent at the limit of development: in the area of the city occupied by poor households, the actual rent schedule is HI . As the location moves inward from the jurisdictional boundary, the actual rent is increasing at rate tM_1/a_1 but the willingness to pay of rich households is increasing at the faster rate tM_2/a_2 ; the distance between HI and GI decreases. At distance x from the city center, the willingness to pay of a rich household to live at x equals the actual rent, and rich households live at the locations closer to the city center.

As the metropolitan population increases, some of the additional rich households locate in the suburb, shifting the limit of development further out and increasing the rent paid at the suburban side of the jurisdictional boundary. The size of the fall FG in the bid of rich households as the location moves across the jurisdictional boundary reflects the change in the consumer surplus of a rich household from the public service and is a constant, independent of the population size. Therefore the additional rich households divide themselves between the city and the suburb so that the bid drops by the constant distance FG . The limit of development in the city moves outward. At a critical population size, there is no undeveloped city land.

If the metropolitan population is further increased, there is no undeveloped land and Case 1.2 applies. The suburban rent schedule continues to be anchored by the reservation rent r_0 at the limit of suburban development. However, the city's rent schedule is no longer anchored by the reservation rent r_0 at the limit of development in the city. Instead, because rich households live in both jurisdictions, the city's rent schedule is anchored by the required drop FG in the bid of rich

households across the jurisdictional boundary. Rents paid by poor households in the city are still sufficiently low that poor households do not wish to migrate into the suburb.

As the metropolitan population increases further, the city fills with poor households, driving rich households out of the city into the suburb. At a critical population, the city is filled with all the poor households; if the metropolitan population increases further, the additional rich and poor households locate in the suburb, and Case 1.3 applies. Poor households live in both locations and the level of the city's rent schedule adjusts to ensure that poor households achieve the same utility in the city as in the suburb: DEK is the bid-rent curve of poor households in the suburb and KH is the rent premium a poor household is willing to pay to acquire its desired public service.

Case 2: rich households are majority in city, poor households are majority in suburb.

Case 2 considers the equilibria in which rich households form the city's majority. If the metropolitan population is small, all rich households live in the city and all poor households live in the suburb. High city taxes (to finance the high public service voted in the city by the rich) deter poor households from wanting to live in the city. This case is denoted Case 2.1 and a typical rent schedule is shown on the left-side of Figure 3. Both the city and the suburban rent schedules are anchored by the rent r_0 paid at the limit of development.

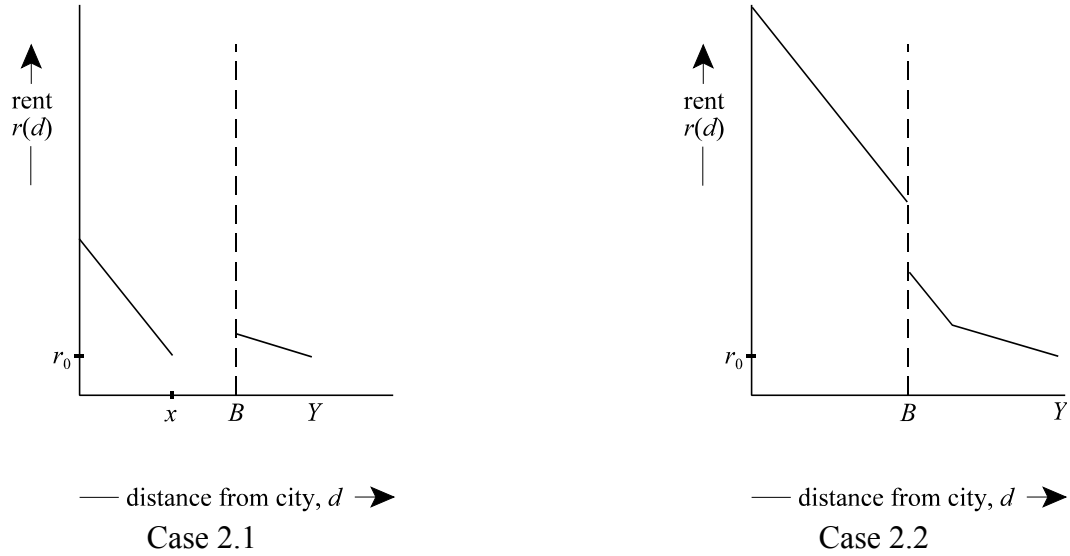


Figure 3: rent schedules for Case 2.

As the metropolitan population increases, the city fills up with rich households and at a critical population some rich households are pushed into the suburb. This corresponds to Case 2.2. Rich households live in both jurisdictions. The suburban rent schedule continues to be anchored by the reservation rent paid at the limit of development, and the rent increases as the location moves across the jurisdictional boundary from the suburb into the city, to reflect the premium a rich household is willing to pay to receive its desired public service.

3. SIMULATIONS

3.1 Analytical Framework

The utility function of a household of income M_i achieving consumption c and receiving public service g is specified as

$$c + \text{sign}(\rho) A (M_i)^\delta g^\rho$$

where A , δ and ρ are parameters, $\delta > 0$, $|\rho| < 1$ and $\rho \neq 0$.¹¹ The household's demand for g is

$$g_i^d = (|\rho| A)^{\frac{1}{1-\rho}} (M_i)^{\frac{\delta}{1-\rho}}.$$

As noted earlier, we represent the income distribution by assuming that there are equal numbers of poor and rich households (technically there is one additional poor household so that the metropolitan population has an overall majority of poor households). We choose the income of households in each group to be (close to) the median income of the bottom- and top- half of the U.S. income distribution in 1990. Table 1 shows the assumed values of the model parameters and, for comparison, the observed values for U.S. metropolitan areas.

Parameter	Variable Name	Model Value	Societal Value
Poor households as fraction of population	θ	0.5	0.5
Income of poor household (\$ per year) ^a	M_1	15,000	16,523
Income of rich household (\$ per year) ^a	M_2	45,000	46,725
Average lot size (acres) ^b		0.3333	0.3402
Lot size of poor household (acres) ^c	a_1	0.2833	
Lot size of rich household (acres) ^c	a_2	0.3833	
Commute time per mile as fraction of work day ^d	t	0.013	
Public service demand parameter ^c	δ	1.4	
Public service demand parameter ^c	ρ	-1.0	
Public service demand parameter ^c	A	3.0	
Reservation rent ^c	r_0	0	

a. Societal figures are total 1990 money income at 25th and 75th percentile in metropolitan income distribution (*Money Income of Households, Families and Persons in the United States 1990*, Table 2) multiplied by $(1-\tau)$ where τ is average federal income tax rate for federal tax return with adjusted gross income equal to money income of respective household (*Individual Income Tax Returns 1990*). Model value represents earned income of a household with zero commuting costs.

b. Societal average lot size is metropolitan land area divided by societal metropolitan population. Metropolitan land area calculated as 1991 occupied housing units in all central cities multiplied by median lot size in central city plus same 1991 figure for suburban units (*American Housing Survey for the United States 1991*, Tables 8-3 and 9-3). Societal metropolitan population is calculated as 1991 number of owner-occupied units in all central cities plus suburbs (*American Housing Survey for the United States 1991*, Tables 8-3 and 9-3).

c. See text

d. Fraction of 8-hour workday spent commuting to metropolitan center if household lives one mile from city center. Figure is based on an average travel speed of 20 miles per hour and a round trip commute.

Table 1: Parameter and Population Values

The population and average lot size are chosen to be close to the values observed in the population. The lot size for poor and rich households is adjusted down and up from the average lot size, respectively, in order to be consistent with a 0.3 income elasticity of demand for land - an elasticity value which is consistent with the recent estimates by Glaeser et al. (2000).¹² The population and housing demand parameters imply that the area of developed land in the metropolitan area is 90,000 acres: this translates to a radius (Y) of 6.7 miles if the city has no vacant land.

The values for ρ and δ are chosen so that the price and income elasticities of demand for the public service are -0.5 and 0.7, respectively, which is consistent with current estimates in the literature (see Ross and Yinger (1999)). These parameter values (and $A=3$) imply that poor households vote a public service level of 1452 (\$ per year) and rich households vote a public service level of 3132 (\$ per year). The "average" calculated value for the share of income spent on public service is therefore $(1452+3132)/(1500 + 4500) = .076$ ¹³; in contrast, the societal value is 0.13.¹⁴ When we raised the value of the parameter A so that the simulated value lay closer to the observed value, there was little effect on the results except that we ceased to find an equilibrium configuration of the form Case 1.2.¹⁵ In order to illustrate an equilibrium with this form, we decided to use the lower value of A .

Our focus is on comparing equilibria and what is important is the relative value of rents. Therefore, for convenience, we set the reservation rent to zero.

4. SUBURBANIZATION IN A GROWING CITY

Current metropolitan areas have grown out of much smaller cities. Does the likely growth path select an equilibrium with a majority of poor or rich households in the city? Our presumption is that marginal increases in the population are accommodated by changes in the boundary between the income groups and not by large population shifts between the communities. Put differently, once an equilibrium configuration of majorities is established, we presume that it is maintained as the population grows (provided the configuration continues to be an equilibrium).

Historically, when metropolitan populations were small compared to the size of the city, all households lived in the city and poor households formed the majority. As the metropolitan population grew, our model predicts that rents increased in the city, the limit of urban development moved outwards, and the poor households who resided at the edge of development were pushed further from the city center. Eventually such growth leads to the development of a suburb. There are two possibilities:

1. Rich households are the first households to move into the suburb. By setting up a new community in the suburb, they obtain their desired public service and face lower rents, but they incur higher commuting costs. If they are sufficiently sensitive to the public service level, they will choose to do this by “jumping over” undeveloped city land¹⁶. The equilibrium corresponds to Case 1.1 in our simulations. In this scenario, rich households establish themselves in the suburb, leaving poor households in control of the city.
2. Poor households are the first households to move into the suburb. Poor households

have no incentive to set up a suburban community while there is still undeveloped city land - if the household at the edge of the city were to move, he would get the same public service, pay the same rent and incur higher commuting costs. Therefore in this scenario poor households enter the suburb only when the limit of development moves across the jurisdictional boundary; at this instant poor households who reside at the edge of urban development spill into the suburb. With poor households leaving the city, rich households become the city majority.¹⁷

Given our presumption that the configuration of majorities does not change spontaneously (provided it continues to be an equilibrium), poor households control the city if Case 1.1 arises as an equilibrium while the metropolitan population is too small to completely fill the city.

We approximate the growth path to be the path traced out by the comparative statics of increasing the metropolitan population N in the presence of a fixed city boundary $B=5.0$ miles;¹⁸ other parameter values are maintained at the values shown in Table 1. The relevant equilibrium configurations are shown in Figure 4. In Figure 4, the central axis shows the metropolitan population N . The equilibrium configuration (at each N) in which poor households are the city's majority is shown above the central axis, and the equilibrium configuration in which rich households are the city's majority is shown below the central axis. For example: for metropolitan populations between 22,491 and 281,368 households, the equilibrium in which poor households are the city's majority has the form of Case 1.1.

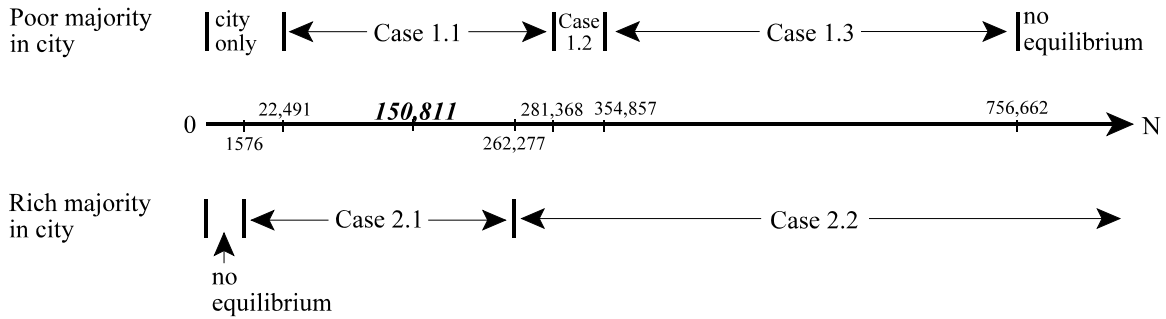


Figure 4: equilibrium configurations

With a boundary at 5.0 miles, the city can physically accommodate all households until the population exceeds 150,811. At metropolitan populations less than 1,576 households, there is only one equilibrium in which all households live in the city, the city's majority is poor and the suburb is uninhabited - households are deterred from living in the suburb by the high cost of commuting. At a population of 1,576 households, there are two potential equilibrium configuration - the configuration in which all households reside in the city and Case 2.1. However the establishment of Case 2.1 requires that poor households migrate to the suburb while there is undeveloped land - a situation in which migrating to the suburb makes a poor household worse-off. We predict therefore that the growing population continues to select the equilibrium in which all households reside in the city.

Above a population of 22,491 households, having only the city inhabited is no longer an equilibrium. There are two possible equilibria: Case 1.1 and Case 2.1. This multiplicity occurs when there is still undeveloped land in the city, and so we predict that Case 1.1 is selected. Put differently, we consider the counter-factual to be the situation in which all households live in the city and the no-migration constraint is not imposed. As the metropolitan population increases, the city's limit of development moves towards the jurisdictional boundary, and the utility change

experienced by a poor and rich household moving to set up a new community in the suburb decreases. These utility changes are shown on Lines 12 and 13a of Table 2 on Page 25. At $N=1,000$ and $N=20,000$ the utility difference is large and negative for poor households (-896 and -620 (\$ per year) respectively¹⁹); for rich households the utility change is large and negative at $N = 1,000$, is very small at $N=20,000$, is zero at $N = 22,491$ (not shown) and is positive thereafter. Therefore, as the metropolitan population expands beyond 22,491, rich households have the incentive to “jump over” the undeveloped land to form a new community in the suburb with the form of Case 1.1. In contrast, the establishment of Case 2.1 requires that some poor households move to the suburb although this makes them worse off. We predict therefore that the growth path selects Case 1.1 as the equilibrium configuration, a situation which establishes rich households in the suburb. As the population further expands, if we rule out the case of a large population shift between the communities, the growth path continues to select the Case 1 equilibrium - poor households form the city's majority and the configuration changes from Case 1.1 to Case 1.2 to Case 1.3.

At population greater than 756,662 households, an equilibrium having a majority of poor households in the city ceases to exist. If an equilibrium with the configuration of Case 1 were to exist, the boundary between rich and poor households in the suburb would be far from the metropolitan center; rich suburban households would have such high rents and commuting costs that they would gain by moving "back" to the city even at the low public service level chosen by the city residents; poor households would be outbid in the city and would move to the suburb. The only equilibrium is Case 2.2. This incentive for rich households to move back to the city as the metropolitan area grows is qualitatively similar to the gentrification that has been observed in

many large U.S. cities over the last couple of decades.

To summarize this section, our simulation suggests that, as the population grows, rich households migrate to form a new community beyond the city's jurisdictional boundary. This in turn suggests that later growth selects the equilibrium in which the poor are the majority in the city. At large populations, Case 1 cannot be supported and rich households move back into the city.

5. WELFARE COMPARISONS

5.1 Positive description of the equilibria

Tables 2 and 3 present the simulation results for the equilibria in which poor and rich households form the city's majority at selected metropolitan population sizes. In each table Line 1 shows the metropolitan population (assigned exogenously). Line 2 shows the equilibrium case (using the labels discussed in Section 2) which arises for the given population. Lines 3 and 4 show the utility achieved by poor and rich households. Lines 5, 6 and 7 show the average commuting cost, the average consumer surplus from the public service and the average rent²⁰. Lines 8, 9, 10 and 11 show the spatial characteristics of the metropolitan area. Lines 12 and 13 show the utility change a poor and rich household would experience if he were to move from the community in which he resides to the other community.

City jurisdictional boundary: $B = 5.0$ (miles)			Equilibrium outcomes							
1. Metropolitan population, (households)	N	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
2. Equilibrium Case Number (label from Section 2)		city only	city only	1.1	1.1	1.2	1.3	1.3	1.3	no equilibrium
3. Utility of poor households (\$ per year)		12,018	11,742	11,488	11,269	10,952	9,607	9,333	9,095	
4. Utility of rich households (\$ per year)		36,584	35,867	35,437	35,031	34,611	34,036	33,391	32,818	
5. Average commuting cost (\$ per year)		95	426	1,445	1,717	1,952	2,303	2,596	2,856	
6. Average consumer surplus from public service (\$/yr)		-5,556	-5,556	- 4,861	- 4,785	- 4,715	- 4,634	- 4,743	- 4,806	
7. Average rent per household (\$ per year)		48	213	232	347	552	1,241	1,298	1,381	
8. Boundary between income groups in city (miles)	x	0.31	1.38	1.65	1.99	1.97	0	0	0	
9. Boundary of city development (miles)	X	0.41	1.82	3.12	4.25	5	5	5	5	
10. Boundary between income groups in suburb (miles)	y			5.64	6.33	7.05	7.95	8.80	9.58	
11. Boundary of suburban development (miles)	Y			5.64	6.33	7.05	8.14	9.55	10.77	
12. Poor household's utility change if he moves from city to suburb (\$ per year)		-896	-620	- 1,392	- 1,309	- 1,132	0	0	0	
13a. Rich household's utility change if he moves from city to suburb (\$ per year)		-773	-56							
13b. Rich household's utility change if he moved from suburb to city (\$ per year)				0	0	0	- 613	- 339	- 89	

Table 2: Equilibria in which the city has a majority of poor households

City jurisdictional boundary: $B = 5.0$ (miles)			Equilibrium outcomes							
1. Metropolitan population, (households)	N	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
2. Equilibrium Case Number (label from Section 2)		no equilibrium	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2
3. Utility of poor households (\$ per year)			11,095	10,993	10,879	10,722	10,509	10,235	9,996	9,782
4. Utility of rich households (\$ per year)			37,929	36,930	36,182	33,213	32,659	31,946	31,325	30,768
5. Average commuting cost (\$ per year)			764	1,122	1,402	1,651	1,906	2,236	2,522	2,779
6. Average consumer surplus from public service (\$/yr)			-4,583	-4,583	-4,583	-4,706	-4,918	-5,092	-5,192	-5,256
7. Average rent per household (\$ per year)			141	333	484	1,676	1,591	1,582	1,625	1,690
8. Boundary between income groups in city (miles)	x		1.38	3.09	4.37	5	5	5	5	5
9. Boundary of city development (miles)	X		1.38	3.09	4.37	5	5	5	5	5
10. Boundary between income groups in suburb (miles)	y		5	5	5	5.35	6.17	7.24	8.17	9.00
11. Boundary of suburban development (miles)	Y		5.14	5.66	6.25	7.05	8.14	9.55	10.77	11.87
12. Poor household's utility change if he moved from suburb to city (\$ per year)			-169	-400	-535	-2,422	-2,618	-2,871	-3,091	-3,289
13. Rich household's utility change if he moved from city to suburb (\$ per year)			-4,100	-3,238	-2,646	0	0	0	0	0

Table 3: Equilibria in which the city has a majority of rich households

Reading across each table, as the metropolitan population increases, households are living further out and commuting costs increase. This in turn causes the utility achieved by poor and rich households to decrease.

The average consumer surplus from the public service is determined by the quality of the match between households and their preferred public service. For the equilibria in which poor households are the city's majority: as the metropolitan population increases from 1,000 to 400,000, the proportion of the rich households who live in the city and obtain the low public service is decreasing, and the average consumer surplus increases (i.e., becomes less negative). At metropolitan populations above 400,000, it is poor households in the suburb who are being mismatched with the public service; as the proportion of poor households who live in the suburb increases, the average consumer surplus decreases. For the equilibria in which rich households are the city's majority: as the metropolitan population increases above 200,000, more rich households live in the suburb and obtain a low public service, and the average consumer surplus decreases accordingly.

Concerning the average rent. For the equilibria in which poor households form the city's majority, the average rent increases, reflecting the greater pressure from commuting. The increase is most dramatic as the equilibrium shifts from Case 1.2 to Case 1.3. With Cases 1.1 and 1.2, city rents are kept low, initially by the reservation rent and then by the low willingness to pay of rich households to live in the city. With Case 1.3, city rents are bid up by poor households in the suburb who are willing to pay a premium for the low public service in the city. For the equilibria in which rich households form the city's majority, there is a large increase in rent as the relevant case shifts from Case 2.1 to Case 2.2, reflecting the increase in city rents as rich

suburban households bid to enter the city. However, at a population of 300,000 households, the average rent is so high that further increases in population cause the average rent to follow a U-curve. There is a trade-off between the increased rent paid by pre-existing households and the low rent paid by the incremental households added in the low-rent suburb. Initially the latter effect dominates and average rent falls, but at large populations the former effect dominates and average rent rises.

5.2 Normative analysis

Section 4 discusses the selection of the equilibrium of a metropolitan area. We now apply normative analysis, asking whether the selected equilibrium is the preferred equilibrium. We find that the answer depends on the population size and on the criterion used for evaluation, and that there is a trade-off between efficiency and equity. We use Table 4 in the following discussion.

City jurisdictional boundary: $B = 5.0$ (miles)

Equilibrium outcomes									
1. Metropolitan population, (households)	1,000	20,000	100,000	200,000	300,000	400,000	550,000	700,000	850,000
<i>SUMMARY IF POOR ARE MAJORITY IN CITY (CASE 1)</i>									
2. Equilibrium Case Number (label from Section 2)	city only	city only	1.1	1.1	1.2	1.3	1.3	1.3	no equil- ibrium
3. Efficiency measure (Average utility plus average rent) (\$ per household per year)	24,389	24,018	23,694	23,497	23,333	23,063	22,660	22,338	
4. Utility of poor households (\$ per year)	12,018	11,742	11,488	11,269	10,952	9,607	9,333	9,095	
<i>SUMMARY IF RICH ARE MAJORITY IN CITY (CASE 2)</i>									
5. Equilibrium Case Number (label from Section 2)	no equil- ibrium	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2
6. Efficiency measure (Average utility plus average rent) (\$ per household per year)		24,653	24,294	24,014	23,643	23,175	22,672	22,286	21,965
7. Utility of poor households (\$ per year)		11,095	10,993	10,879	10,722	10,509	10,235	9,996	9,782
<i>CITY MAJORITY PREFERRED FOR:</i>									
8. City majority for efficiency		rich	rich	rich	rich	rich	rich	rich	poor
9. City majority for equity		poor	poor	poor	poor	rich	rich	rich	

Table 4: Normative evaluation of equilibria as the metropolitan population increases

5.2.1 Efficiency comparisons

Relative efficiency is obtained by comparing the total surplus achieved in the two cases - with poor households being the city's majority and with rich households being the city's majority. Because we have specified an utility function with the property that the marginal utility of income is unity, the "as if" shifting of income from absentee landlords to resident households does not change the total surplus. Therefore our measure of efficiency is the sum of the average utility of resident households plus the average land rent. This is shown in Lines 3 and 6 of Table 4.

Rich households have high commuting costs, so commuting costs are minimized if rich households are predominantly placed in the city; at metropolitan populations of 100,000 and above, commuting cost considerations favor Case 2. However, the other factor affecting efficiency is the matching of households with their preferred public service. We denote the public service preferred by poor (rich) households as g_1 (g_2). Our specific utility function implies that matching is more important for rich households: the benefit to a rich household of being matched with g_2 instead of with g_1 exceeds the benefit to a poor household of being matched with g_1 instead of with g_2 ; or

$$[\beta(M_2)V(g_2) - g_2] - [\beta(M_2)V(g_1) - g_1] > [\beta(M_1)V(g_1) - g_1] - [\beta(M_1)V(g_2) - g_2].$$

If the metropolitan population is small (100,000 and 200,000 households in our simulations), the relevant comparison is between Case 1.1 and Case 2.1. Commuting cost considerations favor Case 2.1. In Case 1.1, all poor households obtain their preferred public service, but some rich households live in the city and obtain "too little" public service. In Case 2.1 all households obtain their preferred public service. Therefore, both commuting and

matching favor Case 2.1, or the efficient equilibrium has rich households being the city's majority.

If the metropolitan population is 300, 000 households, the relevant comparison is between Cases 1.2 and 2.2. Case 2.2 continues to have lower commuting costs. In both configurations, some rich households receive “too little” public service. However, the number of households thus mismatched is smaller in Case 2.2.²¹ Hence commuting and matching considerations continue to ensure that the efficient equilibrium is the equilibrium in which rich households are the city's majority.

If the metropolitan population is larger (400,000 households or greater in our simulations), the relevant comparison is between Case 1.3 and Case 2.2. Case 2.2 is favored for its lower commuting costs. With Case 1.3 it is some poor households who are mismatched; with Case 2.2 it is some rich households who are mismatched and, as noted earlier, matching of rich households is more important than matching of poor households. Consumer surplus is thus higher in Case 1.3. There is therefore a conflict between commuting considerations and matching considerations. At intermediate populations (400,00 and 550,000 households), the number of mismatched households is sufficiently small that commuting considerations dominate and Case 2.2 is efficient. As the metropolitan population increases, the number of mismatched households increases and matching considerations become increasingly important. If the metropolitan population is large (750,000 households), the mismatched households are sufficiently numerous that matching considerations dominate and Case 1.3 is efficient. The efficient equilibrium therefore shifts from having a majority of rich households to having a majority of poor households.

5.2.2 Equity comparisons

Our measure of equity is the Rawlsian welfare function, $\max \min [U_1, U_2]$ where U_i is the utility achieved by a household with income M_i .²² An important difference between the efficiency and equity analyses concerns the treatment of rent. Efficiency is concerned with total surplus: any gain which accrues to landlords is included in the analysis and rent therefore is considered "as if" returned to households. Equity is concerned with the surplus accruing to poor households: rent paid by poor households is lost to them and is not considered "as if" returned.

If the metropolitan population is small (100,000 and 200,000 households), the relevant comparison is between Cases 1.1 and 2.1. With Case 1.1, poor households have low commuting costs. Commuting costs are the only consideration as, in both cases, poor households obtain their preferred public service and their rent schedule is anchored at the reservation rent. Therefore equity favors Case 1.1. Put differently, the equilibrium in which poor households form the city's majority is equitable.

If the metropolitan population is 300,000 households, the relevant comparison is between Case 1.2 and Case 2.2. In Case 1.2: poor households, living closer to the metropolitan center, have lower commuting costs and city rents are kept low by the low bid of rich households to live in the city. Equity favors Case 1.2.

If the metropolitan population is intermediate or large (400,000 households or greater), the relevant comparison is between Case 1.3 and Case 2.2. With Case 1.3, many poor households in the city get their preferred public service but bidding by poor households in the suburb ensures that this gain is capitalized in the rent: the gains of matching poor households with their preferred public service go to landlords. However, in Case 2.2, these gains are not bid away to

landlords. Hence equity favors Case 2.2, or the equilibrium in which rich households are the city's majority.

5.2.3 Conflict between efficiency and equity

In Section 4, we showed that the growth path of the metropolitan area selects the equilibrium in which poor households are the city's majority. In Sections 5.2.1 and 5.2.2 we have implicitly shown that, at many metropolitan populations, there is a conflict between efficiency and equity as to the "goodness" of the selected equilibrium. In addition, when there is no conflict between efficiency and equity, the growth path selects the equilibrium which is both inefficient and inequitable. These findings are summarized in Lines 8 and 9 of Table 4.

If the metropolitan population is small (100,000 - 300,000 households), all (or almost all) the households of one income group can live in the city and be matched with their preferred public service. Commuting considerations favor placing rich households in the city, and the equilibrium in which rich households form the city's majority is efficient. However, city rents are low and the same considerations - low commuting costs - mean that the utility of poor households is greater if poor households live in the city to form the city's majority.

If the metropolitan population is intermediate or large, (400,000 households or greater) the income group which resides in the city must also spill over into the suburb. For efficiency: initially commuting considerations dominate and favor placing rich households in the city. At very large populations, matching considerations dominate and favor placing all rich households together - in the suburb. For equity, it is important that gains to matching are not bid away to landlords, or that all poor households live in the same jurisdiction - the suburb.

6. CONCLUSION

In a monocentric urban model with two jurisdictions - a inner city and a surrounding suburb - there tend to be multiple equilibria: one in which poor households are the inner city's majority and one in which rich households are the inner city's majority. We have suggested that poor households are concentrated in the inner city because this is the equilibrium which is selected by the growth path: as the metropolitan population grows, rich households “jump over” undeveloped land to establish a community in the suburb. For small metropolitan populations, this selected configuration is inefficient but equitable; for intermediate populations, it is inefficient and inequitable; for large metropolitan populations, it is efficient but inequitable. Although the model is necessarily stylized, we believe it highlights important trade-offs in urban policy and in the growth of metropolitan areas.

Our model has focused on the U.S. experience. In doing so, we have assumed that separate communities have considerable autonomy, leading to considerable variation in the public service level across communities. The equilibrium in which rich households congregate in the suburb is selected because of their high willingness to pay for their preferred public service. Breuckner, Thisse and Zenou. (1999) note that, in contrast to the U.S., many European cities have higher average income in the inner city than in the suburbs. We believe that an important difference between the U.S. and Europe is that, in Europe, there is less variation in the public service level across communities. In our model, if a regional government is introduced which prevents large differences in the service level being established between communities, rich households have a smaller incentive to “jump over” undeveloped land to form a new community in the suburb. If the allowed difference is sufficiently small, the equilibrium in which rich

households form the inner city's majority will be selected by the growth path. Thus it seems that, by adding a regional government to our model and imbuing it with different powers or roles, we can explain the difference between the U.S. and the European experience. This is an issue we intend to explore in future research.

APPENDIX: FORMAL PRESENTATION OF THE MODEL

We denote the city as jurisdiction c and the suburb as jurisdiction s . If households of income M_i ($i = 1, 2$) with land demand a_i locate in jurisdiction j ($j = c, s$), they achieve the same utility at all points d at which they locate, or

$$M_i - a_i r(d) - tM_i d - g + \beta(M_i)V(g) = \text{constant}.$$

Hence the land expenditure plus commuting cost paid by a household of income M_i locating at d in the community j is

$$b_{ij} = a_i r(d) + tM_i d$$

where b_{ij} is a constant.²³ Instead of solving for the rent function $r(d)$, it is convenient instead to solve for the constants b_{2c} , b_{1c} , b_{2s} and b_{1s} .

Rent is continuous in a jurisdiction. If both income levels live in the suburb, rent continuity at y implies

$$B < y < Y: \quad \frac{b_{2s} - tM_2 y}{a_2} = \frac{b_{1s} - tM_1 y}{a_1}.$$

If $y = B$, only poor households live in the suburb. If a rich household were to move to the suburb, he would achieve his highest utility by locating just across the jurisdictional boundary, and would pay rent $(b_{1s} - tM_1 B) / a_1$. In this case, we interpret $(b_{2s} - tM_2 B) / a_2$ as the rent he would pay if he were to move and therefore set :

$$B = y: \quad \frac{b_{2s} - tM_2 y}{a_2} = \frac{b_{1s} - tM_1 y}{a_1}.$$

If $y = Y$, only rich households live in the suburb. If a poor household were to move to the suburb, he would achieve his highest utility by locating at the fringe of development, or pay rent $(b_{2s} - tM_2Y)/a_2$. In this case we interpret $(b_{1s} - tM_1Y)/a_1$ as the rent a poor household would pay if he were to move to the suburb, and we set:

$$y = Y: \quad \frac{b_{2s} - tM_2Y}{a_2} = \frac{b_{1s} - tM_1Y}{a_1}.$$

Combining these cases, we can write:

$$B \leq y \leq Y: \quad \frac{b_{2s} - tM_2y}{a_2} = \frac{b_{1s} - tM_1y}{a_1}. \quad (\text{A.1})$$

Similarly, rent continuity at x in the city implies

$$0 \leq x \leq X: \quad \frac{b_{2c} - tM_2x}{a_2} = \frac{b_{1c} - tM_1x}{a_1}, \quad (\text{A.2})$$

where, if $x = 0$, b_{2c}/a_2 is interpreted as the rent a rich household would pay if he were to move to the city; if $x = X$, $(b_{1s} - tM_1X)/a_1$ is interpreted as the rent a poor household would pay if he were to move to the city.

Denoted the public service in the city as g_c and the public service in the suburb as g_s . Equilibrium requires that the rent schedule adjusts so that no household can obtain more utility by moving between jurisdictions. If $x = 0$, there are no rich households in the city or equilibrium requires that a rich suburban household obtains at least as much utility in the suburb as he could obtain if he were to move to the city, or

$$x = 0: \quad M_2 - b_{2s} - g_s + \beta(M_2)V(g_s) \geq M_2 - b_{2c} - g_c + \beta(M_2)V(g_c). \quad (\text{A.3a})$$

Similarly, if there are rich households in the city and in the suburb

$$0 < x \text{ and } B < y: M_2 - b_{2s} - g_s + \beta(M_2) V(g_s) = M_2 - b_{2c} - g_c + \beta(M_2) V(g_c); \quad (\text{A.3b})$$

and if there are no rich households in the suburb

$$B = y: M_2 - b_{2s} - g_s + \beta(M_2) V(g_s) \leq M_2 - b_{2c} - g_c + \beta(M_2) V(g_c). \quad (\text{A.3c})$$

Similar equations apply for poor households:

$$x = X: M_1 - b_{1s} - g_s + \beta(M_1) V(g_s) \geq M_1 - b_{1c} - g_c + \beta(M_1) V(g_c); \quad (\text{A.4a})$$

$$x < X \text{ and } y < Y: M_1 - b_{1s} - g_s + \beta(M_1) V(g_s) = M_1 - b_{1c} - g_c + \beta(M_1) V(g_c); \quad (\text{A.4b})$$

$$y = Y: M_1 - b_{1s} - g_s + \beta(M_1) V(g_s) \leq M_1 - b_{1c} - g_c + \beta(M_1) V(g_c). \quad (\text{A.4c})$$

The rent paid at the limit of development in the suburb is the reservation rent r_0 . If the suburb contains poor households,

$$y < Y: \frac{b_{1s} - tM_1 Y}{a_1} = r_0;$$

if the suburb contains only rich households

$$y = Y: \frac{b_{2s} - tM_2 y}{a_2} = r_0.$$

Using Equation (A.1), this is collapsed to the single relationship:

$$y \leq Y: \frac{b_{1s} - tM_1 y}{a_1} = r_0. \quad (\text{A.5})$$

Similarly, if there is undeveloped land in the city, the rent at the limit of development must be the reservation rent r_0 . If there is no undeveloped land, the rent at the city's boundary must equal or exceed r_0 . If the city contains only rich families

$$x = X < B: \quad \frac{b_{2c} - tM_2X}{a_2} = r_0;$$

$$x = X = B: \quad \frac{b_{2c} - tM_2B}{a_2} \geq r_0.$$

If the city contains poor households

$$x < X < B: \quad \frac{b_{1c} - tM_1X}{a_1} = r_0;$$

$$x < X = B: \quad \frac{b_{1c} - tM_1B}{a_1} \geq r_0.$$

Using Equation (A.2), these equations are collapsed to

$$X < B: \quad \frac{b_{1c} - tM_1X}{a_1} = r_0; \tag{A.6a}$$

$$X = B: \quad \frac{b_{1c} - tM_1B}{a_1} \geq r_0. \tag{A.6b}$$

The public service in each community is determined by voting. We assume that households vote myopically by taking the rent schedules as given. If the suburb has a majority of poor households, voting sets the public service in the suburb g_s to maximize their utility or

$$\frac{\pi(Y^2 - y^2)}{a_1} \geq \frac{\pi(y^2 - B^2)}{a_2}; \beta(M_1) V'(g_s) = 1. \tag{A.7a}$$

Conversely, if the suburb has a majority of rich households, voting sets the public service level

in the suburb as

$$\frac{\pi(Y^2 - y^2)}{a_1} < \frac{\pi(y^2 - B^2)}{a_2}; \quad \beta(M_2) V'(g_s) = 1; \quad (\text{A.7b})$$

Similarly in the city the public service level g_c is set by the majority as

$$\frac{\pi(X^2 - x^2)}{a_1} \geq \frac{\pi x^2}{a_2}; \quad \beta(M_1) V'(g_c) = 1. \quad (\text{A.8a})$$

$$\frac{\pi(X^2 - x^2)}{a_1} < \frac{\pi x^2}{a_2}; \quad \beta(M_2) V'(g_c) = 1. \quad (\text{A.8b})$$

Summarizing, the variables to be solved for are: b_{2c} , b_{1c} , b_{2s} , b_{1s} , g_c , g_s , x , X , y , and Y .

The equation system corresponds to the Equations (2) - (3) and (A.1) - (A.8). The equilibrium outcome is a function of the city boundary B and the other parameters.

The solution to the equation system is only a solution if the ordering of distances satisfies

$$0 \leq x \leq X \leq B \leq y \leq Y.$$

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ENDNOTES

1. Ross and Yinger (1999) survey this literature.
2. Wheaton (1976) and Sasaki (1990) provide a comparative static analysis of this equilibrium.
3. The assumption of fixed lot size greatly simplifies the problem and allows us to avoid well-known existence problems associated with stratified local public-finance equilibria (see Epple, Filimon and Romer (1984, 1993)).
4. For ease of presentation, the community is assumed to provide a public service and not a public good. The public service shows constant returns to community size. It is straightforward to change the publically-provided good from a public service to a public good.
5. Put differently, the commuting benefit to rich households of living closer to the metropolitan center outweighs the benefit they can achieve from the reduced price of land further out.
6. This assumption is consistent with the findings of Glaeser, Kahn, and Rappaport (2000).
7. A formal proof is provided in de Bartolome and Ross (2000).
8. This assumption greatly simplifies the model. We do not believe that our results depend on this assumption - what is important is that the two communities vote different public service levels.
9. Because of the specific form of the utility function, it is straightforward to change this assumption to allow each household to receive an equal share of the total rent paid as a lump-sum transfer. To do this, simply sum the average rent and the quoted utility levels in Tables 2 and 3. We prefer to have rents paid to absentee landlords as the alternative suggests that households in one community benefit from property value increases in the other community.
10. A list of all such possible equilibria is available from the authors on request. de Bartolome and Ross (2000) show that an equilibrium with strict income sorting always exists.
11. Many studies require ρ to be positive, implying a price elasticity that exceeds unity. We generalize the utility function to allow for negative values of ρ and price elasticities that are less than unity.
12. Sensitivity analysis was performed using a lower and higher value (0.1 and 0.5) for the implied income elasticity of land demand, and there was little effect on the results.
13. This figure is a crude average and ignores possible variations in the number of households living in low- and high- service jurisdictions.
14. Societal figure calculated as local government expenditure financed from local government own-revenue (*Census of Governments Volume 4 Number 5* 1992, Table 3) divided by personal income times one minus the average federal income tax rate (*Individual Income Tax Returns* 1990, Table 1.1).

15. Using the labels described later in the text, the equilibrium at $N = 300,000$ in Table 3 changed from Case 1.2 to Case 1.1.
16. Although the causation is quite different, this result resembles the leapfrog development pattern that may appear in models of urban growth where some land is left vacant in the interior because its option value for future development exceeds its value in current use. For some examples in the literature, see Arnott and Lewis (1979), Capozza and Helsley (1989) and Wheaton (1982).
17. With equal numbers of poor and rich households, the rich become the majority in the city immediately the poor spill over into the suburb.
18. The “average” inner city has area 82 sq miles, or has radius 5.1 miles. This is calculated as total land area of all central places inside a metropolitan area (*Census of Population and Housing 1990*, Table 11) divided by number of metropolitan areas (*Statistical Abstract of the United States 1998*, Table 40).
19. At $N=1,000$ and $N=20,000$, the suburb is uninhabited: if a poor household moved to the suburb he would vote a low public service. The utility changes are therefore smaller than the changes of -1,392, -1,309, ... at $N = 100,000$, $N= 200,000$, ... in which the rich are “already” in the suburb and the suburban public service is high.
20. This is the average rent paid *per household*. If the value quoted in the table is T (\$ per household per year): because the average lot size is 0.3333 (acres per household), the average rent per acre per year is $3T$. Because the reservation rent has been set to zero, this is interpreted as the average rent premium (above the reservation rent) paid for one acre of land in the metropolitan area.
21. In Case 1.2: the number of rich households in the city is 20,273. In Case 2.2: the number of rich households in the suburb is 18,861.
22. Equity is normally interpreted as placing more weight on the welfare of the poor. For convenience of exposition, we take an extreme welfare function with all weight being placed on the welfare of the poor. If the equity objective is the utilitarian objective, $U_1 + U_2$, the basic conclusion is unchanged: there is often conflict between efficiency and equity and, when there is no conflict, growth selects the equilibrium which is inefficient and inequitable.
23. If households of income M_i do not live in the community j , we interpret b_{ij} to be the rent plus commuting cost which a household of income M_i would pay if he were to move into the community j .