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The Making of Optimal and Consistent Policy: An Implementation Theory Framework for Monetary Policy

Huiping Yuan
Xiamen University

Stephen M. Miller
University of Connecticut and University of Nevada, Las Vegas

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341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Abstract

This paper shows that optimal policy and consistent policy outcomes require the use of control-theory and game-theory solution techniques. While optimal policy and consistent policy often produce different outcomes even in a one-period model, we analyze consistent policy and its outcome in a simple model, finding that the cause of the inconsistency with optimal policy traces to inconsistent targets in the social loss function. As a result, the social loss function cannot serve as a direct loss function for the central bank. Accordingly, we employ implementation theory to design a central bank loss function (mechanism design) with consistent targets, while the social loss function serves as a social welfare criterion. That is, with the correct mechanism design for the central bank loss function, optimal policy and consistent policy become identical. In other words, optimal policy proves implementable (consistent).

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1. Introduction

The economic literature contains a strand that focuses on the optimality and consistency of decision making. Optimal plans lead inextricably to inconsistencies. An important part of this literature examines the optimality and consistency of macroeconomic policy, especially monetary policy.

Kydland and Prescott (1977) launch this whole literature by arguing that optimal policy proves inconsistent and showing that the inconsistency results from rational expectations. In a simple model of monetary policy making, the central bank needs some commitment technique to achieve optimal monetary policy over time. Absent the commitment technique, optimal monetary policy proves time inconsistent. The Kydland and Prescott (1977) thesis focuses on intertemporal issues and the need for commitment. While most of their analysis considers an intertemporal model, they do explore the issues within a simple sequential decision, one-period model.

Barro and Gordon (1983a) build an analytical model for analyzing the inconsistency issue of monetary policy, by modifying a verbal and graphical model in Kydland and Prescott (1977).[1] Because of rational expectations, an inflation bias prevails under discretion (consistent policy), even though the optimal policy equals zero inflation. Barro and Gordon (1983b) prove that reputation can provide the commitment technique necessary to make consistent policy, optimal, under certain conditions.

Based on the Barro and Gordon (1983a) standard monetary model, much of the literature provides solutions to the inconsistency problem in monetary policy. Before considering our

[1] Barro and Gordon (1983a) modify the social objective function, making both the deviations of inflation and unemployment from target quadratic terms, whereas the implied model in Kydland and Prescott (1977) enters the deviation of unemployment from target as a linear, and not quadratic, term. But, the model in Barro and Gordon (1983a) encompasses the verbal and graphical monetary model in Kydland and Prescott (1977).

solution, we first review and define the concepts of optimal and consistent policies. Then, we compute optimal policy and consistent policy in a simple model, using control-theory and game-theory solution techniques. By analyzing consistent policy and its outcome, we find that the source of inconsistency comes from the social loss function, whose two targets, the inflation rate and the employment level, prove inconsistent. Reconsidering the role of the social loss function, we argue that the social loss function cannot serve as a direct loss function for the central bank. Accordingly, we employ implementation theory to design a central bank loss function (mechanism design), while the social loss function serves as a social welfare criterion. That is, with the correct mechanism design for the central bank loss function, optimal policy and consistent policy become identical. In other words, optimal policy proves implementable (consistent).

More specifically, implementation theory considers how to design institutions (mechanism design) to achieve a socially desirable outcome, given that participants in that society interact with each other and may send false signals.[2] In other words, implementation theory studies how individuals interact within a designed institutional structure to produce the outcomes that achieve the social optimum.

Game theory provides the standard framework for examining issues of implementation theory. Usually, game theory problems examine how players respond within a given game structure. Implementation theory asks a broader question of how to design the game structure to achieve socially optimal outcomes.

In our context, the central bank's loss function captures the central bank mechanism. That is, we design the central bank's loss function so that the equilibrium outcome (i.e., consistent

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[2] Jackson (2001) provides a "crash course" on the theory of implementation.

outcome)[3] resulting from the game between the central bank and the economy, proves optimal, according to the given social welfare criterion. In our context, the social welfare criterion equals the minimization of the social loss function.

Our paper demonstrates, using implementation theory, that appointing a central bank with the correct (optimal) objective function or delegating to the central bank that correct (optimal) objective function will cause a convergence of the consistent to the optimal monetary policy. That is, the correct (optimal) objective function equals the mechanism design that achieves the optimal policy. We apply our method to several different variants of the simple sequential decision, one-period Barro and Gordon (1983a) model with identical results.

The paper unfolds as follows. Section 2 provides a brief review of the inconsistency of optimal plans. Section 3 develops the simple Barro and Gordon (1983a) type model and illustrates how consistent policy proves non-optimal. Section 4 discusses the design of the central bank loss function with implementation theory and demonstrates that central bank mechanism design achieves optimal and consistent policy. Section 5 concludes.

2. Optimal Policy and Consistent Policy: A Review

Strotz (1955-56) first identified the inconsistency of optimal plans. Afterwards, much literature illustrates its existence, attempts to determine its sources or causes, and offers its solutions. To review this literature, we first clarify the concepts of optimal and consistent plans, making it easier to understand the inconsistency of optimal plans.

Definitions of Optimal and Consistent Policy

For optimal plans, the existing literature employs the same implicit definition, but uses different terms, for example, "commitment optimum path" (Pollak, 1968) and "Ramsey policy" (Chari,

[3] See section 2 for more details.

1988). An optimal plan, defined by Strotz (1955-56), implies that an individual chooses over some future period of time to maximize the utility of the plan, evaluated in the present. The individual's choice, of course, satisfies certain constraints. Strotz's definition applies to one-person decision problems. In a game-theory model, especially a model with a social planner (e.g., the government or the central bank), we define the *optimal policy* as the social planner's *ex ante* plan that, if implemented, produces a Pareto efficient outcome, according to some social welfare criterion.

Now, consider a consistent plan. Strotz (1955-1956) defines a consistent plan as the best plan among those that an individual will actually follow. Pollak (1968) argues, however, that Strotz's consistent plan, which corresponds to Pollak's "naïve optimum path", could not actually occur. Pollak uses another term, "sophisticated optimum path," for the correct definition of a consistent plan. The sophisticated optimum path captures the same idea as subgame perfect equilibrium and/or sequential equilibrium in game-theory models, though Pollak's model encompasses only a one-person decision problem. Kydland and Prescott (1977) define consistent policy much like Pollak's "naïve optimum path." Kydland (1977) suggests that "operational characteristics of economics models … point strongly toward an equilibrium concept for dynamic dominant-player models … This solution is called the feedback solution … it has the property that the original plan is consistent under replanning." Chari and Kehoe (1989) define time consistent policy as a sustainable plan. Sustainability closely relates to subgame perfection and sequential equilibrium. In sum, in macroeconomic models with more than one decision maker, we define a *consistent policy* as the government's (or the central bank's) plan, which, together with the strategies of other decision makers, constitutes equilibrium. The equilibrium can include a Nash equilibrium, a subgame perfect equilibrium (Selten, 1965), or a sequential equilibrium (Kreps and Wilson, 1982). At this point, we do not discuss, in detail, which equilibrium concept proves more appropriate, because different equilibrium concepts correspond to different types of game-theory models. Also, the concept of equilibrium continuously evolves. Loosely speaking, equilibrium contains a strategy profile that results in "an outcome that satisfies mutually consistent expectations." (Shubik, 1998, p. 6)

Existence and Sources of Inconsistency

With clear definitions of optimal and consistent plans, we can now more easily grasp the nature of the inconsistency of optimal plans from the perspective of game theory, where equilibria often prove Pareto inefficient. Now, given that optimal plans generally prove inconsistent, the literature studies the sources of inconsistency. For one-person decision problems, inconsistency may arise from an "intertemporal tussle" (Strotz, 1955-1956) and the specific form of the utility function.[4] Thus, for example, Calvo (1978a), Rodriguez (1981), and Leininger (1985) show that consistent and optimal plans exist in an important class of economies with special forms for the utility function (e.g., stationary period or instantaneous utility). Other researchers, such as Dasgupta (1974), demonstrate that an inadequate social welfare criterion can lead to inconsistent optimal plans. In sum, inconsistency can occur for different specific reasons in different specific models. This view, we argue also applies to more-than-one-person decision problems (i.e., game-theory models).

Kydland and Prescott (1977) first identify the inconsistency that resulted from rational expectations.[5] Rational expectations imply the important notion of equilibrium in game theory. As defined above, an equilibrium outcome reflects rational players' mutually consistent

[4] Actually, the "intertemporal tussle" in Strotz (1955-1956) results from the non-exponential discount function, which also captures a specific functional form of utility.

[5] Calvo (1978b) independently identifies this point.

expectations. We argue that they correctly recognize the source of inconsistency of the optimal plan. Then Kydland and Prescott (1977) conclude "there is no way control theory can be made applicable to economic planning when expectations are rational." (p. 473) But in Kydland and Prescott (1980), they also indicate, "Even though there is little hope of the optimal plan being implemented—because of its time inconsistency—we think the exercise is of more than pedagogical interest. The optimal plan's return is a benchmark with which to compare the time consistent solution..." (p. 79) In other words, control theory can identify the optimal plan and, thus, the optimal economic outcomes. Then, we can seek a consistent plan that coincides with the optimal plan through institutional design. That is, the optimal plan can indicate how to design the optimal institution, through which we implement the optimal plan with a consistent plan.

Solutions to Inconsistency of Optimal Plans

We classify the solutions to the inconsistency of optimal plans into three types: rules, reputation, and delegation.

Kydland and Prescott (1977) argue for "rules rather than discretion." That is, rules can provide the commitment technique to achieve optimal policy. And the literature provides many illustrations that economies perform better under rules than under consistent policy (i.e., discretion). As a result, a literature exists on the design of policy rules. In monetary models, they include McCallum (1988), Taylor (1993), Svensson (1999), and so on.

As we know, equilibria often exhibit Pareto inefficiency (i.e., consistent policy generally proves not optimal). Game theory suggests, however, that an equilibrium outcome may prove optimal under certain conditions, if the game repeats and reputation plays a role. Barro and Gordon (1983b) construct such a model to show that optimal policy proves implementable and consistent under certain conditions. Backus and Driffill (1985) demonstrate that reputation, based on the concept of Kreps and Wilson's (1982) sequential equilibrium, makes optimal policy credible. We, however, do not advocate the reputation approach, which we explain below.

In our context and in monetary models, delegation means that the government delegates a monetary policy objective to the central bank. In a broad sense, delegation implies mechanism or institutional design. When establishing a specific institution (e.g., the central bank), the government must delegate an appropriate objective. For example, Rogoff (1985), Walsh (1995), Svensson (1997), and Chortareas and Miller (2003) fall broadly into the delegation approach. Rogoff (1985) argues for the appointment of a "conservative" central banker. Svensson (1997) delegates an inflation target that differs from society's target. Walsh (1995) introduces an incentive contract that penalizes the central banker for deviations from the target inflation rate. Finally, Chortareas and Miller (2003) propose an output contract for the central banker that penalizes deviations of output from the natural level. The proper choice of the penalty rate completely eliminates the inflation bias, even if the government cares about the cost of the contract.

In sum, delegation solutions to the inconsistency problem adopt a central bank loss function that differs from society's loss function. Using implementation theory, we develop a general method of mechanism design or delegation of a correct (optimal) central bank loss function so that the optimal policy proves implementable (i.e. consistent).

3. Optimal and Consistent Policy in a Simple Model

Barro and Gordon (1983a) introduce a basic model for analyzing the inconsistency issue in monetary policy. We adopt a one-period model with complete information. Reasons follow.

First, understanding our analytical method becomes less difficult in the simplest models. Thus, we attack the problem one piece at a time.

Second, for a one-period game, the inconsistency of optimal plans generally exists, no matter whether players' decisions occur simultaneously or sequentially.[6] For example, the prisoner's dilemma provides an example of the simultaneous-decision, one-period game model. Before the game begins, both suspects know that their optimal strategy equals "confess," their rational and consistent strategy equals "defect," once the game starts.

Third, some multi-period models in the literature basically reduce to one-period models for a stationary period function and a discount function δ^t, where $0 \leq \delta \leq 1$ equals the discount factor. Such models include the inflation-unemployment example in Kydland and Prescott (1977) and the Barro and Gordon (1983a,b) model.

Fourth, a repeated game creates other difficulties, making them much different from the one-period game. The Folk Theorem indicates that equilibrium outcomes of the game only require that each player's payoff exceeds the player's max-min payoff. No definite method predicts which equilibrium gets chosen, however. Moreover, the equilibrium outcomes in a repeated game may depend on psychology and culture. As a result, the equilibrium becomes unreliable.[7] But, if we make optimal policy consistent in a one-period game, then in the repeated game, the equilibrium (consistent) policy always proves optimal.

Finally, we can consider a repeated game with incomplete information as a one-period game with complete information. As the game repeats, players adjust their beliefs in a Bayesian fashion and approach complete information as the game progresses.

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[6] Kydland (1977, p313) notes that even in the first period, the dominant player may deviate from the original policy, implying inconsistency even in the one-period game. He refers to the deviant policy as the closed-loop policy. A similar deviation occurs for open-loop policy—the optimal policy.

[7] But Barro and Gordon (1983b) show that reputation plays a role and indicate the conditions under which the consistent policy proves optimal.

The Basic Model

We begin with a standard-version macroeconomic model and a quadratic social loss function in terms of the inflation rate and employment.[8] That is,

(1) $$L = \chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2,$$

where p equals the natural logarithm of the price level, p_0 equals its initial value, ℓ equals the natural logarithm of the employment level, and $\overline{\ell}$ equals the natural logarithm of "full employment," which we assume higher than the natural logarithm of the natural level of employment, $\widetilde{\ell}$. The social loss function implies that the society considers two targets – a zero inflation rate[9] and "full employment." The weight that society places on the inflation target relative to the employment target equals χ, the trade-off parameter. To simplify, we consider only a one-period social loss function, allowing us to omit the time period subscript t. We also assume that the central bank directly controls the price level, p.

Now, we model the structure of the economy with an expectations-augmented Phillips curve and rational expectations. That is,

(2) $$\ell = \widetilde{\ell} - \beta(w - p) - u, \text{ and}$$

(3) $$w = E(p), \text{ or } \min_{w} E(L_{WS}) = E\left[(w - p)^2\right],$$

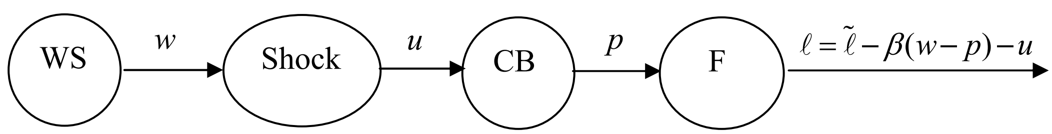
where β equals the responsiveness of employment to unexpected inflation, w equals the natural logarithm of the wage setter's nominal wage rate, u equals an independently and identically

[8] See, for example, Barro and Gordon (1983a), Rogoff (1985), Flood and Isard (1989), Lohnman (1992), Walsh (1995), Persson and Tabellini (1993), Svensson (1997), and so on.

[9] Equation (1) proves equivalent to the other typical specification, where $L = \chi\pi^2 + (\ell - \overline{\ell})^2$, because $\pi = \left(P - P_0\right)/P_0 = d\ln P = dp = p - p_0$, where P and P_0 equal the price level and its initial value.

distributed negative supply shock with mean 0 and variance σ^2, L_{WS} equals the wage setter's loss function, and E equals the mathematical expectation operator.

The private sector's behavioral equations (2) and (3), the firm (F) and the wage setter (WS), respectively, incorporate the following logic. The wage setter and the firm sign a wage contract, where the wage setter sets the nominal wage, w, and the firm sets the amount of labor, ℓ, that it hires. After signing the wage contract, a negative shock, u, may occur. Then, the central bank (CB) implements its policy decision, p. Since the contract fixes the nominal wage, the wage setter must form a rational expectation of the price level before setting the wage rate [i.e., the wage setter uses behavioral equation (3)]. Finally, given the firm's decision, a certain employment level emerges from the firm's behavioral equation (2). The timing of the sequential decisions in this one-period model unfolds according to the following chart:



Further, we assume that the participants in the economy (i.e., central bank, wage setter, and firm) view the model as common knowledge (i.e., the social loss function and the two private-sector behavioral equations).

We combine the above assumptions into the following model:

$$\min_{p} L = \chi(p - p_0)^2 + (\ell - \overline{\ell})^2$$

(4)
$$s.t. \begin{cases} \ell = \tilde{\ell} - \beta(w - p) - u \\ \min_{w} E(L_{WS}) = E\left[(w - p)^2\right] \end{cases} .$$

Now we compute this model's optimal policy and consistent policy with control-theory and game-theory solution techniques.

Optimal Policy

As stated above, control theory can provide a useful benchmark. With control theory, we determine the optimal plan and, thus, the optimal economic outcome. This provides a benchmark for policy making. The benchmark case assumes complete information with decisions made by one person before the game starts. That is, we assume that the optimal policy reflects an ex ante plan made by a social planner with complete information. The optimal policy and outcomes for model (4) reduce to the following results:[10]

(5) $$p = p_0 + \frac{\beta}{\chi + \beta^2} u ,$$

(6) $$\ell = \tilde{\ell} - \frac{\chi}{\chi + \beta^2} u , \text{ and}$$

(7) $$E(L) = \frac{\chi}{\chi + \beta^2} \sigma^2 + k^2 .$$

Consistent Policy

To better facilitate comparison with the analysis of Section 4, we rewrite the problem expressed in model (4) as a two-player, sequential-decision, one-period game as follows, where the wage setter moves first:

Players	The Wage Setter	The Central Bank
Preferences	$E(L_{WS}) = E\left[(w-p)^2\right]$	$L_{CB} = \chi(p - p_0)^2 + \left(\ell - \bar{\ell}\right)^2$
Strategies	w	p
Subject to the Firm's Decision: $\ell = \tilde{\ell} - \beta(w-p) - u$		

[10] See Appendix A for the derivation.

Using backward induction to solve the problem expressed in model (4) and in the above game form, we first must solve for the central bank's optimal decision for the price level. That is, given the nominal wage and the supply shock, the central bank chooses the p to minimize the social loss function, yielding the following relationship:

$$p = \frac{\chi}{\chi + \beta^2} p_0 + \frac{\beta^2}{\chi + \beta^2} w + \frac{\beta}{\chi + \beta^2} k + \frac{\beta}{\chi + \beta^2} u , \tag{8}$$

where $k \equiv \overline{\ell} - \tilde{\ell}$ equals the employment bias.

With the central bank's reaction function in equation (8), the wage setter's expected loss equals the following relationship:

$$E\left(L_{WS}\right) = \left[\frac{\chi}{\chi + \beta^2}\left(w - p_0\right) - \frac{\beta}{\chi + \beta^2} k\right]^2 + \left(\frac{\beta}{\chi + \beta^2}\right)^2 \sigma^2 . \tag{9}$$

Therefore, the equilibrium nominal wage equals[11]

$$w = p_0 + \frac{\beta}{\chi} k . \tag{10}$$

Substituting equation (10) into equation (8) yields the equilibrium price level as follows:

$$p = p_0 + \frac{\beta}{\chi} k + \frac{\beta}{\chi + \beta^2} u . \tag{11}$$

As observed in the standard literature, there exists an inflationary bias, $E\left(p - p_0\right) = \frac{\beta}{\chi} k$. With the equilibrium nominal wage and price level, we get the equilibrium employment:

$$\ell = \tilde{\ell} - \frac{\chi}{\chi + \beta^2} u , \tag{12}$$

which generates the expected social loss as follows:

[11] Equation (10) emerges by setting $dE\left(L_{WS}\right)/dw = 0$.

(13) $$E(L) = \frac{\chi}{\chi+\beta^2}\sigma^2 + \left(1+\frac{\beta^2}{\chi}\right)k^2.$$

Compared with the optimal policy and outcomes in equations (5), (6) and (7), the consistent policy in equation (11) generates the inflationary bias (i.e., a higher price level than the initial one) and a larger social loss in equation (13) results from the inflationary bias.

Two important points deserve comment. First, the two targets in the social loss function, p_0 and $\overline{\ell}$, actually conflict with each other, given the structural model in equations (2) and (3). If the central bank wants to achieve full employment, it must inflate the economy, meaning that the central bank cannot achieve the zero inflation-rate target. If the central bank, on the other hand, wants to hit the zero inflation-rate target, then it cannot raise the employment level above the natural level. So the two targets, p_0 and $\overline{\ell}$, prove incompatible. Does it make sense to delegate incompatible targets to the central bank?

Second, we observe that the employment target, $\overline{\ell}$, proves overambitious and unattainable under the assumptions of the structural model because

(14) $$E(\ell) = E\left[\tilde{\ell} - \beta(w-p) - u\right] = \tilde{\ell} - \beta\left[w - E(p)\right] - E\left[u\right] = \tilde{\ell},$$

which means that the level of employment can only equal the natural level, on average. According to equations (11) and (12), we also know that

(15) $$\begin{aligned} &E(p) = p_0 + \frac{\beta}{\chi}k \neq p_0 \text{ and} \\ &E(\ell) = \tilde{\ell} \neq \overline{\ell} \end{aligned}.$$

The above inequalities mean that, on average, the central bank cannot achieve each of its targets, which seems illogical. Society should not delegate such targets to the central bank. A more sensible approach makes the following assumptions about delegating targets to the central bank

(16) $$p^* = E(p) \text{ and } \ell^* = E(\ell).$$

That is, proper targets should allow the central bank to achieve them. The assumptions in equation (16) prove essential for pinning down the central bank loss function in Section 4. Actually, the assumptions in (16) hold when choosing parameters, p^* and ℓ^*, to minimize the central bank loss function.[12]

Reconsidering the role of the social loss function, it provides a social welfare criterion, reflecting a normative process that does not necessarily prove consistent with the structure of the economy. Consequently, the social loss function cannot serve as a direct loss function for the central bank. We will use implementation theory to design a central bank mechanism (i.e. the central bank loss function) that interacts with the structure of the economy so that the equilibrium outcome proves optimal according to the given social welfare criterion.

4. Central Bank Mechanism Design

Given the social welfare criterion and the macroeconomic structure, we identify the optimal policy and its outcomes (i.e., the social choice function) with the solution techniques in control theory. Then we design the loss function of the central bank that interacts with macroeconomic structure so that the central bank's consistent (i.e., equilibrium) policy and its equilibrium outcomes coincide with the optimal policy and the optimal outcomes. Thus, the optimal policy and its outcomes prove implementable.

Social Welfare Criterion

A popular view takes optimizing the representative household's utility as the social welfare criterion. This view, however, does not permit differences between private and social interests. Social welfare criteria can also capture the ideas of Rawls' maximin criterion (Rawls, 1971), the

[12] See equations (29), (31) and (32).

Bergson-Samuelson social welfare function (Pollak, 1979), or Arrow's social welfare function (Arrow 1951), and so on. Thus, we argue that constructing the social welfare criterion reflects a normative problem in philosophy. In our context, we assume that the social welfare criterion minimizes the social loss function given in equation (1).

Outcomes

The social loss function incorporates $\chi, p_0,$ and $\bar{\ell}$ as parameters and p and ℓ as economic outcomes. We define *A* as the set of economic outcomes:

(17) $$A = \{(p, \ell) | p \ and \ \ell \in \mathbb{R}_+\}.$$

Preferences

We assume that the preference profile, $R = (R_F, R_{WS})$, for the macroeconomic structure (i.e., the firm and the wage setter, respectively) equals equations (2) and (3). That is,

(18) $$\begin{aligned} R_F: &\quad \ell = \tilde{\ell} - \beta(w-p) - u \\ R_{WS}: &\quad \min_w E(L_{WS}) = E\left[(w-p)^2\right] \end{aligned}.$$

Here, the preference of the firm appears in its behavioral equation, which comes from the first-order condition of its profit function with a Cobb-Douglas production function with one variable factor of production (i.e., labor). The preference of the wage setter, who intends to hold the real wage level constant, equals the expected loss function, $E(L_{WS}) = E[(w-p)^2]$.

To illustrate our method for mechanism design, we also assume another preference profile, $R' = (R'_F, R'_{WS})$, as follows:

(19) $$\begin{aligned} R'_F: &\quad \ell = \tilde{\ell} - \beta(w-p) - u \\ R'_{WS}: &\quad \min_w E(L_{WS}) = E\left[-2\gamma(w-p) + (w-p)^2 + \lambda(\ell - \bar{\ell})^2\right] \end{aligned}.$$

Now, $R'_F = R_F$ and we modify the wage setter's preference to include a desire for a higher real wage rate as well as higher employment than the natural level. We specify R'_{WS} to encompass R_{WS} so that we can derive the mechanism, designed for profile $R = (R_F, R_{WS})$, as a special case of this encompassing profile $R' = (R'_F, R'_{WS})$. Moreover, we can compare the mechanisms under these two different preference profiles, R and R'.

We denote the set of admissible preference profiles as $\Re$.

Social Choice Correspondence

A social choice correspondence, *F*, maps the preference profiles into subsets of outcomes. That is, for any preference profile $R \in \Re$, $F(R) \subset A$ represents the set of socially desirable outcomes. A single-valued *F* is referred to as a social choice function.

In many applications, *F* will represent a well-known correspondence, such as the Walrasian, or top-cycle, correspondence, or will represent a social choice correspondence derived from some normative criterion. In our context, the social choice function, *F*, derives from the social welfare criterion (1), using control theory. Specifically, for *R* and $R' \in \Re$, we define:

(20) $$F(R) \equiv \left(\breve{p}(R), \breve{\ell}(R)\right) \equiv \left(p_0 + \frac{\beta}{\chi + \beta^2}u, \tilde{\ell} - \frac{\chi}{\chi + \beta^2}u\right) \in A,$$ and[13]

(21) $$F(R') \equiv \left(\breve{p}(R'), \breve{\ell}(R')\right) \equiv \left(p_0 + \frac{\beta}{\chi + \beta^2}u, \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1 + \lambda\beta^2)} - \frac{\chi}{\chi + \beta^2}u\right) \in A.$$ [14]

Mechanism Design and Equilibrium Outcomes

The central bank's loss function provides a mechanism. Now, we design the central bank's loss function, which takes the following form:

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[13] See Appendix A for the derivation.

[14] See Appendix B for the derivation.

(22) $$L_{CB} = \chi^*\left(p - p^*\right)^2 + \left(\ell - \ell^*\right)^2,$$

where we need to design the three parameters, χ^*, p^*, and ℓ^*.

Under the preference profile, $R = \left(R_F, R_{WS}\right)$, the sequential-play, one-period game between the wage setter and the central bank possesses the following structure:

Players	The Wage Setter	The Central Bank
Preferences	$E\left(L_{WS}\right) = E\left[\left(w - p\right)^2\right]$	$L_{CB} = \chi^*\left(p - p^*\right)^2 + \left(\ell - \ell^*\right)^2$
Strategies	w	p
Subject to the Constraint R_F: $\ell = \tilde{\ell} - \beta(w - p) - u$		

The equilibrium strategy profile chosen equals

(23) $$\left(\hat{w}\left(R\right), \hat{p}\left(R\right)\right) \equiv \left(p^* + \frac{\beta}{\chi^*}\left(\ell^* - \tilde{\ell}\right), p^* + \frac{\beta}{\chi^*}\left(\ell^* - \tilde{\ell}\right) + \frac{\beta}{\chi^* + \beta^2}u\right),$$

and the equilibrium outcome equals

(24) $$\left(\hat{p}\left(R\right), \hat{\ell}\left(R\right)\right) \equiv \left(p^* + \frac{\beta}{\chi^*}\left(\ell^* - \tilde{\ell}\right) + \frac{\beta}{\chi^* + \beta^2}u, \tilde{\ell} - \frac{\chi^*}{\chi^* + \beta^2}u\right).$$ [15]

Similarly, under the preference profile, $R' = \left(R'_F, R'_{WS}\right)$, the sequential-play, one-period game between the wage setter and the central bank possesses the following structure:

[15] The backward solution technique follows that outlined in Section 3 under the *Consistent Policy* subsection.

Players	The Wage Setter	The Central Bank
Preferences	$E\left(L_{WS}\right)=E\left[-2\gamma(w-p)+\left(w-p\right)^{2}+\lambda(\ell-\bar{\ell})^{2}\right]$	$L_{CB}=\chi^{*}\left(p-p^{*}\right)^{2}+\left(\ell-\ell^{*}\right)^{2}$
Strategies	w	p
Subject to the Constraint R_F : $\ell=\tilde{\ell}-\beta(w-p)-u$		

The equilibrium strategy profile chosen equals

(25) $$\left(\hat{w}\left(R'\right),\hat{p}\left(R'\right)\right)\equiv\begin{pmatrix} p^{*}+\dfrac{\beta}{\chi^{*}}\left[\left(\ell^{*}-\tilde{\ell}\right)+\dfrac{\beta(\gamma-\lambda\beta k)}{(1+\lambda\beta^{2})}\right]+\dfrac{(\gamma-\lambda\beta k)}{(1+\lambda\beta^{2})}, \\ p^{*}+\dfrac{\beta}{\chi^{*}}\left[\left(\ell^{*}-\tilde{\ell}\right)+\dfrac{\beta(\gamma-\lambda\beta k)}{(1+\lambda\beta^{2})}\right]+\dfrac{\beta}{\chi^{*}+\beta^{2}}u \end{pmatrix},$$

and the equilibrium outcome equals

(26) $$\left(\hat{p}\left(R'\right),\hat{\ell}\left(R'\right)\right)\equiv\begin{pmatrix} p^{*}+\dfrac{\beta}{\chi^{*}}\left[\left(\ell^{*}-\tilde{\ell}\right)+\dfrac{\beta(\gamma-\lambda\beta k)}{(1+\lambda\beta^{2})}\right]+\dfrac{\beta}{\chi^{*}+\beta^{2}}u, \\ \tilde{\ell}-\dfrac{\beta(\gamma-\lambda\beta k)}{(1+\lambda\beta^{2})}-\dfrac{\chi^{*}}{\chi^{*}+\beta^{2}}u \end{pmatrix}.$$ [16]

Implementation

Implementing the social choice function, *F*, means that $\left(\hat{p}\left(R\right),\hat{\ell}\left(R\right)\right)=\left(\breve{p}\left(R\right),\breve{\ell}\left(R\right)\right)$, that is, the equilibrium outcome equals the socially desirable outcome, for all $R\in\Re$. Under the preference profile, $R=\left(R_F,R_{WS}\right)$, $\left(\hat{p}\left(R\right),\hat{\ell}\left(R\right)\right)=\left(\breve{p}\left(R\right),\breve{\ell}\left(R\right)\right)$ means that

[16] See footnote 16.

(27)
$$\begin{cases} p^* + \dfrac{\beta}{\chi^*}\left(\ell^* - \tilde{\ell}\right) + \dfrac{\beta}{\chi^* + \beta^2}u = p_0 + \dfrac{\beta}{\chi + \beta^2}u \\ \\ \tilde{\ell} - \dfrac{\chi^*}{\chi^* + \beta^2}u = \tilde{\ell} - \dfrac{\chi}{\chi + \beta^2}u \end{cases}.$$

Therefore,

(28)
$$\begin{cases} \chi^* = \chi \\ \\ p^* + \dfrac{\beta}{\chi}\left(\ell^* - \tilde{\ell}\right) = p_0 \end{cases}.$$

An infinite number of combinations of p^* and ℓ^* exist that conform to equations (28). To pin down their values, we use the assumptions in equation (16) and obtain

(29)
$$\begin{cases} p^* = E\left(\hat{p}\right) = p_0 \\ \\ \ell^* = E\left(\hat{\ell}\right) = \tilde{\ell} \end{cases}.$$

Viewing the problem somewhat differently, but leading to the same conclusion, the assumptions in (16) minimize the central bank loss function. To see this, consider the value of the central bank loss function, given the solutions to the minimization of the social loss function. Note that the only ambiguity relates to the values of the target price and employment levels. That is, the first equation in (28) indicates that the weight associated with the inflation term in the social loss function equals the weight in the central bank loss function. Thus, we need to choose the target price and employment levels to minimize the following central bank loss function:

$$\min_{p^*,\ell^*} EL_{CB} = E\left[\chi\left(p_0+\left\{\frac{\beta}{\chi+\beta^2}\right\}u-p^*\right)^2+\left(\tilde{\ell}-\left\{\frac{\chi}{\chi+\beta^2}\right\}u-\ell^*\right)^2\right],$$

(30)

$$= \chi(p_0-p^*)^2+(\tilde{\ell}-\ell^*)^2+\left\{\frac{\chi}{\chi+\beta^2}\right\}\sigma^2.$$

This minimization problem produces the following solutions:

(31) $p^* = p_0$ and

(32) $\ell^* = \tilde{\ell}$.

Thus, the assumptions in (16) hold when the central bank's targets minimize its loss function. As a result, the optimal target values in the central bank loss function minimize the central bank loss function as well as the social loss function. The central bank's minimum loss equals the following:

(33) $$EL_{CB} = \left\{\frac{\chi}{\chi+\beta^2}\right\}\sigma^2.$$

In sum, under the preference profile, $R=\left(R_F, R_{WS}\right)$, the optimal central bank loss function equals the following:

(34) $$L_{CB} = \chi\left(p-p_0\right)^2+\left(\ell-\tilde{\ell}\right)^2.$$

With this designed loss function, the equilibrium outcome equals the optimal outcome:

(35) $$\left(\hat{p}(R),\hat{\ell}(R)\right)=\left(\breve{p}(R),\breve{\ell}(R)\right)\equiv\left(p_0+\frac{\beta}{\chi+\beta^2}u,\tilde{\ell}-\frac{\chi}{\chi+\beta^2}u\right)\in A.$$

That is, the optimal policy proves consistent.

Under the preference profile, $R'=\left(R'_F, R'_{WS}\right)$, implementing the social choice function means that $\left(\hat{p}(R'),\hat{\ell}(R')\right)=\left(\breve{p}(R'),\breve{\ell}(R')\right)$. That is,

(36) $$\begin{cases} p^* + \dfrac{\beta}{\chi^*}\left[\left(\ell^* - \tilde{\ell}\right) + \dfrac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)}\right] + \dfrac{\beta}{\chi^* + \beta^2}u = p_0 + \dfrac{\beta}{\chi + \beta^2}u \\ \tilde{\ell} - \dfrac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)} - \dfrac{\chi^*}{\chi^* + \beta^2}u = \tilde{\ell} - \dfrac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)} - \dfrac{\chi}{\chi + \beta^2}u \end{cases}.$$

Therefore,

(37) $$\begin{cases} \chi^* = \chi \\ p^* + \dfrac{\beta}{\chi}\left[\left(\ell^* - \tilde{\ell}\right) + \dfrac{\beta\left(\gamma - \lambda\beta k\right)}{\left(1+\lambda\beta^2\right)}\right] = p_0 \end{cases}.$$

An infinite number of combinations of p^* and ℓ^* exist that conform to equations (37). To pin down their values, we use the assumptions in equation (16). That is,

(38) $$\begin{cases} p^* = E\left(\hat{p}\right) = p_0 \\ \ell^* = E\left(\hat{\ell}\right) = \tilde{\ell} - \dfrac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)} \end{cases}.$$ [17]

In sum, under the preference profile, $R' = \left(R'_F, R'_{WS}\right)$, the optimal central loss function equals the following:

(39) $$L_{CB} = \chi\left(p - p_0\right)^2 + \left\{\ell - \left(\tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)}\right)\right\}^2.$$

With this designed loss function, the equilibrium outcome equals the optimal outcome:

(40) $$\left(\hat{p}\left(R'\right), \hat{\ell}\left(R'\right)\right) = \left(\breve{p}\left(R'\right), \breve{\ell}\left(R'\right)\right) \equiv \left(p_0 + \frac{\beta}{\chi + \beta^2}u, \tilde{\ell} - \frac{\beta(\gamma - \lambda\beta k)}{(1+\lambda\beta^2)} - \frac{\chi}{\chi + \beta^2}u\right) \in A.$$

That is, the optimal policy proves consistent.

[17] These two target values also minimize the mechanism design central bank loss function.

5. Conclusion

Since the work of Strotz (1955-1956), economists continue to struggle with the consistency of optimal plans. Kydland and Prescott (1977) link the problem to the time consistency of optimal economic policy, showing that consistent policy proves non-optimal, in a game framework. Much work focuses on the simple one-period, game-theory problem developed by Barro and Gordon (1983a,b). Within that model, an inflationary bias exists, since consistent policy proves non-optimal.

Our paper develops a general method for making consistent policy optimal. We utilize implementation theory and demonstrate that with the correct mechanism design for the central bank loss function, optimal policy proves implementable (consistent). That is, appointing a central bank with the correct (optimal) objective function or delegating to the central bank that correct (optimal) objective function will cause a convergence of the consistent to the optimal monetary policy. That is, the correct (optimal) objective function equals the mechanism design that achieves the optimal policy. We apply our method to several different variants of the simple sequential-decision, one-period Barro and Gordon (1983a) model with identical results.

Control theory plays a role and provides a benchmark for mechanism design. Specifically, in our context, the social choice function, F, derives from minimizing the social loss function, using control theory. With the social choice function as a criterion, we design the central bank loss function.

Carefully designing the central bank's loss function can make optimal policy and consistent policy identical. This desirable result requires an understanding the two following points. First, the determination of the social loss function reflects a normative process. The social loss function only provides a criterion for designing a public institution, not the direct loss function for this specific institution. Second, the preferences of the economy prove key to determining the central bank loss function. That is, an optimal loss function for the central bank must depend on the preferences of the economy.

In sum, the correct central bank loss function depends on two factors -- the social welfare criterion and the preferences of the economy. Future research should focus on the following. First, consider our method with alternative normative social welfare criteria and with alternative preferences. Second, and more important, evaluate our method within a dynamic model, since economic variables generally prove persistent.

Appendix A:

For the two-stage stochastic optimization problem associated with model (4) and with the preference profiles contained in expression (18), the first stage solves for the optimal value of deterministic variable, w, under certainty.[18] That is, solving the follow model

$$\text{(A-1)} \qquad \begin{aligned} &\min_{w,p} L = \chi\left(p - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2 \\ &s.t. \begin{cases} \ell = \tilde{\ell} - \beta(w - p) \\ w = p \end{cases} \end{aligned},$$

we find that

$$\text{(A-2)} \qquad w = p = p_0 \quad and \quad \ell = \tilde{\ell}.$$

The second-stage adjusts the price level, p, after a shock, u, occurs. Because the loss function equals a quadratic form with linear constraints, the adjustment of p must include a term linear in u. Thus,

(A-3) $\qquad p = p_0 + au$; and

the second-stage problem equals the following:

$$\text{(A-4)} \qquad \begin{aligned} &\min_{a} EL = E\left[\chi\left(p_0 + au - p_0\right)^2 + \left(\ell - \overline{\ell}\right)^2\right] \\ &s.t. \begin{cases} \ell = \tilde{\ell} - \beta\left[w - \left(p_0 + au\right)\right] - u \\ w = p_0 \end{cases} \end{aligned},$$

which generates $a = \dfrac{\beta}{\chi + \beta^2}$. That is, the optimal policy equals that reported in equations (5) and (6) and in equation (20).

[18] See Kolbin (1977) and Marti (2005) for the two-stage solution technique with a stochastic optimization. In sum, the approach solves the optimization under certainty and then incorporates the random shock in the second stage as done in Appendix A and B.

Appendix B:

For the two-stage stochastic optimization problem associated with the preference profiles contained in expression (19), the first-stage solves for the optimal value of deterministic variable, *w*, under certainty. That is, solving the following model

(B-1)
$$\min_{w,p} E(L) = E\left[\chi\left(p-p_0\right)^2 + \left(\ell-\overline{\ell}\right)^2\right]$$
$$s.t.\begin{cases} \ell = \tilde{\ell} - \beta(w-p) \\ -\gamma + \left(w-p\right) - \lambda\beta\left(\ell-\overline{\ell}\right) = 0 \end{cases},$$

with the constraint, $-\gamma + \left(w-p\right) - \lambda\beta\left(\ell-\overline{\ell}\right) = 0$, coming from the first-order condition of the wage-setter's loss function, we find that

(B-2) $$w = p_0 + \frac{(\gamma-\lambda\beta k)}{(1+\lambda\beta^2)},\ p = p_0,\ \text{and}\ \ell = \tilde{\ell} - \frac{\beta(\gamma-\lambda\beta k)}{(1+\lambda\beta^2)}.$$

The second-stage adjusts the price level, *p*, after a shock, *u*, occurs. Similarly, the adjustment of *p* must include a linear term in *u* as follows:

(B-3) $p = p_0 + bu$; and

now, the second-stage problem equals the following:

(B-4)
$$\min_{b} EL = E\left[\chi\left(p_0+bu-p_0\right)^2 + \left(\ell-\overline{\ell}\right)^2\right]$$
$$s.t.\begin{cases} \ell = \tilde{\ell} - \beta\left[w-\left(p_0+bu\right)\right]-u \\ w = p_0 + \dfrac{(\gamma-\lambda\beta k)}{(1+\lambda\beta^2)} \end{cases},$$

which generates $b = \dfrac{\beta}{\chi+\beta^2}$. That is, the optimal policy equals that reported in equation (21).

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