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**Consistent Targets and Optimal Monetary Policy: Conservative
Central Banker Redux**

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Abstract

Kydland and Prescott (1977) consider the issue of the time-inconsistency of optimal policy and its source. Our paper provides additional insight on this issue. They develop a simple model of monetary policy making, where the central bank needs some commitment technique to achieve optimal monetary policy over time. Although not their main focus, they illustrate the difference between consistent and optimal policy in a sequential-decision one-period world. In our solution, the government appoints a central bank or delegates to the central bank an objective function that differs from the social welfare function. The central bank's welfare function causes the consistent policy implemented by the central bank to prove optimal for society. The optimal institutional design for the Kydland-Prescott sequential-decision one-period model requires the appointment or delegation to a completely conservative central banker.

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1. Introduction

The economic literature considers the optimality and consistency of decision making. Optimal plans lead inextricably to inconsistencies. An important part of this literature examines the optimality and consistency of monetary policy.

Kydland and Prescott (1977) launch this whole literature by showing that optimal policy proves inconsistent because of rational expectations. They argue that the central bank needs some commitment technique to achieve optimal monetary policy over time. Absent the commitment technique, optimal monetary policy proves time inconsistent.

Barro and Gordon (1983a) provide another important benchmark in the development of this literature, by modeling the verbal and graphical story in Kydland and Prescott (1977).¹ Because of rational expectations, an inflation bias prevails under discretion (consistent policy), even though the optimal policy equals zero inflation. Based on Barro and Gordon's (1983a) monetary model, much of the literature provides solutions to the inconsistency problem in monetary policy.

Before considering our method, we briefly review and define the concepts of optimal and consistent policies. Then, we compute optimal policy and consistent policy in the simple Kydland-Prescott sequential-decision one-period model, using control-theory and game-theory solution techniques. We find that the source of inconsistency comes from an *inconsistent weight* in the social loss function. That is, the relative weight on the trade-off between deviations of the inflation and unemployment rates from their targets in the social loss function proves inconsistent. As a result, the central bank should not adopt the social loss function as its own.

¹ Barro and Gordon (1983a) modify the social objective function, making both the deviations of inflation and unemployment from target quadratic terms, whereas the implied model in Kydland and Prescott (1977) enters the deviation of unemployment from target as a linear, and not quadratic, term. The model in Barro and Gordon (1983b), however, encompasses the verbal and graphical monetary model in Kydland and Prescott (1977).

Accordingly, we design the central bank loss function with a consistent weight. Under the designed loss function, the optimal policy and consistent policy prove identical. The optimal institutional design for the Kydland-Prescott sequential-decision one-period model requires the appointment or delegation of a completely conservative central banker, an extreme version of the Rogoff (1985) solution.

Returning to Kydland and Prescott (1977), they illustrate the time inconsistency of optimal policy, whereby in an intertemporal model. But, they also illustrate the difference between consistent and optimal policy in a sequential-decision, one-period framework, where that difference hinges on whether the central bank incorporates how the private sector responds to changes in central bank actions. In other words, Kydland and Prescott (1977) insightfully note the source of the inconsistency of optimal policy—“economic planning is not a game against nature but, rather, a game against rational economic agents” (p. 473).

Our paper provides additional insight on this issue of the inconsistency of optimal policy. In our solution, the government appoints a central bank with the correct (optimal) objective function that includes a consistent weight or delegates to the central bank that correct (optimal) objective function, which causes a convergence of the consistent to the optimal monetary policy.²

2 Basic Model

Kydland and Prescott (1977, pp. 477-480) develop a one-period exposition of their point about the difference between consistent and optimal policy with their inflation-unemployment model.

They begin with an expectations augmented Phillips curve as follows:

² In Yuan, Miller, and Chen (2009) consider this issue in the Barro-Gordon (1983a) model. There, the problem does not relate to an incorrect weight. Rather, inconsistent targets prove the cause of the problem of achieving optimal policy. In this paper, the targets of policy in the social welfare function prove consistent.

$$(1) \quad u_t = \lambda(x_t^e - x_t) + \tilde{u},$$

where u equals the unemployment rate, x equals the inflation rate, x^e equals the expected inflation rate, and $\tilde{u}$ equals the natural rate of unemployment.

Next, they impose rational expectations or, given that no stochastic elements exist in the model, perfect foresight on the formation of inflation expectations. Thus,

$$(2) \quad x_t^e = Ex_t = x_t.$$

They verbally describe the social objective function, given as follows:

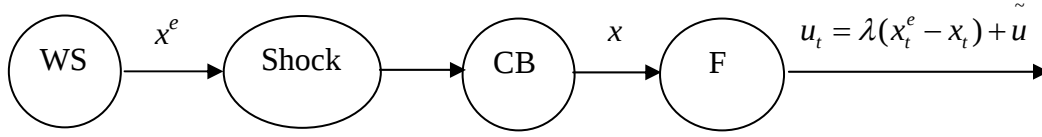
$$(3) \quad S(x_t, u_t),$$

which they illustrate in Figure 1 (Kydland and Prescott, 1977, p. 479). The following social loss function captures the characteristics of their description and Figure 1:

$$(4) \quad L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u}),$$

where α equals the weight that society gives to combating inflation versus unemployment relative to their targets of zero inflation and natural unemployment rates. The private sector's behavioral equations (1) and (2), the firm (F) and the wage setter (WS), respectively, conform to the following sequence of events. The wage setter and the firm sign a wage contract, where the wage setter sets the nominal wage and the firm sets the quantity of labor that it hires. After signing the wage contract, a (negative) shock occurs. Then the central bank (CB) implements its policy decision, x , minimizing the social loss function. Since the contract fixes the nominal wage, the wage setter must form a rational expectation of the inflation rate, x^e , before setting the wage rate, contingent on that inflation forecast (i.e., the wage setter uses behavioral equation 2). Finally, given the firm's decision, a certain unemployment level emerges from the firm's behavioral equation (1). The timing of the sequential decisions in this one-period model unfolds

according to the following chart:



Consistent Policy

We solve this one-period game with perfect information through backward induction. The firm behaves according to equation (1). The central bank minimizes the social loss by setting the inflation rate, x , given the wage setter's expectation of inflation rate, x^e , and shocks, if any occur. The key assumption to this optimization is that the central bank takes the wage setter's expected inflation rate as given and does not try to incorporate how the central bank's policy may affect the wage setter's expected inflation forecast.

The optimizing problem equals the following:

$$\begin{aligned}
 (5) \quad & \underset{x}{\text{Min}} \quad L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u}) \\
 & \text{Subj to} \quad \begin{cases} u_t = \lambda(x_t^e - x_t) + \tilde{u}; \text{ and} \\ x_t^e = x_t. \end{cases}
 \end{aligned}$$

Since the central bank takes the expected inflation rate as given, we do not endogenize the expected inflation rate by substituting the second constraint into the social loss function. We only use the first constraint.

The first-order condition for an optimum, assuming that the central bank takes the expected inflation rate as given, yields the following:

$$(6) \quad \frac{\partial L_t^S}{\partial x_t} = 2\alpha x_t - 2\lambda = 0.$$

Thus, the inflation rate that emerges as the solution to this problem equals the following:

$$(7) \quad x_t = \lambda / \alpha.$$

With perfect information and rational expectations, the wage setter forms its expectation as follows:

$$(8) \quad x_t^e = \lambda / \alpha.$$

As a result, the realized unemployment level and the social loss equal the following:

$$(9) \quad u_t = \tilde{u} \text{ and } L_t^S = \lambda^2 / \alpha.$$

This solution implicitly matches that in Kydland and Prescott (1977).

In sum, equilibrium or consistent policy, as defined by Kydland and Prescott (1977), produces an inflationary bias at the natural rate of unemployment. As such, the result matches the Barro and Gordon (1983a, 1983b) bias.

Optimal Policy

Optimal policy chooses the inflation rate to minimize the social loss function subject to the Phillips curve and the perfect foresight inflation rate of the private sector, where the central bank now does consider how the perfect foresight inflation rate of the wage setter responds to the central bank's policy choice. That is, we substitute the Phillips curve and the perfect foresight inflation rate into the social loss function and take the derivative with respect to the inflation rate.

The optimization problem equals the following:

$$(10) \quad \begin{aligned} \underset{x}{Min} \quad & L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u}) \\ \text{Subj to} \quad & \begin{cases} u_t = \lambda(x_t^e - x_t) + \tilde{u}; \text{ and} \\ x_t^e = x_t. \end{cases} \end{aligned}$$

The first-order condition for an optimum, assuming that the central bank takes the expected inflation rate as endogenously responding to its decision, yields the following:

$$(11) \quad \frac{\partial L_t^S}{\partial x_t} = 2\alpha x_t = 0.$$

The solution of the optimization yields the following:

$$(12) \quad x_t = 0; \quad x_t^e = 0; \quad u_t = \tilde{u}; \quad \text{and} \quad L_t^S = 0.$$

The optimal policy outcomes do not include an inflationary bias and the social loss equals zero, smaller than the social loss under consistent policy.

Therefore, the inconsistency of optimal policy exists in this simple model. The inconsistency emerges because the central bank shares the same loss function with society, which we show in the next section reflects an inconsistent weight. Given the economic structure, equations (1) and (2), the unemployment level always equals the natural rate. The central bank, however, with a loss function linear in the unemployment rate decreases the unemployment rate to as low a level as possible³ and, thus, always possesses the incentive to inflate. At the same time, the central bank uses a zero inflation rate target. Thus, the central bank faces the trade-off of reducing its loss by lowering the unemployment below the natural rate at the cost of higher inflation, which conflicts with the target inflation and unemployment rates of zero and the natural rate.

In addition, the inflation bias, λ/α , persists, despite the targets of a zero inflation rate and the natural unemployment rate. Logic dictates that the government delegate to the central bank achievable targets. We note that the optimal policy outcomes prove consistent with the targets in the social welfare function. Thus, the difference between the consistent and optimal outcomes must come from another source, that is, the relative weight on the deviations of the inflation rate

³ The loss function implies that the loss decreases as the unemployment rate moves toward minus infinity. As a practical matter, negative unemployment cannot occur and the actual unemployment rate proves positive because of the penalty imposed by higher inflation as the unemployment rate falls.

and the unemployment rate from their targets.

In sum, the government should choose a central bank, or delegate to the central bank, a loss function that differs from the social loss function. But, how precisely should the delegated loss function differ from the social loss function? The next section answers this question.

3. Optimal Objective and Consistent Targets

As we shall see, the central bank confronts the problem of an inconsistent weight in the above optimization when delegated the social loss function contained in equation (4). We hypothesize a central bank loss function of the same form, but with potentially different choices for the target values and the trade-off parameter. We then choose the optimal targets and trade-off parameter for the central bank loss function that minimizes the social loss function. The following system captures the complete problem:

$$(13) \quad \begin{aligned} \min_{a^*, x^*, u^*} L_t^S &= \alpha(x_t)^2 + 2(u_t - \tilde{u}) \\ \text{Subj to: } &\begin{cases} \min_{x_t} L_t^{CB} = \alpha^*(x_t - x^*)^2 + 2(u_t - u^*) \\ \text{Subj to: } \begin{cases} u_t = \lambda(x_t^e - x_t) + \tilde{u}; \text{ and} \\ x_t^e = x_t. \end{cases} \end{cases} \end{aligned}$$

The optimization proceeds in two steps.⁴ First, we minimize the central bank loss function subject to the Phillips curve and the perfect foresight inflation rate of the private sector, but where the central bank does not consider how the perfect foresight inflation rate responds to its policy choice (i.e., the central bank employs consistent policy). That is,

⁴ Hughes Hallett and Weymark (2005, 2006) present a similar two-stage optimization. They model the government as setting the degree of central bank independence and the weight on output fluctuations in the central banks loss function at optimal levels. Then, in the second stage, the government and the central bank choose fiscal and monetary policies in a non-cooperative game, where each player takes as given the policy choice of the other player.

$$\begin{aligned}
(14) \quad & \min_{x_t} L_t^{CB} = \alpha^* (x_t - x^*)^2 + 2(u_t - u^*) \\
& \text{Subj to: } \begin{cases} u_t = \lambda(x_t^e - x_t) + \tilde{u}; \text{ and} \\ x_t^e = x_t. \end{cases}
\end{aligned}$$

The first-order condition for an optimum, assuming that the central bank takes the expected inflation rate as given, yields the following:

$$(15) \quad \frac{\partial L_t^S}{\partial x_t} = 2\alpha^* (x_t - x^*) - 2\lambda = 0.$$

Solving for the equilibrium values produces the following outcomes:

$$(16) \quad x_t = x^* + \lambda / \alpha^*;$$

$$(17) \quad x_t^e = x^* + \lambda / \alpha^*;$$

$$(18) \quad u_t = \tilde{u}; \text{ and}$$

$$(19) \quad L_t^S = \alpha \left(x^* + \lambda / \alpha^* \right)^2$$

These outcomes correspond to the findings for consistent policy in the prior section, when the target inflation rate equals zero (i.e., $x^* = 0$) and $\alpha = \alpha^*$.

Second, we substitute the solutions from the first optimization into the social loss function and optimize the social loss function by choosing optimal values for the target inflation rate, the target unemployment rate, and the trade-off parameter. That is,

$$\begin{aligned}
(20) \quad & \min_{\alpha^*, x^*, \tilde{u}} L_t^S = \alpha(x_t)^2 + 2(u_t - \tilde{u}) \\
& \text{Subj to: } \begin{cases} x_t = x^* + \lambda / \alpha^*; \text{ and} \\ u_t = \tilde{u}. \end{cases}
\end{aligned}$$

Now, the second optimization reduces to the following:

$$(21) \quad \min_{\alpha^*, x^*, u^*} L_t^S = \alpha \left(x^* + \lambda / \alpha^* \right)^2.$$

The first-order conditions for an optimum yield the following:

$$(22) \quad \frac{\partial L_t^S}{\partial x^*} = 2\alpha \left[x^* + \lambda / \alpha^* \right] = 0; \text{ and}$$

$$(23) \quad \frac{\partial L_t^S}{\partial \alpha^*} = 2\alpha \left[x^* + \lambda / \alpha^* \right] \left(-\lambda / \alpha^{*2} \right) = 0.$$

The target unemployment rate does not appear in the social loss function, once the central bank optimizes. Thus, no first-order condition exists for the unemployment target.

Optimization with respect to the target inflation rate (i.e., x^*) and the trade-off parameter (i.e., α^*) gives an infinite number of combinations for the target inflation rate and the trade-off parameter as follows:

$$(24) \quad x^* = -\lambda / \alpha^*, \text{ where } \alpha^* \neq 0.$$

In other words, the target inflation rate does not equal zero. Rather, it equals a negative value that exactly off-sets the in the consistent policy solution (i.e., $x_t = x^* + \lambda / \alpha^*$). This outcome proves similar to Svensson's (1997) inflation targeting solution. Substituting the solution for the target inflation rate into the solutions of the first-stage optimization produces the overall solution as follows:

$$(25) \quad x_t = 0; \ x_t^e = 0; \ u_t = \tilde{u}; \text{ and } L_t^S = 0.$$

In other words, consistent policy in the Kydland and Prescott (1977) definition yields the optimal policy, when the central banker receives the delegated optimal targets and trade-off parameter as follows:

$$(26) \quad x^* = -\lambda / \alpha^*; \text{ and } \alpha^* \succ 0.$$

Note again that an infinite number of combinations of the inflation rate and the trade-off parameter exist. Can we reduce the degree of uncertainty and pin down precise values? We need another condition. We impose consistent targets, where consistent targets, as opposed to consistent policy, mean that the expected values of the target variables equal the targeted values. That is,

$$(27) \quad Ex_t = x^* \text{ and } Eu_t = u^* .$$

Thus,

$$(28) \quad \alpha^* = \infty, x^* = 0, \text{ and } u^* = \tilde{u} .$$

The first condition in equation (27) pins down the value of α^* , which must equal infinity, so that $x^* = Ex_t = 0$ (see equation 25). That is, the optimal central bank loss function only includes the square of the deviation of the inflation rate from its target of zero. Deviations of unemployment from its target play no role in the minimization of the central bank's loss function. The central bank targets the inflation rate. As a side result, the consistent targets minimize the central bank loss function, in this case $L_t^{CB} = 0$ (see the equation in model 14). Finally, once $\alpha^* = \infty$, no need exists for the delegated central bank loss function to specify a target unemployment rate (u^*).

If society desires an unemployment rate as low as possible, as in equation (4), then the optimal central bank loss function cannot exhibit any weight on the deviations of the unemployment rate from its target, otherwise the central bank always possesses an incentive to inflate. As a result, the optimal loss function for the central bank puts infinite weight on the inflation component relative to the unemployment component (or zero weight on the unemployment component relative to the inflation component) to counteract the social loss function.

This finding may appear obvious. That is, to make consistent policy, optimal, just

eliminate the unemployment component of the loss function. But remember, we illustrate our method within a simple model. Simple models lead to obvious solutions. More complex models do not necessarily lead to intuitively obvious outcomes (see Yuan, Miller, and Chen, 2009).

5. Conclusion

Economists continue to struggle with the consistency of optimal plans. Kydland and Prescott (1977) highlight the time consistency of optimal economic policy, showing that consistent policy proves non-optimal, in a game-theory framework. Our paper applies a general method for making consistent policy, optimal.⁵ We illustrate the method in the simple Kydland and Prescott (1977) sequential-decision, one-period framework. We adopt institutional design or central bank delegation to solve the inconsistency problem. That is, we propose that society (government) delegates an objective function to the central banker that differs from the social loss function. As such, consistent policy becomes optimal.

The control-theory solution provides the benchmark optimal policy with which we evaluate the consistent policy emerging from our delegated (designed) central bank loss function. Carefully designing the central bank's loss function can make optimal policy and consistent policy identical. This desirable result requires an understanding of two points. First, the social loss function comes out of a normative process. The social loss function only provides a criterion for designing a public institution, not the direct loss function for this specific institution.⁶ Second, the economic structure proves key to determining the central bank loss function, since an optimal loss function for the central bank must depend on the structure of the economy.

⁵ Yuan, Miller, and Chen (2009) provide details about the general solution method.

⁶ A popular view uses the representative household's utility as the social welfare criterion. This view, however, does not permit differences between private and social interests. We argue that the social welfare criterion captures a normative problem in philosophy. Our method requires that the delegated central bank welfare function must differ from the social welfare function.

In the present paper, the problem with the social loss function relates to the relative weight on deviations of the inflation and unemployment rates from their target values. The target values themselves prove consistent. In Yuan, Miller, and Chen (2009), the social welfare function incorporates inconsistent targets and the relative weight proves optimal.

The optimal institutional design requires a central bank loss function that only includes deviations of the inflation rate from its zero target value. Deviations of the unemployment rate from its target do not appear. In other words, the optimal institutional design requires a completely conservative central banker, an extreme version of the Rogoff (1985) solution.

What moral can we draw from our story? If the social welfare criterion incorporates an inconsistent weight, then the central bank can improve social welfare by adopting a different welfare criterion with a consistent weight, a second-best solution. The first-best outcome occurs if society replaces its weight-inconsistent welfare function with a weight-consistent welfare function. The third-best solution occurs when the central bank uses the weight-inconsistent social welfare criterion as its own.

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