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Who's in Charge in the Inner City? The Conflict Between Efficiency and Equity in the Design of a Metropolitan Area

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Abstract

A circular metropolitan area consists of an inner city and a suburb. Households sort over the two jurisdictions based on public service levels and their costs of commuting to the metropolitan center. Using numerical simulations, we show (1) there typically exist two equilibria: one in which the poor form the majority in the inner city and the other in which the rich form the majority in the inner city; (2) there is an efficiency vs. equity trade-off as to which equilibrium is preferred; and (3) if the inner city contains only poor households, equity favors expanding the inner city to include rich households.

Journal of Economic Literature Classification: H73, R12, R14

Keywords: urban, equilibria, welfare

1. INTRODUCTION

In Tiebout's (1956) seminal model of fiscal competition, households sort themselves between communities based on their benefits from the public service, and the resulting allocation of households to communities is first-best efficient.[1] In Tiebout's model, the public service is financed by a residency tax and community boundaries are flexible. Hamilton (1975, 1976) shows that, if the residency tax is replaced by a property tax, zoning can allow households to sort across local jurisdictions to achieve first-best efficiency. Alternatively, Elickson (1971), Yinger (1982), Epple, Filimon and Romer (1984, 1993) and Epple and Romer (1991) describe communities as having fixed boundaries and a property tax, but no zoning powers. In such models, there is still sorting between communities but the public service and the property tax of a community are capitalized into the community's land price, distorting the location and housing decisions of individual households.[2] In general, the equilibrium outcomes in such economies are not first-best efficient.

In contrast to Tiebout's model, Alonso (1964), Mills (1967) and Muth (1969) downplay the fiscal difference between communities and instead consider a metropolitan area to be "as if" a single monocentric city to whose center households commute. The commuting disadvantage of a location further from the city's center is capitalized into its land price, so that the land price declines as locations move away from the city center. Income sorting still occurs, but it now arises from the interaction of commuting costs and land demand. If land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city's center is greater for the rich households and compensates them for the associated increase in commuting cost.[3] In this case poor households win the bid for land near the city's center and vote for low levels of public services. Conversely, if commuting costs increase with income and if land demand is unresponsive to income changes, rich households outbid poor households for locations closer to the city's center.[4] Regardless of the sorting outcome, the urban equilibrium is efficient (Fujita (1989), and neither the poor nor the rich can be made better off by reversing the sorting outcome.

In general, households are likely to choose communities based on both public services and commuting costs, and so we combine the two approaches into a single model. A circular city has an exogenous boundary and is surrounded by a suburb. The public service in each community is determined by voting and all households must commute to the central business district which is located at the center of the inner city. The model has two income-classes. Rich households have a higher demand than poor households for the public service which favors fiscal sorting between communities: *ceteris paribus* different income groups prefer to live in different communities. Rich households have higher commuting costs per mile than poor households and land demand is relatively income inelastic. This gives rise to spatial sorting within each community as rich households outbid poor households for land nearer the city's center.

Fiscal sorting implies that different income groups try to separate from each other, but there is no prediction as to which income group locates in the inner city. Spatial sorting does not eliminate this potential multiplicity of equilibria. In particular, we find two equilibria. In one equilibrium, it is the poor households who form the majority in the inner city, voting low public services in that city; in the second equilibrium, it is the rich households who form the majority in the inner city, voting high public services there. A shift from one equilibrium to the other affects the welfare of poor and rich households by changing the equilibrium level of rents.

Which equilibrium is preferred? If the city boundary were free to adjust, efficiency would require it to be set so that the city contains all the rich households (and no poor households): there is perfect matching of households with their desired public service and commuting costs are minimized. From an equity standpoint, however, poor households obtain the most utility if they form the majority in the city and if the city is large enough to have vacant land at its jurisdictional boundary - this arrangement decreases competition for urban land and thereby lowers the rent that the poor must pay for living in the city.

If the city boundary is fixed, the form of the efficient equilibrium depends on the city size, and the efficiency-equity trade-off continues to be present. In our model, the matching of a rich household with his preferred public service level gives a larger surplus than the matching of a poor household with his preferred public service level. Therefore, if the city's area is sufficiently small that it cannot contain all the rich households, efficiency favors the equilibrium in which only poor households live in the city: all the rich households live in the suburb, enjoying their preferred service level, and the benefit of this fiscal matching dominates the additional commuting cost. However, bidding by poor suburban households anxious to escape the high suburban public service and taxes causes rent paid by poor city households to be high in this equilibrium. Therefore, with a small city, equity favors the equilibrium in which the city contains only rich households; in this equilibrium there are no poor city households bidding up suburban rents, so poor households get their preferred public service at a low rent. If the city's area is large so that it can contain all the households of one income group, the result is reversed: efficiency favors the equilibrium in which the rich are the city's majority and equity favors the equilibrium in which the poor are the city's majority.

Our results are of interest because the jurisdictional boundary between the inner city and the suburb varies dramatically between and even within regions. Table 1 shows the land area and number of households for major central cities and their surrounding metropolitan areas for New England and Texas. The fifth and sixth columns contain the ratio of inner city land area or number of households to the metropolitan area variables. The New England city area ratios are always less than 3 percent while the area ratios for the major Texas central cities fall between 5 and 12 percent. In terms of household numbers, the New England ratios fall between 10 and 25 percent while the Texas ratios fall between 31 and 72 percent.

Regions/Cities	Metropolitan Area		Inner city		Ratio	
	Area	Population	Area	Population	Area	Population
New England						
Boston, MA	6,452.22	5,559,103	48.43	589,141	0.008	0.106
Hartford, CT	1,591.52	1,148,618	17.31	121,578	0.011	0.106
Providence, RI	940.87	962,886	18.47	173,618	0.020	0.180
Springfield, MA	1,147.43	608,479	32.10	152,082	0.028	0.250
Texas						
Houston	5,920.12	4,177,646	579.42	1,953,631	0.098	0.468
Dallas	6,185.84	3,519,176	342.54	1,188,580	0.055	0.338
Fort Worth	2,917.96	1,702,625	292.54	534,694	0.100	0.314
San Antonio	3326.40	1,592,383	407.56	1,144,646	0.123	0.719
Austin	4,240.41	1,249,763	251.52	656,562	0.059	0.525

Source Information: Census 2000 Summary File 1 (ST 1) 100 Percent Sample, County and Place Tables, 1990 PMSA, MSA, and NECMA definitions

Table 1: Relative Size of Central Cities by Square Miles and Number of Households

Moreover, the jurisdictional boundary is potentially a variable that is chosen by the policy-makers. For example, in 1993 Memphis (USA) proposed merging with its near suburbs and a similar exercise was completed by Toronto (Canada) in 1998. One motivation for such a change is strategic: by merging with its suburbs an inner city is able to enlarge its commercial tax base and reduce tax competition. Our model suggests another, perhaps more fundamental, feature of boundary changes: the change in the sorting of households by income between the inner city and suburbs induces a change in rents. Our simulations show that, if the status-quo is the equilibrium in which only poor households live in the inner city, expanding the city so that it contains some rich households benefits poor households. In our model, taxes are residency taxes so that this benefit to poor households does not arise because rich households pay more taxes: poor households benefit because their rents fall.

This paper extends the model of de Bartolome and Ross (2002). In de Bartolome and Ross (2002), we consider a metropolitan area to be growing and discuss which equilibrium is likely to be observed. Here we consider an existing metropolitan area and compare the equilibria obtained with different city boundaries. This highlights the conflict between efficiency and equity. Because comparisons are made between different equilibria, it is difficult to do calculus-based comparisons. We therefore use a computable general equilibrium model. We also use a very simple utility function so that the intuition is highlighted.

The paper is structured as follows: Section 2 presents the theoretical model and the possible equilibria. Section 3 presents the simulations. Section 4 discusses efficiency and equity. Section 5 concludes.

2. THE MODEL

2.1 The model structure

A household lives in a community and obtains utility U from consuming a privately-provided good c and from a public service g provided by the community: $U(c, g)$. The privately-provided good is the numeraire good. The household's demand for lot size a is assumed to be exogenous[5] and the non-land components of housing are included as part of the private good: therefore housing *per se* does not enter the utility function.

Our interest is in the welfare comparisons of different equilibria. This comparison is made particularly simple if we restrict attention to a utility function of consumer surplus form:

$$U = c + \beta V(g).$$

Because we want the public service to appear normal or to be more valued by households of higher income, we make β to be a function of endowed income M :

$$\beta \equiv \beta(M) \ ; \quad \beta'(M) > 0 \ .$$

In this description, households differ in their tastes for the public service, and their tastes vary systematically with endowed income.

All households commute to the central business district which is located at the metropolitan center and, for analytical convenience, is assumed to have no area. A household has a fixed time endowment which he can use either for working or for commuting. If he lives at the metropolitan center, he spends no time commuting and his income is M . If he lives at distance d

from the metropolitan center, his income is reduced by the opportunity cost of the commute. The time spent commuting is proportional to d and the opportunity cost of a unit of his time is proportional to M , so that his commuting cost is tMd . The price of a unit of land at d is $r(d)$. The community provides the public service g . The public service shows constant returns to community size, and the cost of providing a unit of the public service to each resident is one unit of numeraire per resident [6]; the public service is financed by a residency tax g. Therefore the consumption of the private good by the household if he locates distance d from the metropolitan center is

$$c = M - tMd - a\,r(d) - g \,;$$

his utility is

$$M - tMd - ar(d) - g + \beta(M)V(g).$$

There are two income levels. Poor households have income M_1 and rich households have income M_2: $M_1 < M_2$. Associated with each income level, M_1 and M_2, is a lot size a_1 and a_2 respectively: as noted earlier, a_1 and a_2 are exogenous and $a_1 < a_2$. At equilibrium, a household of income M_i achieves utility $W(M_i)$. His bid for land which is distance d from the metropolitan center and which is in jurisdiction j providing public service g_j is $R_{ij}(d)$, where

$$M_i - tM_i d - a_i R_{ij}(d) - g_j + \beta(M_i)V(g_j) \equiv W(M_i).$$

Hence, within the jurisdiction

$$tM_i d + a_i R_{ij}(d) \; = \; constant\,.$$

Differentiating with respect to *d*, the slope of the household's bid-rent curve in jurisdiction *j* is

$$\frac{dR_{ij}(d)}{dd} \; = \; - \frac{tM_i}{a_i}\,. \tag{1}$$

The household's willingness to pay for a location decreases as the location moves further from the metropolitan center, reflecting the greater commuting costs. We assume

$$\frac{tM_2}{a_2} > \frac{tM_1}{a_1}\ ,$$

or the bid-rent curve of a rich household is steeper than the bid-rent curve of a poor household. In consequence, if both income-groups are present in a community, rich households outbid poor households for the locations in the community which are closer to the metropolitan center. Put differently, there is income sorting within the jurisdiction: rich households live on the "inside" (closer to the metropolitan center) and poor households live on the "outside" (further from the metropolitan center) of the community.[7]

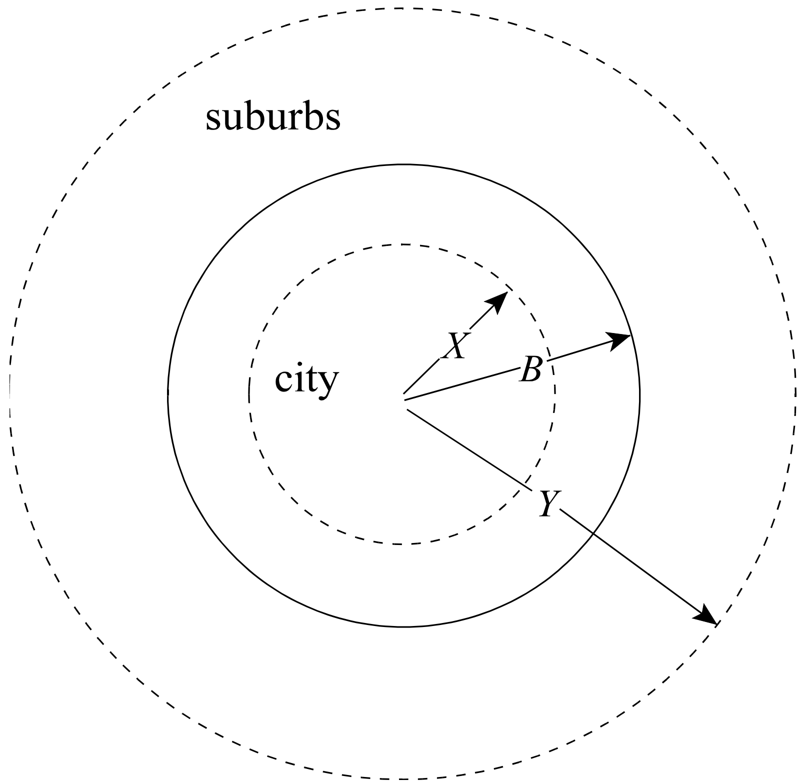


Figure 1: the metropolitan area

The metropolitan area is comprised of a circular inner city (henceforth termed "the city") surrounded by the suburb. The jurisdictional boundary between the city and the suburb has radius B . There is the possibility of undeveloped land at the fringe of the city if either poor or rich households are unwilling to pay more than the reservation rent to live at the outside of the city. The limit of development in the city has radius X . If

$X < B$: there is undeveloped land at the fringe of the city;

$X = B$: there is no undeveloped land in the city.

Remembering that rich households live on the inside of the city and poor households live on the outside, denote the boundary between rich and poor households in the city as occurring at distance *x* from the center. If

$x = 0$:	only poor households live in the city;
$0 < x < X$:	rich and poor households live in the city;
$x = X$:	only rich households live in the city.

In the suburb, the limit of development is distance *Y* from the city center and we assume that the outer jurisdictional boundary is sufficiently distant from the metropolitan center that all households live in the city or in the suburb. The boundary between the rich and poor households in the suburb occurs at distance *y* from the center. If

$y = B$:	only poor households live in the suburb;
$B < y < Y$:	rich and poor households live in the suburb;
$y = Y$:	only rich households live in the suburb.

There are *N* households of which a fraction θ are poor. Equating land demand and land supply requires:

$$\pi X^2 + \pi (Y^2 - B^2) = \theta N a_1 + (1 - \theta) N a_2 \; ; \tag{2}$$

$$\pi x^2 + \pi (y^2 - B^2) = (1 - \theta) N a_2 . \tag{3}$$

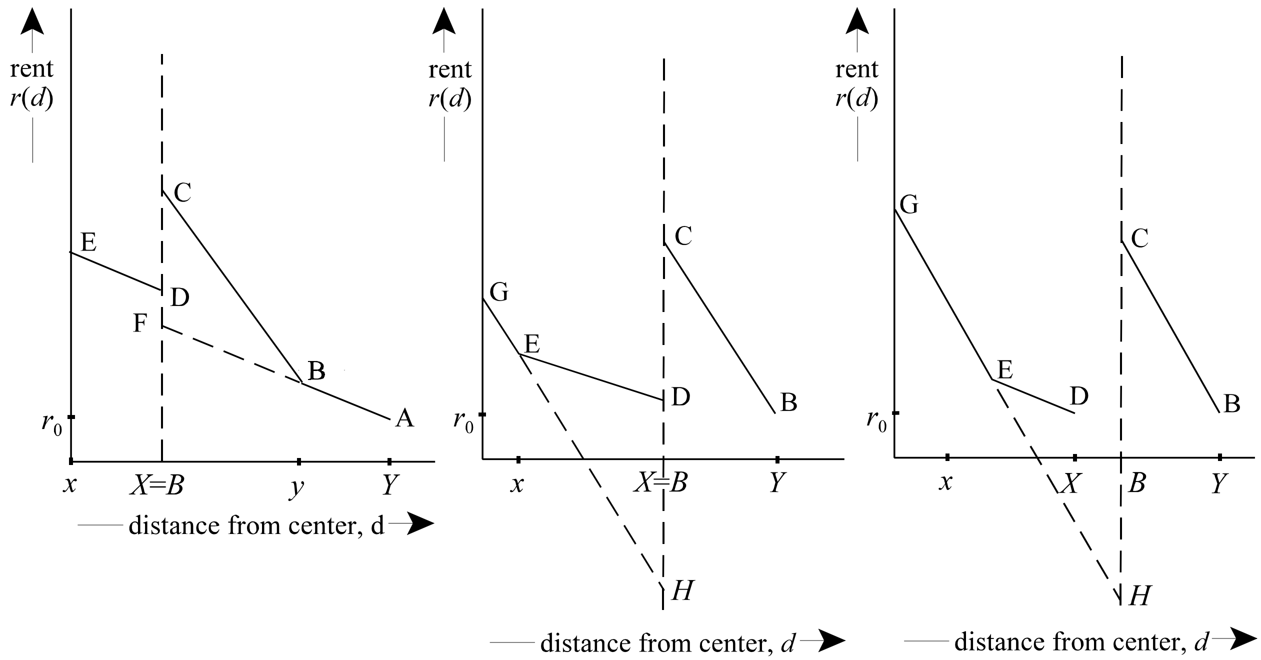
The model is now summarized descriptively; the formal algebraic formulation is presented in the Appendix.

1. Rent continuity: rent is continuous in a jurisdiction. If it were discontinuous, a household living on the side of relatively high rent could increase his utility by moving across the discontinuity to the side of low rent: his rent would decrease discontinuously but his commuting cost would increase only marginally.[8]
2. No migration: no household can achieve higher utility by moving to another location. This implies that, if an income class resides in both communities, the rents are such that a household in that income class is indifferent between the communities. If an income class does not reside in a community, rents are such that a household in that income class cannot increase his utility by moving into the community.
3. Reservation land price: the reservation price of land is r_0. If a community contains no undeveloped land, the rent at the limit of development is at least r_0. If a community contains undeveloped land, the rent at the limit of development is r_0.
4. Determination of the public service level. The public service level in each community is determined by majority voting; households vote myopically, taking the rent schedules as given.[9]
5. Model closure. We assume that rent is paid to absentee landlords.[10]
6. The population in each community is considered to be a continuous variable.

We restrict attention to the case where the metropolitan population is sufficiently large that not all households can live in the city and both communities must be occupied. Our focus is on how the equilibrium outcomes change as the city's jurisdictional boundary, *B*, is increased. We set the numbers of poor and rich households to be equal. For each value of *B*, we found two equilibria:[11] an equilibrium in which poor households are the majority in the city and rich households are the majority in the suburb, *and* an equilibrium in which the majorities are reversed. The former configurations are grouped together as Case 1 and the latter configurations are grouped together as Case 2. We consider Case 1 first.

2.2 Case 1: poor households are majority in city, rich households are majority in suburb

As the city's boundary moves outwards, the equilibrium configuration in which the city has a majority of poor households shifts from Case 1.1 through Case 1.2 to Case 1.3. The rent profiles of these cases are shown in Figure 2. Using Equation (1), the rent profile has slope tM_1/a_1 (tM_2/a_2) in the region occupied by poor (rich) households, so that we can use the slope of the rent schedule at a location to infer the income of a resident at that location: poor (rich) households live at locations where the rent schedule is flat (steep).



(a) Case 1.1 (b) Case 1.2 (c) Case 1.3

Figure 2: rent schedules for Case 1.

If the city area is small, all the city is occupied by poor households, and some poor households live in the suburb. All rich households live in the suburb. This is Case 1.1 and its typical rent profile is shown in Figure 2(a). The rent at the limit of suburban development is r_0: this rent anchors the suburban rent schedule. Poor households live at the outside of the suburb so that, as the location moves inwards from *Y*, the commuting advantage to poor households is capitalized in the rent and the rent rises at rate tM_1/a_1 . At distance *y* from the metropolitan center, residents become rich and the rent gradient rises to tM_2/a_2 . *ABF* is interpreted as the bid-rent curve of a poor household in the suburb, and *BC* is interpreted as the bid-rent curve of a rich household in the suburb.

As the location moves across the jurisdictional boundary, the public service changes from the level desired by rich households to the level desired by poor households: poor suburban households are willing to pay the premium *FD* to move across the jurisdictional boundary. This premium supports the city's rent schedule; it also ensures that the rent in the city exceeds the reservation rent r_0 and that there is no undeveloped land. The rent schedule in the city is *DE* and along *DE* the rent rises at rate tM_1/a_1. Rich households do not live in the city because the rent exceeds their willingness to pay for the lower public service (i.e., their bid-rent curve lies below *DE*).

As the city's jurisdictional boundary expands, rent in the city exceeds the willingness to pay of rich households to live in the city, and so it is the poor households who migrate from the suburb to fill the enlarged city. At a critical jurisdictional size, all poor households have migrated from the suburb. If the city's boundary further increases, it is rich households who are indifferent between communities. As the city's boundary moves further out, rich households migrate into the city and the city's rent schedule adjusts so that the rent they pay equals the bid of a rich suburban household to live in the city with its low public service. Rents therefore fall relative to Case 1.1. The city contains rich and poor households, and the suburb contains only rich households; there is no undeveloped land in the city. This case is denoted as Case 1.2.

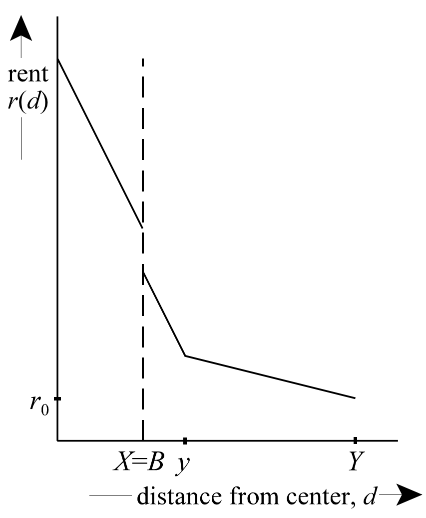
A typical rent schedule for Case 1.2 is shown in Figure 2(b). In particular: the rent at the suburban fringe is r_0 and rises at rate tM_2/a_2 along *BC* as the location moves towards the metropolitan center. As the location crosses the jurisdictional boundary, the rent would have to fall by *CH* (and become negative in this example) if a rich household were to be willing to live on the city side of the boundary. *HEG* is the bid-rent curve of a rich household in the city. *DE* is the actual rent schedule paid in the area of the city occupied by poor households. Reflecting the difference in the commuting costs, the actual rent gradient along *DE* is tM_1/a_1 but the bid of rich households rises at the faster rate tM_2/a_2 as the location moves inwards. At distance *x* from the metropolitan center, the bid of a rich suburban household to live at *x* equals the actual rent, and rich households live at the locations closer to the metropolitan center.

As the city's boundary further expands, rich households are migrating into the city, *x* is increasing and poor households are getting pushed further and further out. In Case 1.2 there is no undeveloped land in the city; the total inhabited land in the metropolitan area is unchanged, or *Y* is unchanged and hence rents paid at locations in the suburb do not change. This in turn implies the rent schedule at the locations in the city which are inhabited by rich households does not move, or *G* does not move. Therefore, as *x* increases, *E* moves down *GH*, lowering the rent at the city-side of the jurisdictional boundary. At a critical jurisdictional size, the rent at the city's side of the boundary is the reservation rent r_0 . If the boundary further expands, undeveloped land opens up in the city and the relevant case shifts to Case 1.3.

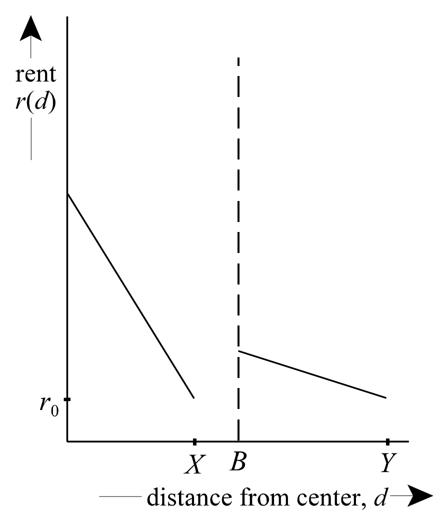
Figure 2(c) shows a typical rent schedule for Case 1.3. In Case 1.3, some rich households continue to inhabit the suburb, attracted by the high public service there. The undeveloped land in the city causes the limit of development *Y* to move out, but the rent at *Y* is still the reservation rent and the rent schedule in the suburb still has slope tM_2/a_2. As the location moves across the jurisdictional boundary, the bid of rich households - determined by their willingness to pay for the low public service in the city - falls by the same distance *CH* as in Figure 2(b). There is undeveloped city land and the limit of development in the city is *X* ($X < B$). As the location moves in from *X*, the rent rises initially at rate tM_1/a_1. At *x*, the actual rent equals the bid of rich households, rich households reside at the locations closer to the metropolitan center and the rent gradient steepens accordingly.[12]

2.3 Case 2: rich households are majority in city, poor households are majority in suburb.

The alternative equilibrium has rich households forming the city's majority and poor households forming the majority in the suburb. If the city is small, it contains only rich households; the city cannot contain all the rich households so that some live in the suburb with the poor households. This case is denoted Case 2.1 and a typical rent schedule is shown in Figure 3(a). The suburban rent schedule is anchored by the rent r_0 paid at the limit of development. In the area between Y and y occupied by poor households, the rent rises at rate tM_1/a_1. At y, the rent gradient steepens to tM_2/a_2. Rich households are indifferent between the communities; because rich households control the public service level in the city, rich households in the suburb are willing to pay a premium to move to the city so that the rent increases as the location crosses the jurisdictional boundary into the city.



(a) Case 2.1



(b) Case 2.2

Figure 3: rent schedules for Case 2.

As the city's boundary moves out, rich households migrate into the city until all rich households live in the city. If the boundary is further increased, poor households continue to live in the suburb to benefit from their desired public service (and avoid the city's high taxes). Undeveloped city land opens up and each community becomes homogeneous in income. This case is denoted Case 2.2 and its rent schedule is shown in Figure 3(b). Compared to Case 2.1, rent in the city is no longer being bid up by suburban rich households, and city rents therefore fall.[13]

3. SIMULATIONS

3.1 Analytical Framework

The utility function of a household with income M_i is specified as

$$c + sign(\rho)\ A\ (M_i)^{\delta}\ g^{\rho}$$

where A, δ and ρ are parameters, $\delta > 0$, $|\rho| < 1$ and $\rho \neq 0$.[14] The household's preferred public service is

$$g_i = (|\rho| A)^{\frac{1}{1-\rho}}\ (M_i)^{\frac{\delta}{1-\rho}}.$$

We represent the income distribution by assuming that there are equal numbers of poor and rich households, and choose the income of households in each group to be (close to) the median income of the bottom- and top- half of the U.S. income distribution in 1990. Table 2 shows the assumed values of the model parameters and , for comparison, the observed values for U.S. metropolitan areas.

Parameter	Variable Name	Model Value	Societal Value
Poor households as fraction of population	θ	0.5	0.5
Income of poor household ($ per year) [a]	M_1	15,000	16,523
Income of rich household ($ per year) [a]	M_2	45,000	46,725
Metropolitan population (households) [b]	N	270,000	266,389
Average lot size (acres) [c]		0.3333	0.3402
Lot size of poor household (acres) [d]	a_1	0.2833	
Lot size of rich household (acres) [d]	a_2	0.3833	
Commute time per mile as fraction of work day [e]	t	0.013	
Public service demand parameter [d]	δ	1.4	
Public service demand parameter [d]	ρ	-1.0	
Public service demand parameter [d]	A	3.0	
Reservation rent [d]	r_0	0	

a. Societal figures are total 1990 money income at 25th and 75th percentile in metropolitan income distribution (*Money Income of Households, Families and Persons in the United States 1990*, Table 2) multiplied by $(1-\tau)$ where τ is average federal income tax rate for federal tax return with adjusted gross income equal to money income of respective household (*Individual Income Tax Returns* 1990). Model value represents earned income of a household with zero commuting costs.

b. Societal metropolitan population calculated as 1991 number of owner-occupied units in all central cities plus suburbs (*American Housing Survey for the United States 1991*, Tables 8-3 and 9-3), divided by number of metropolitan areas (*Statistical Abstract of the United States 1998*, Table 40)

c. Societal average lot size is metropolitan land area divided by societal metropolitan population. Metropolitan land area calculated as 1991 occupied housing units in all central cities multiplied by median lot size in central city plus same 1991 figure for suburban units (*American Housing Survey for the United States* 1991, Tables 8-3 and 9-3), divided by the number of metropolitan areas *(Statistical Abstract of the United States 1998, Table 40)*. For societal metropolitan population, see note b.

d. See text

e. Fraction of 8-hour workday spent commuting to metropolitan center if household lives one mile from city center. Figure is based on an average travel speed of 20 miles per hour and a round trip commute.

Table 2: Parameter and Population Values

The population and average lot size are chosen to be close to the values observed in the population. The lot size for poor and rich households is adjusted down and up from the average lot size, respectively, in order to be consistent with a 0.3 income elasticity of demand for land - an elasticity value which is consistent with the recent estimates by Glaeser, Kahn, and Rappaport (2000).[15] The population and housing demand parameters imply that the area of developed land in the metropolitan area is 90,000 acres: this translates to a radius (Y) of 6.7 miles if the city has no vacant land.

The values for ρ and δ are chosen so that the price and income elasticities of demand for the public service are -0.5 and 0.7, respectively, which is consistent with current estimates in the literature (see Ross and Yinger (1999)). These parameter values (and $A=3$) imply that poor households vote a public service level of 1452 ($ per year) and rich households vote a public service level of 3132 ($ per year). The "average" calculated value for the share of income spent on public service is therefore (1452+3132)/(1500 + 4500) = .076 [16]; in contrast, the societal value is 0.13.[17] When we raised the value of the parameter A so that the simulated value lay closer to the observed value, there was little effect on the results except that we ceased to find an equilibrium configuration in which both poor and rich households live in the city and in which all city land is developed.[18] In order to illustrate an equilibrium with this form, we decided to use the lower value of A.

Our focus is on comparing equilibria and what is important is the relative value of rents. Therefore, for convenience, we set the reservation rent to zero.

3.2 Results

In this section we investigate the equilibria as the city's jurisdictional boundary, *B*, is increased from 0.5 miles to 6.5 miles in increments of 1 mile. Our main interest in doing this is to illustrate the efficiency *v.* equity trade-off discussed in the next section.

3.2.1 Case 1: Poor households are majority in city, rich households are majority in suburb

Table 3 presents the simulation results for the equilibrium in which poor households are the city's majority and rich households are the suburban majority. This might be considered the common U.S. outcome, with high levels of public services in the suburbs. Line 1 shows the city's jurisdictional boundary (assigned exogenously). Line 2 shows the equilibrium case (using the labels discussed in Section 2) which arises for the given boundary size. Line 3 shows the average rent. [19] Lines 4 and 5 show the utility achieved by poor and rich households. Lines 6-11 show some characteristics of the metropolitan area. Lines 12 and 13 show the utility change a poor and rich household would experience if he were to move from the community in which he resides to the other community.

Metropolitan population, *N = 270,000*

Attributes of metropolitan area	Symbol	Equilibrium values						
1. City jurisdictional boundary (miles from center)	B	0.5	1.5	2.5	3.5	4.5	5.5	6.5
2. Equilibrium Case Number (label from Section 2)		1.1	1.1	1.1	1.1	1.2	1.3	1.3
3. Average rent per household ($ per year)		786	813	880	996	611	416	438
4. Utility of poor households ($ per year)		9,891	9,891	9,891	9,891	10,905	11,109	11,035
5. Utility of rich households ($ per year)		35,334	35,272	35,155	34,992	34,823	34,627	34,306
6. Boundary between income groups in city (miles)	x	0	0	0	0	1.11	2.58	3.26
7. Boundary of city development (miles)	X	0.5	1.5	2.5	3.5	4.5	5.07	5.45
8. Boundary between income groups in suburb (miles)	y	5.10	5.29	5.66	6.16	6.69	7.02	7.57
9. Boundary of suburban development (miles)	Y	6.69	6.69	6.69	6.69	6.69	7.02	7.57
10. City population of poor households		1,774	15,969	44,357	86,940	135,000	135,000	135,000
11. City population of rich households		0	0	0	0	6,443	34,831	55,822
12. Poor household's utility change on moving from city to suburb ($/yr)		0	0	0	0	-1,014	-1,284	-1,316
13. Rich household's utility change on moving from suburb to city ($/yr)		-1,528	-1,466	-1,349	-1,186	0	0	0

Table 3: Case 1 equilibria as the city's boundary moves out.

For small and medium sized inner cities (*B* less than or equal to 3.5 miles), the equilibrium has the form of Case 1.1. The rent paid at the suburban fringe, r_0, anchors the utility achieved by poor households, and their utility does not change as the boundary expands. As the city radius increases from 0.5 to 3.5 (miles), poor households move from the suburb into the city to fill the new city land: poor households are willing to pay a premium for this move and the average rent increases accordingly. Because rich households are being pushed away from the center and have high commuting costs, their utility falls.

As the jurisdictional boundary moves outwards from 3.5 to 4.5 (miles), the city's area has grown sufficiently large that it can contain all poor households: the equilibrium structure changes to Case 1.2. As noted in Section 2, the level of the rent schedule in the city is now determined by the rent required to make a rich household indifferent between the communities. Rich households in the city need to be compensated for the lower public service, and average rents fall accordingly. The utility of poor households is no longer determined by what happens at the suburban edge; the lower city rent raises their utility. The utility of rich households is determined by the utility they obtain in the suburb: they are being pushed further out and their utility falls.

When *B* moves from 4.5 to 5.5 (miles), vacant land appears near the city boundary, and the equilibrium moves to the form of Case 1.3. Poor households located at the city's edge of development pay the reservation rent. The rent paid by poor households falls faster than their commuting costs increase. The overall effect is a decrease in average rent and an increase in utility of poor households. The utility of rich households continues to be determined by what happens at the suburban edge: their commuting costs are increasing and their utility falls.

Finally, when B increases from 5.5 to 6.5 (miles), rich households in the suburb are being pushed further out; this increases their willingness to pay to live in the city and average rents increase again. The rent of poor households is anchored by the reservation price of land. Poor households are being pushed further out and, with the rent at the edge of development fixed, their utility falls. Rich households in the suburb continue to incur higher commuting costs and the utility of rich households, determined by what happens in the suburb, continues to decrease.

3.2.2 Case 2: Rich households are majority in city; poor households are majority in suburb

Table 4 presents the simulation results for the equilibrium in which rich households are the city's majority and poor households are the suburban majority. When the jurisdictional boundary B is set to values between 0.5 and 4.5 (miles), Case 2.1 arises.

Metropolitan Population, *N = 270,000*

Attributes of metropolitan area	Symbol	Equilibrium values						
1. City jurisdictional boundary (miles from center)	*B*	0.5	1.5	2.5	3.5	4.5	5.5	6.5
2. Equilibrium Case Number (label from Section 2)		2.1	2.1	2.1	2.1	2.1	2.2	2.2
3. Average rent per household ($ per year)		793	868	1,019	1,246	1,548	566	557
4. Utility of poor households ($ per year)		10,792	10,792	10,792	10,792	10,792	10,729	10,571
5. Utility of rich households ($ per year)		33,397	33,397	33,397	33,397	33,397	35,768	35,768
6. Boundary between income groups in city (miles)	*x*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
7. Boundary of city development (miles)	*X*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
8. Boundary between income groups in suburb (miles)	*y*	5.07	5.07	5.07	5.07	5.07	5.5	6.5
9. Boundary of suburban development (miles)	*Y*	6.69	6.69	6.69	6.69	6.69	7.02	7.83
10. City population of poor households		0	0	0	0	0	0	0
11. City population of rich households		1,311	11,803	32,785	64,258	106,222	135,000	135,000
12. Poor household's utility change on moving from suburb to city ($/yr)		-3,425	-3,187	-2,950	-2,712	-2,475	-523	-365
13. Rich household's utility change on moving from city to suburb ($/yr)		0	0	0	0	0	-2,595	-3,130

Table 4: Case 2 equilibria as the city's boundary moves out.

As the city's boundary moves out from 0.5 to 4.5 (miles), rich households in the suburb move into the city: because rich households control the public service level in the city, rents on the city side of the jurisdictional boundary exceed rents on the suburban side. Average rents therefore increase. However, utilities do not change: the boundary between the rich and poor households in the suburb does not move, so the utilities of both rich and poor households stay anchored at their suburban levels.

When B is set to 5.5 and 6.5 miles, the city's area exceeds the land demanded by rich households and vacant land appears in the city. The equilibrium structure changes to Case 2.2. There are no longer rich households in the suburb bidding to get into the city, and rents in the city fall: the rent decrease increases the utility achieved by rich households. Of course, as B increases in this range, poor households commute further and their utility decreases.

4. WELFARE COMPARISONS

4.1 Efficiency comparisons

Because we have specified an utility function with the property that the marginal utility of income is unity, the "as if" shifting of income from absentee landlords to resident households does not change the total surplus. Therefore our measure of efficiency is the sum of the average utility of resident households plus the average land rent.

The first-best efficient outcome is to separate the rich and poor households into different communities, so that there is perfect sorting of households by their taste for the public service (fiscal sorting). Because commuting costs increase with income, rich households should be placed in the city and poor households should be placed in the suburb. The city boundary should be set at 5.1 (miles) and there should be marginal undeveloped city land. The efficiency measure (average utility plus average land rent) for this equilibria is 23,850 ($ per household per year).

If the jurisdictional boundary is not 5.1 miles, should the city have a majority of rich households or a majority of poor households? Relative efficiency is determined by considerations of commuting cost and by the total surplus gained from the public service. The potential trade-off is shown in Table 5 overleaf. Commuting costs are shown on Lines 2 and 6. Commuting costs are lower in Case 2: because rich households have higher commuting costs, commuting costs are lower when rich households live nearer the metropolitan center.

Lines 3 and 7 show the surplus obtained from the public service. If the city radius is small, the income class which lives in the city is also present in the suburb where it obtains a non-optimal public service level. In contrast, *all* households of the income class which forms the majority in the suburb live in the suburb and obtain their preferred public service level. We denote the preferred public service of poor (rich) households as g_1 (g_2). Our specific utility function implies that matching is more important for rich households: the benefit to a rich household of being matched with g_2 instead of with g_1 exceeds the benefit to a poor household of being matched with g_1 instead of with g_2 ; or

$$[\beta(M_2)V(g_2) - g_2] - [\beta(M_2)V(g_1) - g_1] > [\beta(M_1)V(g_1) - g_1] - [\beta(M_1)V(g_2) - g_2] .$$

In Case 1, when the city radius is small (4.4 miles or less), some poor households live in the city and enjoy their desired public service and some poor households live in the suburb, obtaining a higher-than-desired public service level; in contrast, all rich households live in the

Metropolitan population, *N = 270,000*

Attributes of metropolitan area	Equilibrium values						
1. City jurisdictional boundary, *B* (miles from center)	0.5	1.5	2.5	3.5	4.5	5.5	6.5
CASE 1 (poor are majority in city)*:*							
2. Average commuting cost ($ per year)	1,574	1,627	1,711	1,819	1,895	1,881	1,906
3. Average consumer surplus from public service ($ per year)	- 5,028	- 4,981	- 4,886	- 4,744	- 4,630	- 4,834	- 4,986
4. Efficiency measure (Average utility plus average rent) ($ per household per year)	23,398	23,394	23,403	23,437	23,475	23,284	23,109
5. Utility of poor households ($ per year)	9,891	9,891	9,891	9,891	10,905	11,109	11,035
CASE 2: (rich are majority in city)							
6. Average commuting cost ($ per year)	1,566	1,566	1,566	1,566	1,566	1,603	1,690
7. Average consumer surplus from public service ($ per year)	- 5,547	- 5,471	- 5,320	- 5,093	- 4,790	- 4,583	- 4,583
8. Efficiency Measure (Average utility plus average rent) ($ per household per year)	22,887	22,693	23,114	23,341	23,643	23,814	23,727
9. Utility of poor households ($ per year)	10,792	10,792	10,792	10,792	10,792	10,728	10,571
CITY MAJORITY PREFERRED FOR:							
10. City majority for efficiency	poor	poor	poor	poor	rich	rich	rich
11. City majority for equity	rich	rich	rich	rich	poor	poor	poor

Table 5: Efficiency comparison of Case 1 and Case 2 equilibria

suburb and enjoy their desired public service level. There is therefore a perfect match of rich households with their desired public service (and a bad match of some poor households with their desired public service). Conversely, in Case 2, some rich households live in the city, but some rich households and all poor households live in the suburb; matching with desired public services is therefore bad for rich households and perfect for poor households. Summarizing, for small cities, Case 1 has a better match of rich households to their desired public service; it also has higher commuting costs. Lines 4 and 8 in Table 5 present the efficiency measures for the equilibria achieved under Cases 1 and 2, as well as the number of poor and rich households living in the city at each boundary size. If the city radius is small (3.5 miles or less), efficiency is higher when the poor are the majority in the city. If the city boundary is 0.5 miles, the efficiency difference is 511 ($ per households per year). We conclude that, for small cities, good matching of rich households with their desired public service is more important than considerations of commuting cost.

As the jurisdictional boundary expands, the number of households living in the city increases, fewer suburban households are being mismatched with their desired public service, and the importance of matching decreases. However, for Case 1, commuting costs increase as all (or almost all) rich households are being forced to live further out. In consequence, the welfare difference between the two cases falls. When the jurisdictional boundary is 4.5 miles, in the Case 2 equilibrium 80% of rich households are able to live in the city and obtain their desired public service; commuting cost considerations dominate and efficiency is higher under Case 2.

For jurisdictional boundaries of 5.5 and 6.5 miles, Case 2 - with two separated communities - has perfect matching of households with their preferred public service, and lower commuting costs even if there is undeveloped land. Therefore higher efficiency is obtained with Case 2.

4.2 Equity comparisons

Our measure of equity is the Rawlsian welfare function, *max min* $[U_1, U_2]$ where U_i is the utility achieved by a household with income M_i.[20] Comparison of Lines 5 and 9 of Table 5 shows that the best outcome for poor households is for poor households to be the majority in the city (Case 1) and for the city boundary to be 5.5 miles. The utility of a poor household in this equilibria is 11,109 ($ per year) while the utility of a poor household in the first-best efficient outcome is only 10,792 ($ per year).

If the city radius is small (3.5 miles or less), Table 5 shows that poor households achieve utility 9,891 ($ per year) in the Case 1 equilibrium and 10,792 ($ per year) in the Case 2 equilibrium. Equity therefore places rich households in the city and makes poor households the majority in the suburb. An important difference between the efficiency and equity analyses concerns the treatment of rent. Efficiency is concerned with total surplus: any gain which accrues to landlords is included in the analysis and rent therefore is considered "as if" returned to households. Equity is concerned with the surplus accruing to poor households: rent paid by poor households is considered lost and not considered "as if" returned. If the city radius is 3.5 miles or less, there are poor households in the suburb in both equilibria. Poor households at the suburban fringe pay the same rent and have the same commuting costs in both equilibria. Under Case 2, poor households in the suburb obtain their preferred public service. Under Case 1, it is the poor households living in the city who receive their preferred public service, but they are prevented from getting the benefit by the competition of poor suburban households; the benefit is capitalized into land rents and paid to absentee landlords. Poor households have higher utility, therefore, in the Case 2 equilibrium.

If the city radius is large (4.5 miles or larger), Case 1 has all poor households living in the city and it is rich households who compete for city land. The low public service in the city makes living in the city relatively unattractive to rich households, creating a "barrier to entry" for rich households. This drives down the rent to benefit poor households. The benefit of good matching of poor households with their desired public service is no longer captured by landlords as rent. Case 1 is preferred for equity.

Our equity finding has a policy implication. Many U.S. metropolitan areas stylistically resemble Case 1.1 in which only poor households reside in the city and in which rich and poor households reside in the suburb. In this case, expanding the city boundary to include the near suburb (i.e. to shift the equilibrium from Case 1.1 to Case 1.2 or Case 1.3) is good policy for poor city households who rent as it increases their utility. It should be stressed that this utility gain accrues to poor households not because of tax shifting - all city households pay the same tax - but because of rent changes. Our model also suggests why such expansion is likely to be politically difficult: the price of land falls precipitously as it is incorporated into the city, so that the expansion is likely to be strongly opposed by the landlords who own this land (and who in practice are likely to be the rich households who live there).

4.3 Conflict between efficiency and equity

The discussion of the two previous subsections has shown that, if the city's boundary can be freely chosen, a boundary of near five miles is preferred for both equity and efficiency reasons. However, there is conflict as to which income class should form the city's majority. Efficiency favors the equilibrium in which rich households fill the city: there is perfect matching of households with their public service levels and commuting costs are minimized. In contrast, equity favors the equilibrium in which all the poor households live in the city: poor households have low commuting costs and obtain their desired public service level, and the presence of a few rich households in the city lowers city rents.

If the city boundary is fixed, Lines 10 and 11 of Table 5 show that there is a conflict between efficiency and equity at all the city sizes considered. This conflict is driven by the scarcity of city space. At small city sizes, efficiency stresses the importance of good matching of rich households with their preferred public service level and places rich households as the majority in the suburb. However, this arrangement is not favored for equity as it has high city rents, which prevent poor city households from capturing the benefit of their preferred public service. At large city sizes, all rich households can live in the city so that matching becomes unimportant and efficiency requires that rich households be the majority in the city to minimize commuting costs. However, this arrangement is not favored for equity as it increases the commuting cost of poor households.

The conflict between efficiency and equity is a general tendency and only in a narrow range of boundary sizes (in our simulations, for B lying between 3.94 and 4.4 miles) is there no conflict. In this narrow range both efficiency and equity are obtained if rich households are made the majority in the city. For efficiency to have rich households be the city's majority, the boundary must be sufficiently large that almost all rich households can live in the city; with only a few rich households living in the suburb, the cost of mismatching them with their preferred public service is dominated by the benefit of the lower commuting cost. For equity to have rich households be the city's majority, the boundary must be sufficiently small so that, if instead poor households were made the city's majority, there would be poor households living in the suburb, keeping city rents high.

5. CONCLUSION

This paper has examined a monocentric urban model in which the metropolitan area is divided into two jurisdictions -an inner city and a surrounding suburb - and there are two income classes. At each jurisdictional size, there are two equilibria: one in which poor households are the city's majority and one in which rich households are the city's majority. We find that there is often a conflict as to which equilibrium is preferred. For small inner cities, efficiency favors the poor being the city's majority and equity favors the rich being the city's majority. For large inner cities, the ordering is reversed. Because there is wide variation in the size of U.S. cities, we believe our model, although it is necessarily stylized, highlights important trade-offs in urban policy.

APPENDIX: FORMAL PRESENTATION OF THE MODEL

We denote the city as jurisdiction *c* and the suburb as jurisdiction *s*. If households of income M_i $(i = 1,2)$ with land demand a_i locate in jurisdiction j $(j = c, s)$, they achieve the same utility at all points *d* at which they locate, or

$$M_i - tM_i d - a_i r(d) - g + \beta(M_i)V(g) = constant .$$

Hence the land expenditure plus commuting cost paid by a household of income M_i locating at *d* in the community *j* is

$$b_{ij} = tM_i d + a_i r(d)$$

where b_{ij} is a constant.[21] Instead of solving for the rent function $r(d)$, it is convenient instead to solve for the constants b_{2c} , b_{1c} , b_{2s} and b_{1s}.

Rent is continuous in a jurisdiction. If both income levels live in the suburb, rent continuity at *y* implies

$$B < y < Y: \qquad \frac{b_{2s} - tM_2 y}{a_2} = \frac{b_{1s} - tM_1 y}{a_1} \quad .$$

If $y = B$, only poor households live in the suburb. If a rich household were to move to the suburb, he would achieve his highest utility by locating just across the jurisdictional boundary, and would pay rent $(b_{1s} - tM_1 B) / a_1$ In this case, we interpret $(b_{2s} - tM_2 B) / a_2$ as the rent he would pay if he were to move and therefore set :

$$B = y: \qquad \frac{b_{2s} - tM_2 y}{a_2} = \frac{b_{1s} - tM_1 y}{a_1}.$$

If $y = Y$, only rich households live in the suburb. If a poor household were to move to the suburb, he would achieve his highest utility by locating at the fringe of development, or pay rent $(b_{2s} - tM_2 Y)/a_2$. In this case we interpret $(b_{1s} - tM_1 Y)/a_1$ as the rent a poor household would pay if he were to move to the suburb, and we set:

$$y = Y: \qquad \frac{b_{2s} - tM_2 Y}{a_2} = \frac{b_{1s} - tM_1 Y}{a_1}.$$

Combining these cases, we can write:

$$B \leq y \leq Y: \qquad \frac{b_{2s} - tM_2 y}{a_2} = \frac{b_{1s} - tM_1 y}{a_1}. \tag{A.1}$$

Similarly, rent continuity at x in the city implies

$$0 \leq x \leq X: \qquad \frac{b_{2c} - tM_2 x}{a_2} = \frac{b_{1c} - tM_1 x}{a_1}, \tag{A.2}$$

where, if $x = 0$, b_{2c} / a_2 is interpreted as the rent a rich household would pay if he were to move to the city and, if $x = X$, $(b_{1s} - tM_1X) / a_1$ is interpreted as the rent a poor household would pay if he were to move to the city.

Denoted the public service in the city as g_c and the public service in the suburb as g_s Equilibrium requires that the rent schedule adjusts so that no household can obtain more utility by moving between jurisdictions. If $x = 0$, there are no rich households in the city or equilibrium requires that a rich suburban household obtains at least as much utility in the suburb as he could obtain if he were to move to the city, or

$$x = 0: \qquad M_2 - b_{2s} - g_s + \beta(M_2) V(g_s) \geq M_2 - b_{2c} - g_c + \beta(M_2) V(g_c). \tag{A.3a}$$

Similarly, if there are rich households in the city and in the suburb

$$0<x \text{ and } B<y: \quad M_2 - b_{2s} - g_s + \beta(M_2)V(g_s) = M_2 - b_{2c} - g_c + \beta(M_2)V(g_c); \qquad \text{(A.3b)}$$

and if there are no rich households in the suburb

$$B = y: \quad M_2 - b_{2s} - g_s + \beta(M_2)V(g_s) \leq M_2 - b_{2c} - g_c + \beta(M_2)V(g_c). \qquad \text{(A.3c)}$$

Similar equations apply for poor households:

$$x = X: \quad M_1 - b_{1s} - g_s + \beta(M_1)V(g_s) \geq M_1 - b_{1c} - g_c + \beta(M_1)V(g_c); \qquad \text{(A.4a)}$$

$$x<X \text{ and } y<Y: \quad M_1 - b_{1s} - g_s + \beta(M_1)V(g_s) = M_1 - b_{1c} - g_c + \beta(M_1)V(g_c); \qquad \text{(A.4b)}$$

$$y=Y: \quad M_1 - b_{1s} - g_s + \beta(M_1)V(g_s) \leq M_1 - b_{1c} - g_c + \beta(M_1)V(g_c). \qquad \text{(A.4c)}$$

The rent paid at the limit of development in the suburb is the reservation rent r_0. If the suburb contains poor households,

$$y < Y: \quad \frac{b_{1s} - tM_1 Y}{a_1} = r_0;$$

if the suburb contains only rich households

$$y = Y: \quad \frac{b_{2s} - tM_2 y}{a_2} = r_0.$$

Using Equation (A.1), this is collapsed to the single relationship:

$$y \leq Y: \quad \frac{b_{1s} - tM_1 Y}{a_1} = r_0. \qquad \text{(A.5)}$$

Similarly, if there is undeveloped land in the city, the rent at the limit of development must be the reservation rent r_0. If there is no undeveloped land, the rent at the city's boundary must equal or exceed r_0. If the city contains only rich families

$$x = X < B: \qquad \frac{b_{2c} - tM_2X}{a_2} = r_0 \; ;$$

$$x = X = B: \qquad \frac{b_{2c} - tM_2B}{a_2} \geq r_0 \; .$$

If the city contains poor households

$$x < X < B: \qquad \frac{b_{1c} - tM_1X}{a_1} = r_0;$$

$$x < X = B: \qquad \frac{b_{1c} - tM_1B}{a_1} \geq r_0.$$

Using Equation (A.2), these equations are collapsed to

$$X < B: \qquad \frac{b_{1c} - tM_1X}{a_1} = r_0; \tag{A.6a}$$

$$X = B: \qquad \frac{b_{1c} - tM_1B}{a_1} \geq r_0. \tag{A.6b}$$

The public service in each community is determined by voting. We assume that households vote myopically by taking the rent schedules as given. If the suburb has a majority of poor households, voting sets the public service in the suburb g_s to maximize their utility or

$$\frac{\pi(Y^2 - y^2)}{a_1} \geq \frac{\pi(y^2 - B^2)}{a_2}: \qquad \beta(M_1)V'(g_s) = 1 \; . \tag{A.7a}$$

Conversely, if the suburb has a majority of rich households, voting sets the public service level in the suburb as

$$\frac{\pi(Y^2 - y^2)}{a_1} < \frac{\pi(y^2 - B^2)}{a_2}: \qquad \beta(M_2)V'(g_s) = 1 . \tag{A.7b}$$

Similarly in the city the public service level g_c is set by the majority as

$$\frac{\pi(X^2 - x^2)}{a_1} \geq \frac{\pi x^2}{a_2}: \qquad \beta(M_1)V'(g_c) = 1 ; \tag{A.8a}$$

$$\frac{\pi(X^2 - x^2)}{a_1} < \frac{\pi x^2}{a_2}: \qquad \beta(M_2)V'(g_c) = 1 . \tag{A.8b}$$

Summarizing, the variables to be solved for are: b_{2c} , b_{1c} , b_{2s} , b_{1s} , g_c, g_s, x, X, y, and Y. The equation system corresponds to the Equations (2) - (3) and (A.1) - (A.8). The equilibrium outcome is a function of the city boundary B and the other parameters.

The solution to the equation system is only a solution if the ordering of distances satisfies

$$0 \leq x \leq X \leq B \leq y \leq Y .$$

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ENDNOTES

1. Ross and Yinger (1999) survey this literature.

2. Inefficiencies include the impact of the property tax on housing consumption and the influence of community heterogeneity on voting outcomes.

3. Wheaton (1976) and Sasaki (1990) provide a comparative static analysis of this equilibrium.

4. Mills (1967), Muth (1969), and Mills and Lubuele (1997) suggest that this sorting mechanism is a key factor in the concentration of the poor into U.S. central cities and in the resulting low level of public-service quality in those cities. On the other hand, Wheaton (1977) and Glaeser, Kahn, and Rappaport (2000) find evidence that the income elasticity of land demand is quite small and as a result conclude that this type of sorting cannot play a major role in explaining the centralization of the poor.

5. The assumption of fixed housing size greatly simplifies the problem and allows us to avoid well-known existence problems associated with stratified local public-finance equilibria (see Epple, Filimon and Romer (1984, 1993)).

6. For ease of presentation, the jurisdiction is assumed to provide a public service and not a public good. The public service shows constant returns to community size. It is straightforward to change the publically-provided good from a public service to a public good.

7. This assumption is consistent with the findings of Glaeser, Kahn, and Rappaport (2000).

8. A formal proof is provided in de Bartolome and Ross (2000).

9. This assumption simplifies the model. We do not believe that our results depend on this assumption - what is important is that the two jurisdictions vote different public service levels.

10. Because of the specific form of the utility function, it is straightforward to change this assumption to allow each household to receive an equal share of the total rent paid as a lump-sum transfer. To do this, simply sum the average rent and the quoted utility levels in Tables 3 and 4. The basic conclusion is unchanged that there is often a conflict between efficiency and equity. We prefer to have rents paid to absentee landlords as the alternative suggests that households in one community benefit from property value increases in the other community.

11. There are of course many potential configurations. A list of all such possible equilibria is available from the authors on request. de Bartolome and Ross (2000) show that an equilibrium with strict income sorting (Case 2) always exists. However in our simulations at each value of B we found only one equilibrium with a majority of poor households in the city, and only one equilibrium with a majority of rich households in the city. We confine our discussion to the equilibria we found.

12. If the boundary continues to be moved outwards, rich households in the suburb continue to be pushed outwards and their commuting costs increase. In consequence, some rich households

migrate to the city. When *B* is sufficiently large, all rich households move to the city and the suburb is uninhabited.

13. As the city's boundary continues to expand, poor households get pushed further and further out, incurring higher commuting costs and at a critical boundary size they start to migrate back into the city. If the boundary is very large, all households live in the city and the suburb is uninhabited.

14. In many studies, ρ is required to be positive, which implies a price elasticity that exceeds unity. We generalize the utility function to allow for negative values of ρ and price elasticities that are less than one.

15. Sensitivity analysis was performed using a lower and higher value (0.1 and 0.5) for the implied income elasticity of land demand, and there was little effect on the results.

16. This figure is a crude average and ignores possible variations in the number of households living in low- and high- service communities.

17. Societal figure calculated as local government expenditure financed from local government own-revenue (*Census of Governments Volume 4 Number 5* 1992, Table 3) divided by personal income times one minus the average federal income tax rate (*Individual Income Tax Returns* 1990, Table 1.1).

18. Using the labels described later in the text, the equilibrium at $B = 4.5$ miles in Table 3 changed from Case 1.2 to Case 1.3.

19. This is the average rent paid per household. If the value quoted in the table is T ($ per household per year): because the average lot size is 0.3333 (acres per household), the average rent per acre per year is $3T$. Because the reservation rent has been set to zero, this is interpreted as the average rent premium (above the reservation rent) paid for one acre of land in the metropolitan area.

20. Equity is normally interpreted as placing more weight on the welfare of the poor. For convenience of exposition, we take an extreme welfare function with all weight being placed on the welfare of the poor.

21. If households of income M_i do not live in the community j, we interpret below b_{ij} to be the rent plus commuting cost which a household of income M_i would pay if he were to move into the community j.