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Asset Pricing with Heterogeneous Agents, Incomplete Markets and Trading Constraints

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Abstract

The consumption capital asset pricing model is the standard economic model used to capture stock market behavior. However, empirical tests have pointed out its inability to account quantitatively for the high average rate of return and volatility of stocks over time for plausible parameter values. Recent research has suggested that the consumption of stockholders is more strongly correlated with the performance of the stock market than the consumption of non-stockholders. We model two types of agents, non-stockholders with standard preferences and stock holders with preferences that incorporate elements of the prospect theory developed by Kahneman and Tversky (1979). In addition to consumption, stockholders consider fluctuations in their financial wealth explicitly when making decisions. Each agent faces idiosyncratic shocks to his labor income as well as aggregate shocks to the per-share dividend but markets are incomplete and agents cannot hedge consumption risks completely. In addition, consumers face both borrowing and short-sale constraints. Data from the Panel Study of Income Dynamics are used to calibrate the labor income processes of the two types of agents. Our results show that in equilibrium, agents hold different portfolios. Our model is able to generate a time-varying risk premium of about 6explanation for the equity premium puzzle reported by Mehra and Prescott (1985).

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Keywords: asset pricing, equity premium puzzle, prospect theory, heterogeneous agents

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I Introduction

The Consumption Capital Asset Pricing Model (CCAPM) developed by Lucas (1978) and Breeden (1979) is the standard economic framework for modeling security prices. However, empirical tests based on the resulting Euler equations have failed to validate the model (Hansen and Singleton, 1982; Hansen and Jagannathan, 1991; Ferson and Constantinides, 1991). Perhaps the most striking failure of the model has been demonstrated by Mehra and Prescott (1985). While the historical real rate of return on a market portfolio of risky assets (such as Standard and Poor's 500 Composite Stock Index) has exceeded the real rate of return on relatively riskless assets (such as 3-month T-bills) by about 6% per year, Mehra and Prescott demonstrate that the CCAPM cannot generate a risk premium of more than 0.35% for the range of plausible parameter values that they assume. The result of their empirical test of the CCAPM was so striking, that they termed it the "equity premium puzzle". Subsequent empirical tests have shown that the equity premium puzzle is very robust; it is neither a sample-period phenomenon (Siegel, 1992; Mehra, 2003), nor a country-specific phenomenon (Dimson, Marsh, and Staunton, 2006; Campbell, 2003; Mehra and Prescott, 2003; however, Jorion and Goetzmann, 1999, find that the equity premium puzzle is largely a US phenomenon).

The seminal work of Mehra and Prescott has stemmed a large volume of theoretical and empirical studies. Kocherlakota (1996), Cochrane (1997), and Mehra and Prescott (2003) offer insightful surveys of the literature. As pointed out by Kocherlakota, most of these studies can be grouped by the assumptions of Mehra and Prescott that they relax: time-nonseparable preferences (Hansen and Constantinides, 1991; Heaton, 1993); recursive preferences (Weil, 1989; Epstein and Zin, 1991); state-nonseparable preferences (Nason, 1988; Abel, 1990); rare-event declines in aggregate consumption (Rietz, 1988); transaction costs (Luttmer, 1993); combined assumptions of consumer heterogeneity and incomplete consumption insurance (Mehra and Prescott, 1985). However, none of these alternatives can overcome the drawbacks of the original Lucas model without posing further complications.

While a number of modifications of the representative-agent model have been proposed, the literature on asset pricing with heterogeneous agents has started to grow only recently (see, for example, Constantinides, Donaldson, and Mehra, 2002; Marcet and Singleton, 1999; Heaton and Lucas, 1996; Constantinides and Duffee, 1996). In a complete market, representative-agent model, individuals completely insure the idiosyncratic shocks to their income and individual consumption is perfectly correlated with the aggregate per capita consumption. Interestingly, asset prices in an incomplete market setting with aggregate uncertainty and transitory idiosyncratic shocks do not differ significantly from those in complete market models (Telmer, 1993; Lucas, 1994) because agents are able to smooth consumption through trading in financial markets and accumulating buffer stock saving. However, Constantinides and Duffie (1996) show that when idiosyncratic shocks are permanent, trade does not take place and the volatility of consumption increases in equilibrium. Evidence also suggests that in the presence of short-sale and liquidity constraints, the equity premium rises when the constraints are binding (see, for example, Marcet and Singleton, 1999). Thus, in an incomplete market setting the predictions of the model depend critically on the underlying market structure as well as the size and persistence of idiosyncratic shocks.

Mankiw and Zeldes (1991) raise an important objection to the empirical tests of the CCAPM. In the United States roughly only a third of the population holds stocks. Therefore, empirical tests of CCAPM based on the Euler equations, which employ aggregate consumption data, are doomed to fail unless the consumption processes of stockholders and non-stockholders are highly correlated. They find that the two consumption processes differ substantially and conclude that "failures of the consumption CAPM might be rationalized by a model with two groups of consumers: stockholders and non-stockholders" (p. 99).

Clearly, a representative agent model cannot explain why stock holdings are concentrated in a small proportion of the population. A related issue is that models, which do allow for agent heterogeneity, typically use the same utility function to describe individual preferences. However, there is no a priory reason that would lead us to believe that stockholders and non-stockholders have identical preferences. Barberis, Huang, and Santos (2001) suggest that stockholders have preferences which differ from the preferences of non-stockholders. They introduce elements of prospect theory developed by Kahneman and Tversky (1979) into a standard asset pricing model and are successful in generating stock returns which are more volatile than the underlying dividends.

Prospect theory is one of the well-regarded alternatives to expected utility theory. It is based on the assumption that agents derive utility not from levels in wealth, but rather from *changes* in wealth. Further, agents are more sensitive to losses than to gains in wealth: a property known as "loss aversion". Prospect theory was developed based on laboratory experiments but recent empirical work supports the presence of loss aversion in decision-making in financial markets (see, for example, Genesove and Mayer, 2001, for evidence from the housing market and Coval and Shumway, 2005, for evidence from the futures market).

The reason why accounting for loss aversion may enable us to explain the equity premium is that loss aversion introduces a new source of risk. In the core of the equity premium puzzle is the definition of risk. While the Capital Asset Pricing Model (CAPM), developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), defines risk as the covariance of stock returns with the return on a market portfolio, CCAPM defines risk as the covariance of consumption growth with the stock market return. Empirically, the puzzle is driven by the low correlation of stock market returns with the aggregate consumption or the low "quantity" of risk. Thus, stocks are not sufficiently risky to generate the high historical return and therefore, the price of risk or the coefficient of risk aversion must be high to reconcile the risk premium generated by the model with its historical counterpart. For the loss averse agent in the model of Barberis, Huang, and Santos, stocks are risky not only because they covary with consumption but because they expose the investor to potential losses in the stock market, which are painful. In addition, loss aversion is time-varying: it increases after prior gains and decreases after prior losses. Thus, similarly to Campbell and Cochrane (1999), Barberis, Huang, and Santos introduce a time-varying risk aversion but in contrast to Campbell and Cochrane the source of this time-varying risk aversion is not consumption.

The purpose of this paper is to explore the equilibrium distributions of asset prices and portfolio allocations in an incomplete-market economy with two types of agents where one type of agent has standard preferences and the other type is loss averse. Agents are heterogeneous along two dimensions: labor incomes and preferences. Type A agent, the representative non-stockholder, has standard preferences used in macroeconomics and Type B agent, the representative stockholder, exhibits preferences suggested by Barbaris, Huang, and Santos. In addition to consumption, Type B agent derives direct utility from changes in his financial wealth and he is loss averse to these changes (the prospect theory element in preferences). In addition, prior stock market performance affects the decision-making of Type B agent. To our knowledge, this preference heterogeneity is an unique feature of this model. We allow for aggregate uncertainty in the form of shocks to the aggregate per share dividend and idiosyncratic labor income shocks, which have both permanent and transitory components. We use data from the Panel Study of Income Dynamics (PSID) to calibrate the individual income processes. Markets are incomplete because there are three sources of uncertainty in this economy but there are only two markets, the bond and the stock market, where agents can trade to smooth their consumption.

We examine whether the results of Barberis, Huang, and Santos, derived based on a complete market model with a loss averse representative agent, carry through in our more general model. The stockholders in our model are only a small proportion of the population. Most of the models with limited stock market participation are representative agent models and the models that allow for agent heterogeneity typically impose the non-participation constraint exogenously (see, for example, Saito, 1995, Basac and Cuoco, 1998, Guvenen, 2005). We allow agents to self-select endogenously as stockholders and non-stockholders based on their preferences, wealth, and liquidity constraints. We are interested in whether individuals with loss averse preferences will self-select as investors. We do not impose any exogenous constraint on stock holdings, thus both agents can trade in the stock market. We also do not allow for transaction costs. Heaton and Lucas (1996) show that sizable transaction costs or limited quantity of tradable securities generate about half of the observed risk premium. As a result, the equity premium generated by our model may be biased downward in the sense that if we account for transaction costs as well, the equity premium should rise.

Our results suggest that heterogeneous preferences and idiosyncratic labor income shocks induce agents to hold different portfolios in equilibrium. Agents trade extensively in both the stock and bond markets to smooth their consumption. However, the idiosyncratic income shocks are persistent and agents cannot completely diversify away the non-systematic risk. Our model generates a time-varying risk premium of stocks over bonds which matches the empirical value of the equity premium while maintaining a low risk-free rate and a low correlation between individual consumption and stock market returns. Compared to the representative agent model of Barberis, Huang, and Santos, we are able to match the empirical value of the equity premium while maintaining loss aversion and risk-free rate consistent with empirical data. In the model of Barberis, Huang, and Santos, the risk free rate is exogenously set equal to 3.79% and they require high loss aversion to match the empirical value of the equity premium. In contrast to them, we are also able to match the mean and volatility of the price-dividend ratio.

We are able to match the historical equity premium even though the loss averse agent is concerned predominantly with consumption smoothing and to a lesser extent, with fluctuations in financial wealth. We find that accounting for loss aversion enables us to generate a 20% higher equity premium compared to a heterogenous-agent model where agents have identical standard preferences. This is a significant improvement given that in our baseline model, on average the prospect term in the utility function of Type B agent accounts for abut 20% of the overall utility. Thus, loss aversion is preserved in the aggregate, even though investors represent a small share of the population. However, the stock and bond volatilities predicted by our model exceed the corresponding volatilities observed in historical data suggesting that our model understates the amount of diversification which takes place in financial markets.

We show that loss averse preferences will not induce individuals to hold stocks. On the contrary, individuals with the preference specification introduced by Barberis, Huang, and Santos would prefer to hold bonds because holding stocks, on average decreases their overall utility. The reason lies in the asymmetry with which gains and losses affect the utility of the representative stockholder: losses are more painful that gains and on average, there are as many losses as gains.

The paper is organized as follows: Section II presents the model; Section III discusses the model parametrization and Section IV discusses the solution algorithm; Section V presents the results; Section VI presents the sensitivity analysis and Section VII concludes.

II The Model

1. The Economy

We model a pure exchange economy with three competitive markets and complete information. There are two types of infinitely-lived agents in this pure exchange economy that differ with respect to their preferences and idiosyncratic labor incomes. The agents are price-takers in goods and securities markets. There are three sources of uncertainty in this economy: shocks to the aggregate dividend and idiosyncratic shocks to the individual labor incomes. However, markets are incomplete and agents can only partially insure against consumption risks. The market incompleteness arises from both missing markets (e.g. agents cannot purchase insurance against idiosyncratic labor income shocks) and borrowing and short-sale constraints.

There are two assets in the economy: a long-lived risky asset (stock), which is a claim to a stream of stochastic dividends, and a short-lived riskless asset (default-free discount bond), which is a sure claim to one unit of the consumption good in period $t+1$. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. Each agent enters the initial period with no bond holdings and half of the market equity in the economy.

1.1. Preferences

For clarity, the preferences of the two types of agents are discussed separately below.

A. Type A Agent

We can think of Type A agent as a representative agent for non-stockholders. He is the average non-stockholder in the economy. Non-stockholders either do not hold stocks or their stock holdings represent a small proportion of their wealth. A Type A agent maximizes an additively-separable utility function which exhibits constant relative risk aversion:

$$E\left[\sum_{t=0}^{\infty} \rho^t \frac{(C_t^A)^{1-\gamma}}{1-\gamma} \mid \Omega(t)\right] \tag{1}$$

where C_t^A is the consumption of Type A agent at time t; $0 < \rho < 1$ is the subjective discount factor; and $\Omega(t)$ denotes the time t information set which is generated by the state variables in the model and is common to both agents. The coefficient of relative risk aversion, $0 < \gamma < \infty$, controls for the curvature of the utility function. The utility function reduces to $\ln C_t^A$ when $\gamma = 1$. It is continuous, concave and obeys the Inada conditions, i.e. $\lim_{C \to 0} U'(C_t) = \infty$ and $\lim_{C \to \infty} U'(C_t) = 0$.

B. Type B Agent

A Type B agent is the representative stockholder or invester. He is the average stockholder in the economy. He derives utility from both consumption and anticipated fluctuations in financial wealth. His instantaneous utility function is additive in these two sources of utility. The idea that individuals derive utility from changes in wealth rather than wealth levels was first postulated by

Kahneman and Tversky (1979). The prospect theory that they developed is a positive theory of choice under uncertainty derived on the base of experimental evidence. The major building block of prospect theory is the assumption that individuals derive more dissatisfaction from a loss than satisfaction from a gain of an equal size, termed "loss aversion". It has been suggested that together with risk aversion and probability weights, loss aversion is a major component of risk attitudes (see for example Köbberling and Wakker, 2005). In the literature, several different ways of modelling loss aversion have been suggested. Based on experimental evidence, Kahneman and Tversky (1992) suggest the following form for the utility from gains and losses:

$$U(X) = \begin{cases} X^{\alpha} & \text{for} \quad X \geq 0 \\ -\lambda(-X)^{\beta} & \text{for} \quad X < 0 \end{cases} \tag{2}$$

where X denotes changes in wealth with respect to a reference point and $\lambda > 1$ is a measure of loss aversion. Thus, the utility function is a piece-wise function which is steeper for losses ($X < 0$) than for gains ($X > 0$) with a discontinuity at zero. Figure 1 plots the utility from gains and losses for $\alpha = \beta = 0.88$ and $\lambda = 2.25$, the parameter values Kahneman and Tversky obtained based on experimental data. The function is slightly concave in the positive domain (risk aversion) and slightly convex (risk seeking) in the negative domain with a kink at 0 (loss aversion). Notice that the function becomes nearly linear in its argument for large gains and losses, i.e. the agent is risk-neutral for large changes in wealth.

In addition to consumption, a Type B agent explicitly takes into consideration expected fluctuations in financial wealth in his decision-making. In line with prospect theory, we assume that his preferences exhibit loss aversion with respect to changes in financial wealth. The prospect theory, however, is a static model of choice under uncertainty and to incorporate it into a dynamic general equilibrium model, additional assumptions on whether and how prior gains and losses affect decision-making have to be made. Barberis, Huang and Santos (2001) show that loss aversion by itself cannot explain the large equity premium of stocks over bonds observed in historical data within the frames of a representative agent, complete markets model. However, allowing for prior investment performance to influence current and future investment decisions improves the performance of their model.

With small modifications to be discussed below, we adopt the preference specification suggested by Barberis, Huang, and Santos for Type B agent. For simplicity, they assume that $\alpha = \beta = 1$, i.e. in line with prospect theory the utility from gains and losses exhibits the loss aversion property (the utility function from losses is steeper than that from gains and the function is kinked at 0). However, the utility function is linear in gains and losses, i.e. a Type B agent is risk-neutral over gains and losses. In order to incorporate prospect theory in a dynamic model, the authors assume that the extent to which an investor is loss averse depends on his prior stock market performance. Thus, people are more willing to gamble after prior gains and more conservative after prior losses. This is the "house money" effect coined by Thaler and Johnson (1990). Thus, the Type B investor maximizes the following utility function suggested by Barberis, Huang, and Santos:

$$E\left[\sum_{t=0}^{\infty}\left(\rho^t \frac{(C_t^B)^{1-\gamma}}{1-\gamma} + b_0\rho^{t+1}v(X_{t+1}, z_t)\right) \mid \Omega(t)\right] \tag{3}$$

where C_t^B is the consumption of Type B agent in time t; S_t^B is the risky asset holdings of Type B agent in time t; z_t is a state variable that measures gains and losses prior to time t; $v(\cdot)$ is the utility the investor derives from financial gains or losses; X_{t+1} is the gain or loss from the risky asset holdings between t and $t+1$; b_0 is an exogenous scaling factor that controls for the relative importance of the "prospect theory" term in the utility function. If $b_0 = 0$, the model reduces to the standard preferences defined in Equation 1. Barberis, Huang and Santos scale the prospect theory term by $b_0\bar{C}_t^{-\gamma}$ where $\bar{C}_t^{-\gamma}$ is the aggregate per capita consumption at time t, exogenous to the investor in their model. As they allow for a constant growth rate in consumption and dividends, determined exogenously, this adjustment is necessary to ensure that the prospect utility term will not have an explosive impact on the utility function as the wealth in the economy grows. However, in our model this is not necessary as consumption is an endogenous process and we are looking for a stationary equilibrium where consumption is determined endogenously. In our model the individual and aggregate wealth are stationary over time.

The reference level with respect to which gains and losses are measured is usually assumed to be the status quo, in our case the value of the risky asset in period t. The gain or loss between t and $t+1$ is the difference between the value of the risky asset holdings in $t+1$ and t adjusted for the asset value in $t+1$ if instead, the value of the risky asset in t were invested in the risk-free asset:

$$X_{t+1} = P_t S_t^B (R_{t+1} - R_{f,t+1}) \tag{4}$$

where P_t and $P_{f,t}$ are respectively the (ex-dividend) price of the risky asset and the price of the risk-free asset measured in units of the consumption good at time t, R_{t+1} is the real gross rate of return on the risky asset between t and $t+1$, $R_{f,t+1}$ is the real gross rate of return on the risk-free asset between t and $t+1$ and and and d_t is the per-share dividend of the risky asset at time t.

Agents are identical in terms of their coefficient of relative risk aversion and discount factor. The first term in the utility function of Type B agent is identical with the instantaneous utility function of Type A agent. However, a Type B agent is also loss averse where the loss aversion property is captured by the second term in his utility function. Thus, the two types of agents essentially differ in their attitudes towards risk: a Type A agent dislikes only fluctuations in consumption while in addition, a Type B agent dislikes fluctuations in financial wealth as well.

Barberis, Huang, and Santos further assume that a Type B agent keeps track of losses and gains over time. In line with the "house money" effect, losses are more painful when they occur after prior losses than after prior gains. We define the "historical benchmark level", Z_t, as the per unit price of the risky asset that the investor remembers. If $Z_t > P_t$, the investor has realized a loss in time t and future losses will be more painful. Conversely, if $Z_t < P_t$, the investor has realized a gain in the stock market and $S_t(P_t - Z_t)$ serves as a cushion for future losses. For simplicity, we define $z_t = Z_t/P_t$. The investor has had prior losses if $z_t > 1$ and prior gains if $z_t < 1$. The cushion of prior gains increases with the increase of the rate of return on the risky asset. The law of motion for z_t is given by:

$$z_t = \eta \left(z_{t-1} \frac{\bar{R}}{R_t} \right) + (1 - \eta) \tag{5}$$

where $\bar{R}$ is chosen in such a way that in equilibrium, the median value of z is 1; η can be thought of as a proxy for the investor's memory. If $\eta = 0$, the investor has "no memory" and the historical benchmark level adjusts immediately to changes in the price of risky assets. In contrast, if $\eta = 1$, the investor has a long memory and prior losses and gains affect his decisions for a long period of time.

Barberis, Huang, and Santos assume that the price-dividend ratio is only a function of z_t and thus, the (endogenous) rate of return on the risky asset in period t is only a function of z_t as well. As a result, z_t is an endogenous state variable. Note that the meaning of endogenous state variable in this case has a slightly different meaning from what is typically meant by the term. Conventionally, the term "endogenous state variables" is used to describe members of the state vector in period t which are endogenous variables in period $t-1$. One of the difficulties in solving the model of Barberis, Huang, and Santos is that we have to solve simultaneously for the endogenous state variable z_t and the price-dividend ratio in period t. In our model we do not impose any restrictions on the price-dividend ratio and z_t is an endogenous state variable in the conventional sense: the state variable z_{t-1} is used to solve for the values of the endogenous variables in time t. z_t is found from Equation 5 after solving for the stock price (and thus, the rate of return on stocks for a given realization of d_t) in period t.

The utility from gains and losses depends on the prior stock market performance of Type B agent. Let $Y_{t+1} = [P_t, P_{f,t}, P_{t+1}, d_{t+1}]$ denote the array of variables that affect the utility from gains and losses. Let

$$v(X_{t+1}(Y_{t+1}, S_t^B), z_t) = v(Y_{t+1}, S_t^B, z_t) \tag{6}$$

If $z_t = 1$ (neither prior gains nor losses), the utility from gains and losses is given by:

$$v(Y_{t+1}, S_t^B, z_t = 1) = \begin{cases} X_{t+1} & \text{for} \quad X_{t+1} \geq 0 \\ \lambda X_{t+1} & \text{for} \quad X_{t+1} < 0 \end{cases} \tag{7}$$

where X_{t+1} is defined in Equation 4. Thus, the utility from a gain is given by the gain itself and the disutility from a loss is equal to the value of the loss penalized by a factor of $\lambda > 1$. λ is a measure of loss aversion; it indicates how much more painful a loss is than a corresponding gain. This utility function is a close approximation to the utility function suggested by Kahneman and Tversky for larger gains and losses.

If $z_t < 1$, the investor has accumulated prior gains that serve as a cushion if future losses occur. Losses which are completely cushioned by prior gains are not very painful but losses in excess of prior gains are penalized more severely. Thus, if the cushion created in period t is equal to or greater than the loss realized between t and $t+1$, i.e., if $S_t Z_t \leq S_t(P_{t+1} + d_{t+1})$, the disutility from a loss is equal to the loss itself. Losses in excess of prior gains are penalized more severely, by a factor of λ. More formally, if we update the cushion created in period t, $S_t(P_t - Z_t)$, by the risk free rate, for $z_t \leq 1$ we obtain:

$$v(Y_{t+1}, S_t^B, z_t) = \begin{cases} P_t S_t^B (R_{t+1} - R_{f,t+1}) & \text{for} \quad R_{t+1} \geq z_t R_{f,t+1} \\ P_t S_t^B (z_t - 1) R_{f,t+1} + \lambda P_t S_t^B \left(R_{t+1} - z_t R_{f,t+1}\right) & \text{for} \quad R_{t+1} < z_t R_{f,t+1} \end{cases} \tag{8}$$

Notice that Equation 8 reduces to Equation 7 when $z_t = 1$.

If the investor has accumulated prior losses on the stock market ($z_t > 1$), subsequent losses are more painful and are penalized more severely than when the investor has had prior gains. The penalty factor in this case or alternatively, the measure of loss aversion, $\lambda(z_t) > \lambda$, is increasing in prior losses:

$$\lambda(z_t) = \lambda + k(z_t - 1) \tag{9}$$

where $k > 0$.

The utility from gains and losses in the case of prior losses ($z_t > 1$) is given by:

$$v(Y_{t+1}, S_t^B, z_t) = \begin{cases} P_t S_t^B (R_{t+1} - R_{f,t+1}) & \text{for} \quad R_{t+1} \geq R_{f,t+1} \\ \lambda(z_t) P_t S_t^B \left(R_{t+1} - R_{f,t}\right) & \text{for} \quad R_{t+1} < R_{f,t+1} \end{cases} \tag{10}$$

Figure 2 shows Type B agent's utility from gains and losses for different values of z_t. When there are no prior gains or losses, $z_t = 1$, the disutility from a loss is greater than the utility from a gain of an equal magnitude as the utility from losses is steeper than the utility from gains, i.e. the utility from gains and losses exhibits the loss aversion property. The utility from gains is the same regardless of Type B agent's prior stock market performance. However, the disutility from a loss differs depending on whether the investor has had prior gains, losses or neither gains or losses. When a loss comes on the heels of prior losses, it is more painful than when there are neither prior gains nor losses. The dashed green line on Figure 2 shows the utility from gains and losses when there are prior losses. It is drawn for $z_t = 1.25$. Compared to the case of $z_t = 1$, the slope of the dashed green line is steeper for losses implying that losses are more painful when there are prior losses. When there are prior gains, how painful a subsequent loss is depends on how large the created cushion and the incurred loss are. The red dash-dotted line is drawn for $z_t = 0.5$, e.g. the investor has had substantial prior gains. In this case losses which are completely cushioned by prior gains are not penalized, i.e. the disutility from the loss is equal to the loss itself. However, losses in excess of the cushion are penalized by a factor of λ.

1.2. Endowments

In addition to preferences, agents are heterogeneous with respect to their labor income as well. In each period t, a Type i agent receives an exogenous labor income y_t^i for $i = A, B$, which is subject to idiosyncratic shocks. In addition, agents receive income if they have invested in stocks and/or bonds. The stock is a claim to a stochastic stream of dividends and agents face aggregate uncertainty if they invest in stocks.

The one-period zero-coupon bond yields one unit of the consumption good in period $t+1$ with certainty. Agents of Type A and B face standard budget constraints:

$$C_t^A + P_t S_t^A + P_{f,t} B_t^A = y_t^A + (P_t + d_t) S_{t-1}^A + B_{t-1}^A \tag{11}$$

$$C_t^B + P_t S_t^B + P_{f,t} B_t^B = y_t^B + (P_t + d_t) S_{t-1}^B + B_{t-1}^B \tag{12}$$

where B_t^i is the riskless asset holdings of Type i agent for $i = A, B$ at time t and y_t^i is the stochastic labor income of agent i for $i = A, B$ at time t.

We assume that there is no population growth and normalize the size of the population to 1. Thus, the aggregate income in the economy y_t at any t is given by:

$$y_t = \theta y_t^A + (1-\theta) y_t^B + d_t \tag{13}$$

where θ is the share of Type A agents in the economy.

1.3. Borrowing and Short-Sale Constraints

Agents can trade securities to transfer wealth across states and time in order to smooth their consumption. However, there are two assets in the economy, a riskless bond and a risky stock, while there are three exogenous shocks. As a result, agents cannot diversify away all risks as markets are incomplete. In addition, individuals face state-dependent short sale and borrowing constraints in the asset markets, which may limit their ability to smooth consumption in bad states of the world and thus, induce them to save more. The constraints are exogenously specified as it is common in the literature (Aiyagari, 1995; Heaton and Lucas, 1996; Marcet and Singleton, 1999).

The short sale constraint, $K_{s,t}^i$ faced by agent i for $i = A, B$ in time t depends on the agent's income. In each period t the short-sale constraint is given by:

$$S_t^i \geq K_{s,t}^i \quad \text{where} \quad K_{s,t}^i = m y_t^i \tag{14}$$

where $m \leq 0$. In our basic model, we rule out short sales, i.e. $m = 0$ irrespective of the state of the economy. However, we test our results for sensitivity to this assumption.

Individuals may not be able to smooth their consumption over time because of credit rationing. In each period t agents can borrow only a fraction $h \leq 0$ of their income:

$$B_t^i \geq K_{b,t}^i \quad \text{where} \quad K_{b,t}^i = hy_t^i \tag{15}$$

where $K_{b,t}^i$ is the state-dependent borrowing constraint faced by agent i for $i = A, B$. The borrowing constraint is binding in some states but not in others. Besides being a realistic feature of financial markets, the borrowing constraint ensures that consumers will not rollover debt, or get involved in Ponzi schemes.

2. Market Equilibrium

The equilibrium consumption and asset holdings as well as asset prices are determined endogenously in our model. Each consumer maximizes his stochastic consumption stream subject to the budget and portfolio constraints for a given stream of prices $\{P_t\}_{t=0}^{\infty}$ and $\{P_{f,t}\}_{t=0}^{\infty}$. Employing the Kuhn-Tucker conditions, the relevant stochastic Euler equations for consumer i's maximization problem for $i = A, B$ are given by:

- Bonds

Either

$$(C_t^i)^{-\gamma} = \rho E\left[\left(C_{t+1}^i\right)^{-\gamma} R_{f,t+1} \mid \Omega(t)\right] \quad \text{and} \quad B_t^i > K_{b,t}^i \tag{16}$$

or

$$(C_t^i)^{-\gamma} \geq \rho E\left[\left(C_{t+1}^i\right)^{-\gamma} R_{f,t+1} \mid \Omega(t)\right] \quad \text{and} \quad B_t^i = K_{b,t}^i \tag{17}$$

for $i = A, B$

- Stocks

Either

$$(C_t^A)^{-\gamma} = \rho E\left[\left(C_{t+1}^A\right)^{-\gamma} R_{t+1} \mid \Omega(t)\right] \quad \text{and} \quad S_t^A > K_{s,t}^A \tag{18}$$

or

$$(C_t^A)^{-\gamma} \geq \rho E\left[\left(C_{t+1}^A\right)^{-\gamma} R_{t+1} \mid \Omega(t)\right] \quad \text{and} \quad S_t^A = K_{s,t}^A \tag{19}$$

Either

$$(C_t^B)^{-\gamma} = \rho E\left[(C_{t+1}^B)^{-\gamma} R_{t+1} \mid \Omega(t)\right] + b_0 \rho E\left[\hat{v}(Y_{t+1}, z_t) \mid \Omega(t)\right] \quad \text{and} \quad S_t^B > K_{s,t}^B \tag{20}$$

or

$$(C_t^B)^{-\gamma} \geq \rho E\left[(C_{t+1}^B)^{-\gamma} R_{t+1} \mid \Omega(t)\right] + b_0 \rho E\left[\hat{v}(Y_{t+1}, z_t) \mid \Omega(t)\right] \quad \text{and} \quad S_t^B = K_{s,t}^B \tag{21}$$

where for $z_t \leq 1$

$$\hat{v}(Y_{t+1}, z_t) = \begin{cases} R_{t+1} - R_{f,t+1} & \text{for} \quad R_{t+1} \geq z_t R_{f,t+1} \\ R_{f,t+1}(z_t - 1) + \lambda\left(R_{t+1} - z_t R_{f,t+1}\right) & \text{for} \quad R_{t+1} < z_t R_{f,t+1} \end{cases} \tag{22}$$

and for $z_t > 1$

$$\hat{v}(Y_{t+1}, z_t) = \begin{cases} R_{t+1} - R_{f,t+1} & \text{for} \quad R_{t+1} \geq R_{f,t+1} \\ \lambda(z_t)\left(R_{t+1} - R_{f,t+1}\right) & \text{for} \quad R_{t+1} < R_{f,t+1} \end{cases} \tag{23}$$

If the short sale and borrowing constraints are non-binding, the Euler equations are given by Equations 16, 18, and 20. The Euler equations for bonds are standard: if the consumer decreases incrementally his consumption in period t and invests his savings in the riskless asset, his utility cost in t should be equal to the discounted expected value of the utility benefit in $t+1$ adjusted for the rate of return on the riskless asset between t and $t+1$. This is a necessary condition for optimality for any t. Similarly, the Euler equation for stocks for Type A agent is standard and has a similar interpretation. However, the Euler equation for stockholders has a different interpretation. If the stockholder reduces his consumption by an infinitesimal amount in time t and invests the savings in the risky asset, his utility cost in t should be equal to the discounted value of the expected utility benefit in the next period adjusted for the expected rate of return on the risky asset plus the expected change in the value of the risky assets. When the investor realizes a loss, $\hat{v}(\cdot)$ is negative implying that he would require a higher expected rate of return to invest in stocks. How high the required expected rate of return would be depends on whether the investor has had prior losses or gains. If he has had prior losses, he is more loss averse and he would require a higher rate of return on the risky asset to invest in it and conversely, if he has had prior gains, he would require a lower rate of return.

For simplicity, the outstanding shares of the risky asset are normalized to one. We only allow for private borrowing and lending and therefore, bonds are in zero net supply. Thus, the market clearing conditions for stocks and bonds in each period t are given by:

$$\theta S_t^A + (1-\theta) S_t^B = 1 \tag{24}$$

$$\theta B_t^A + (1-\theta) B_t^B = 0 \tag{25}$$

We assume that there is no population growth and the population size is normalized to one. Walras law guarantees that the goods market clear, i.e. $\theta C_t^A + (1-\theta) C_t^B = y_t$ is satisfied for each

t. The aggregate income in the economy y_t is given by Equation 13.

Each agent faces idiosyncratic shocks to his labor income as well as aggregate shocks to the per share dividend. Markets are incomplete as while there are three sources of uncertainty, there are only two markets, the bond and the stock markets, to hedge consumption risks. In addition, borrowing and short sale constraints limit the agents' ability to smooth consumption across states and time.

Information is complete and symmetric, i.e. both agents know the past realizations of stock prices as well as shocks to their individual incomes and the per share dividend. There are eight endogenous variables in our model in each t: P_t, $P_{f,t}$ and C_t^i, B_t^i, S_t^i for $i = A$, B. We use the four (relevant) Euler equations, the equilibrium conditions (Equations 24 and 25), the income process (Equation 13) and the budget constraint for Type A agent to find the equilibrium distributions of the endogenous variables as a function of the state vector. Because of Walras Law, the budget constraint for Type B agent is redundant. For convenience, we discuss the state vector separately below.

3. State Variables

3.1. Exogenous State Variable

The exogenous state of the economy at every t is given by $[\ln(y_t^A)\ln(y_t^B)\ln(d_t)]'$ where ln denotes the natural logarithm. The income of Type i agent for $i = A, B$ is assumed to have a stationary distribution, i.e. we rule out endowment growth. It follows a stationary first-order autoregressive process:

$$\ln(y_t^i) = \omega_1^i + \omega_2^i \ln(y_{t-1}^i) + e_t^i \tag{26}$$

where e_t^i for $i = A, B$ is the stochastic component of the labor income of stockholders and non-stockholders.

There is a huge body of literature on the dynamic process that governs the individual earnings recorded in longitudinal studies. While this process does not seem to be clearly understood as yet, it is clear that shocks to individual earnings are highly persistent and follow a complex dynamic structure. Similarly to Deaton (1991), we assume that the labor incomes of stockholders and non-stockholders are subjected to three different transitory shocks which are orthogonal to each other: an aggregate shock $\varepsilon_{1t} \sim Niid(0, \sigma_1^2)$ which is common to both types of agents; a type- or group-specific shock $\varepsilon_{2t}^i \sim Niid(0, \sigma_{2i}^2)$; and an idiosyncratic shock $\varepsilon_{3t}^i \sim Niid(0, \sigma_{3i}^2)$ where $i = A, B$:

$$e_t^i = \varepsilon_{1t} + \varepsilon_{2t}^i + \varepsilon_{3t}^i \tag{27}$$

An agent of Type i for $i = A, B$ observes e_t^i but not its components. The variances of the shocks to the labor incomes of stockholders and non-stockholders, denoted with σ_i^2 for $i = A, B$, are given by:

$$\sigma_i^2 = \sigma_1^2 + \sigma_{2i}^2 + \sigma_{3i}^2 \tag{28}$$

For identification purposes, we further assume that the group-specific shocks are completely diversified away in the aggregate:

$$\pi\varepsilon_{2t}^A + (1-\pi)\varepsilon_{2t}^B = 0 \tag{29}$$

Therefore, as suggested by empirical data (see for example, Marcet and Singleton, 1999), shocks to the labor incomes of stockholders and non-stockholders are correlated and their covariance is given by $E[e_t^A e_t^B] = \sigma_1^2 - \frac{\pi}{1-\pi}\sigma_{2A}^2$.

The aggregate dividend follows a stationary first-order autoregressive process:

$$\ln(d_t) = \varphi_1 + \varphi_2 \ln(d_{t-1}) + e_t \tag{30}$$

The aggregate dividend is subjected to the aggregate shock in the economy as well as an idiosyncratic shock:

$$e_t = \varepsilon_{1t} + \varepsilon_{3t} \tag{31}$$

where $\varepsilon_{3t} \sim Niid(0, \sigma_3^2)$. Therefore, the variance of e_t, σ^2, is given by

$$\sigma^2 = \sigma_1^2 + \sigma_3^2 \tag{32}$$

As both the individual labor incomes and the dividend process are subject to the aggregate shock in the economy, the dividend process covaries with the individual income processses. The variance-covariance matrix of the vector of innovations to the exogenous state variables in the economy is given by:

$$\begin{pmatrix} \sigma_A^2 & \sigma_1^2 - \frac{\pi}{1-\pi}\sigma_{2A}^2 & \sigma_1^2 \\ \sigma_1^2 - \frac{\pi}{1-\pi}\sigma_{2A}^2 & \sigma_B^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma^2 \end{pmatrix} \tag{33}$$

The section on calibration below provides details on data estimation and calibration.

3.2. Endogenous State Variable

The state vector contains endogenous variables as well. These are the elements of wealth defined in the previous period as well as prior investment outcomes, i.e. $B_{t-1}^A, S_{t-1}^A, z_{t-1}$.

III Calibration

1. Law of Motion for the Exogenous State Variables

We use annual data from the Panel Study of Income Dynamics (PSID) to calibrate the income processes of Type A and Type B agents and data from NIPA accounts to calibrate the aggregate dividend process. Our results are consistent with results reported by Abowd and Card (1989), Heaton and Lucas, and Marcet and Singleton, which suggest that aggregate variables have little impact on the conditional mean of individual incomes. As a result, we assume that lagged values of the aggregate dividend have no impact on the individual income processes.

We use PSID data to classify the families in our sample as stockholders and non-stockholders. In our sample of 903 families, 566 (about 65 %) own shares of stock and 337 (about 35 %) do not. Thus, we set the proportion of non-stockholders in the population $\theta = 0.65$ and the proportion of stockholders to 0.35.

To estimate the income processes of the"representative" stockholder and non-stockholder, we use the aggregation method suggested by Heaton and Lucas. We first compute the real labor income per capita by deflating the family income from PSID by the aggregate price level and accounting for the number of family members. For each household in our sample we use linear regression to estimate Equation 26. The parameters in Equation 26 as well as the standard deviation of the residual are the cross-sectional means of the corresponding parameter estimates for the non-stockholders and the stockholders in our sample. Appendix 2 provides details on our estimation procedure. The results are summarized in Panel A of Table 1.

Income shocks to the individual labor incomes are highly persistent with the shocks to the labor income of non-stockholders slightly more persistent than the shocks to the labor income of stockholders. The estimated cross-sectional mean of the standard deviation of the idiosyncratic shocks to the labor income of non-stockholders is $\sigma_A = 0.3$ with a standard deviation of 0.12 while the corresponding value for stockholders is $\sigma_B = 0.28$ with a standard deviation of 0.11.

Our results are consistent with empirical estimates based on microeconomic data. For example, based on PSID data MaCurdy (1982) finds that the standard deviation of the residual in a regression with real labor income per capita in logarithms as the dependent variable is 0.58. However, Deaton (1991) argues that MaCurdy's estimate overstates the true volatility of innovations to the individual labor income because of measurement errors. Deaton suggests that this volatility for shocks to the logarithm of income in first differences should be between 0.1 and 0.15. Assuming that innovations to data in first differences are twice as volatile as innovations to data in levels, Deaton's argument implies that the volatility of shocks to the logarithm of individual income in levels should be between 0.05 and 0.08. As a result, we scale down the estimated volatility of the shocks to the individual income processes that we estimate by about 2/3, i.e. we assume that $\sigma_A = 0.09$ and $\sigma_B = 0.08$ thus placing the volatility of shocks to the individual incomes of Type A and B agents just above the upper bound suggested by Deaton.

The PSID does not provide data on the dividend income for the whole sample period. We use annual data on the net dividend from the National Income and Product Account (NIPA) tables published by the Bureau of Economic Analysis to calibrate the process of the aggregate dividend. To increase the precision of our estimates, we use post-war dividend data, which spans the period from 1947 to 2006. We weigh the dividend by the CPI and the U.S. population in a given year to obtain the real dividend per capita. Data on the U.S. population is obtained from the U.S. Census Bureau.

The regression estimates of the parameters in Equation 30 for the detrended series of the real dividend in logarithms are presented in Panel B of Table 1. The estimated standard error of the shock to the dividend process σ is 0.06. We ajust the intercept to match the dividend share in aggregate income in the data. The aggregate income in the economy in any given year is the weighted sum of the individual labor incomes of the two agents and the aggregate per capita dividend. The aggregate income is normalized, so that its average is 1.

The variance-covariance matrix of the vector of innovations to the exogenous state variabes in the economy is given by:

$$\begin{pmatrix} 0.0275 & 0.0021 & 0.0022 \\ 0.0021 & 0.0236 & 0.0022 \\ 0.0022 & 0.0022 & 0.0034 \end{pmatrix} \tag{34}$$

For non-stockholders and stockholders, idiosyncratic income shocks account for most of the volatility of transitory shocks (92 % and 90 %, respectively). Group-specific shocks are more important for stockholders than for non-stockholders but for both types of agents they account for less than 1 % of the total volatility of transitory shocks. Systematic risk in the economy effects non-stockholders more than stockholders and accounts for most of the volatility in the aggregate dividend.

2. Structural Parameters

Table 2 summarizes the chosen parameters for the model. The discount factor ρ is set equal to 0.96. There is still an ongoing debate on the average value of the coefficient of risk aversion (see, for example, Kocherlakota, 1997). As discussed in the Introduction, the equity premium puzzle exists only if we assume that values of γ greater than 10 are implausible. We set $\gamma = 2$, well within the plausible region suggested by Mehra and Prescott.

The importance of the prospect theory term in the overall utility of Agent B is controlled by b_0 and η is a proxy for investor's memory. For our baseline model we adopt the parameter values of η and the lower bond of b_0 suggested by Barberis, Huang, and Santos. However, we test the sensitivity of our model to these parameter values. λ penalizes losses when there are no prior gains or losses while k is the penalty factor for losses when they occur after prior losses. We set $k = 1.4$ and $\lambda = 2.25$, so that in equilibrium the average loss aversion coefficient is close to 2.25, the value estimated by Kahnemnan and Tversky (1992) based on experimental data.

While it is intuitive that the borrowing constraint is a function of the individual income, it is not immediately clear what the lower bound of the constraint is. For our baseline model we set h = -1/3 and therefore, the state-dependent borrowing constraint is given by $K^i_{s,t} = -1/3y^i_t$ for $i = A, B$. Even though our results presented below show that the borrowing constraint is rarely binding, we test the sensitivity of our results to this assumption. In our baseline model we rule out short sales, $m = 0$.

IV Solution Algorithm

We solve for the stationary stochastic equilibrium numerically using a modification of the parameterized expectations algorithm (PEA) developed by Marcet (1988) and den Haan and Marcet (1990, 1994). Marcet and Singleton (1999) extend the algorithm to account for agent heterogeneity. Appendix 1 offers a concise discussion of the numerical algorithm. We simulate the equilibrium path of the economy for 2,000 periods and exclude the first 100 periods to eliminate any impact of the initial conditions on our results. All computations are executed in Matlab.

V Results

1. Representative Agent Models

To evaluate the performance of our model, we have to compare our estimates to the corresponding values reported in empirical studies. While an average equity premium of 6% and a risk free rate of return of 1% in real terms are widely cited in the literature, these empirical values (and their volatility) are not robust to the sample period. Siegel (1999), for example, reports an equity premium of 4.1% based on U.S. data for the period 1802-1998. However, the equity premium has been more pronounced during the post World War II period predominantly due to a decrease in the risk-free rate.

The moments of asset returns observed in historical data that we use as a benchmark for comparison with our results are obtained from data provided by Robert Shiller on his website. The risk-free rate is the annualized real rate of return on Treasury bills and the stock return is the annual rate of return on the Real S&P 500 Stock Price Index. We estimate the moments of asset returns for two data samples: the Full Sample covers the period from 1871 to 1997 and the Postwar Sample covers the period from 1947 to 1997. We use the empirical values of the moments of asset returns computed based on the Postwar Sample as a benchmark because this period more closely matches the sample period of the data we use to estimate the law of motion of the individual income and aggregate dividend processes.

We first solve the complete-market model to check whether accounting for idiosyncratic income shocks and loss aversion has the potential to improve the results obtained in a representative-agent model. To isolate the impact of idiosyncratic income shocks on asset returns from the impact of loss aversion, we solve three different models:

- **Model 1:** The representative agent's preferences in this economy are standard, given by Equation 1. In a complete market, non-systematic risk will be diversified away and only systematic risk will be priced. Therefore, we assume that idosyncratic and group-specific shocks are diversified away and thus, the volatility of the shock to the aggregate labor income in the economy is set equal to the standard deviation of the aggregate shock in the economy, 0.046. The aggregate labor income is set equal to the labor income of Type B agent, which is only subject to the aggregate shock in the economy. Essentially, we solve the model of Mehra and Prescott. However, in contrast to Mehra and Prescott, who set the consumption of the representative agent equal to the aggregate per share dividend, we set the consumption equal to the aggregate income in the economy, which is the sum of the per share dividend and the aggregate labor income.

- **Model 2:** Model 2 is identical to Model 1, except for the fact that the aggregate labor income in the economy is set equal to the labor income of Type B agent, i.e. in this model we account for idiosyncratic income shocks. However, we do not account for loss aversion.

- **Model 3:** This model is identical to Model 2 except for the fact that the representative agent's preferences account for loss aversion (Equation 3). This model is similar to the model solved by Barberis, Huang, and Santos. However, there are several notable difference between the two models. First, they allow for growth in the economy while we do not to be consistent with our heterogenous agent models. Second, in their model the risk-free rate is exogenous while the price-dividend ratio used to compute stock returns is a function of z_t, which, in turn, is a function of stock returns. Thus, in their model z_t is an endogenous state variable. The endogeneity of the state variable is a source of additional volatility in their model which enables them to generate stock returns which are much more volatile than the underlying dividend process. In our model, we do not place any restrictions on the price-divided ratio and z_t is an exogenous state variable while the endogenous variables are security prices. If we were to use the same exogenous income process as Barberis, Huang, and Santos do, our model should predict lower return volatility compared to their model.

Results are presented in Table 3. Column 4 reports the simulated moments of asset returns and consumption implied by Model 1. The results are an illustration of the well-known puzzles associated with the Consumption Capital Asset Pricing Model. Compared to their empirical counterparts, the average risk-free rate is too high (the risk-free rate puzzle), the average stock return is too low and thus, the equity premium is too low (the equity premium puzzle). The model also fails to account for the second moments of the empirical distributions of asset returns. The stock return is much less volatile than its empirical counterpart (the volatility puzzle) and thus, the Sharpe ratio is only a fraction of its empirical value.

The results for Model 2 are consistent with results obtained by Heaton and Lucas for a complete market economy with income processes estimated from PSID data. Not surprisingly, the equity premium generated by the model is higher than the premium generated by models based on aggregate data because microeconomic data are more volatile than aggregate data thus introducing more uncertainty in this economy. Therefore, idiosyncratic income shocks have the potential to generate high and volatile excess stock returns if there are no mechanisms in the economy which allow for risk diversification.

Our results suggest that Model 3 outperforms Model 2. Introducing loss aversion as another source of risk generates higher and more volatile equity premium. While the excess equity premium of about 10% generated by Model 3 compared to Model 2 may appear trivial, we need to keep in mind that in our benchmark parameterization on average, the loss aversion term in the utility function accounts for about 20% of the overall utility. The major source of utility for the representative agent is consumption and to a lesser extent, fluctuations in financial wealth. Therefoe, allowing for heterogeneity in preferences may enable us to obtain a better match to the equity premium observed in historical data.

2. Heterogenous Agents: Loss Aversion

The sample moments of the distributions of asset returns for our calibrated economy are reported in Table 4. Our model matches the first moments of the distributions of asset returns very closely and thus, it is able to generate an equity premium of about 6%, the historical risk premium widely cited in the literature, while maintaining a low risk-free rate. In line with historical data, our model generates a time-varying risk premium (see Figure 3). Stocks in this economy are more volatile than bonds and as a result, they offer higher returns. However, the average stock and bond volatilities predicted by our model exceed the corresponding volatilities observed in historical data suggesting that our model understates the amount of diversification which takes place in financial markets.

Our model also matches closely the mean and volatility of the price-dividend ratio. It is an important result as the representative agent model of Barberis, Huang, and Santos fails to match the volatility of the price-dividend ratio. For example, their representative agent model with $b_0 = 0.7$ and $k = 3$, the parameter values closest to our parameterization, generates an equity premium of 1.3% with a standard deviation of 17.39%, and a mean price-dividend ratio of 29.8 with a standard deviation of 2.9. The reason is that in the representative agent model of Barberis, Huang, and Santos, the price-divdidend ratio is a function of the state variable z_t alone, $f(z_t)$. Thus, the price-dividend ratio (and its growth rate) is not sufficiently volatile because the volatility of f is low. Nonetheless, they are still able to match the volatility of stock returns because of the high positive correlation (in fact, perfect conditional correlation) of the price-dividend ratio growth rate with the shock to the aggregate dividend. In our model the stock price is endogenous, a function of the state variables in the model. It clears the stock market which is used by both agents for consumption smoothing. The stock price generated by our model is procyclical and quite volatile as Figure 4 shows. The stock price tends to be high relative to the per-share dividend thus pushing up the price-dividend ratio. In addition, the high relative volatility of the stock price and the low relative correlation of the stock price with the aggregate per share dividend enable us to match the volatility of the price-dividend ratio.

The consumption processes for the two agents for 1,000 periods are depicted in Figure 5. Consistent with empirical evidence (Mankiw and Zeldes, 1991), the consumption of Type B agent has higher mean and volatility than the consumption of Type A agent. The result is not surprising as Type B agent has higher average income and higher income volatility as well (see Figure 10). In addition, Type B agent uses the more volatile market, the stock market, to smooth consumption. Furthermore, prior gains and losses in the stock market are an additional source of consumption volatility for Type B agent.

When markets are complete all non-systematic risk is diversified away. Thus, one of the implications of the complete market model, where the representative agent has CRRA utility function, is that in equilibrium, individual consumption is perfectly correlated with aggregate income. Therefore, one measure for the extent of market incompleteness is the departure of the correlation of individual consumption with aggregate income from 1. Our results are in line with previous studies (see for example, Telmer, 1993; Lucas, 1994), which indicate that even when markets are incomplete, agents can diversify away the idiosyncratic risk through trading in financial markets and accumulating precautionary savings. In this economy, individual consumptions are more strongly correlated with aggregate income than with individual incomes suggesting that idiosyncratic shocks are diversified away to some extent. However, in out model idiosyncratic income shocks are volatile and persistent. As a result, part of the idiosyncratic risk cannot be diversified away and the consumption of both types of agents is less than perfectly correlated with the aggregate income. The consumption of Type A agent is more strongly correlated with the aggregate income than the consumption of Type B agent, suggesting that non-stockholders in this economy are better able to diversify away idiosyncratic shocks. There are two mechanism in our model which drive this result. First, non-stockholders hold most of the income in the economy and therefore, their individual income is more highly correlated with the aggregate income. Stockholders hold a disproportionate share of aggregate income: while they represent only 35% of the population, they hold about 45% of the aggregate income. Nonetheless, the income of Type B agent is not as strongly correlated with the aggregate income as the income of Type A agent. Second, Type B agent values fluctuations in stock returns by themselves, not only as an intertemporal hedge against consumption risks.

Consistent with historical data, our model also predicts a low correlation between the individual consumption processes and the real rate of return on stocks. The consumption of non-stockholders is more highly correlated over time with stock returns than the consumption of stockholders. This result is at odds with empirical data suggesting the opposite. One possibility is that the result is driven by the different income processes. As the income of Type A agent has a lower mean, on average stock returns represent a larger share of his liquid wealth in a given period compared to Type B agent. Thus, his consumption more closely tracks stock returns. Another possibility is that it is driven by the preference heterogeneity. While for Type A agent the source of risk is the correlation of his consumption process with the rate of return on the risky asset, Type B agent faces a second source of risk as well, namely fluctuations in his financial wealth. Thus, the consumption process of Type B agent is sensitive not only to fluctuations in the rate of return on risky assets but to changes in his financial wealth as well as prior losses and gains.

Agents trade on the bond and stock markets to smooth their consumption. The volume of trade in both stocks and bonds is high as Figures 6, 7, 8, and 9 show. Through trading, both agents achieve smoother consumption than their individual income processes even though idiosyncratic risks are not completely diversified away. The standard deviation of Agent A's consumption is 0.12 while the standard deviation of his individual income is 0.15. The standard deviation of Type B's consumption is 0.19 while his income volatility is 0.22.

The consumption of Type B agent is negatively correlated with prior stock market performance implying that consumption tends to be high when the investor has had prior gains in the stock market (see Figure 11). In fact, prior gains and losses affect the consumption of Type A agent as well through general equilibrium effects.

The equilibrium distribution of z is depicted on Figure 12. $\bar{R}$ is set equal to 1.04 to ensure that in equilibrium, the median value of z is 1, i.e. half of the time Agent B has losses and half of the time he has gains. z appears to be normally distributed. It is quite volatile with a standard deviation of 0.25. The distribution of z as well as the relationship between z_t and the price-dividend ratio shown in Figure 13 are consistent with the results of Barberis, Huang, and Santos in a complete market economy. As expected, z_{t-1} and R_t in our data have a high positive correlation (see Figure 14). In the presence of prior losses ($z > 1$), higher expected rates of return on the risky asset are necessary to induce Type B agent to invest in the stock market.

The bond holdings and the bond constraints for Type A and B agents are depicted in Figures 6 and 7. The bond constraint is weak in the sense that it is rarely binding for both types of agents. On average, Type A agent is a lender and Type B agent is a borrower. The stock market is more volatile than the bond market as Figures 8 and 9 show. While Type B agents (stockholders) represent only 35% of the population, they hold about 50% of the stocks in the economy.

Figure 15 shows the income, bond, and stock holdings of Type A agent for 200 periods. Even though the borrowing constraint, which is state-dependent, is less severe for Type B agent, he is more often closer to his constraint than Type A agent. Stock holdings are highly positively correlated with the individual income process of Type B agents (a correlation coefficient of 0.5) and virtually uncorrelated with the individual income process of Type A agent (a correlation coefficient of 0.04). Bond holdings are positively correlated with the income of non-stockholders (a correlation of 0.53) and negatively correlated with the income of stockholders (correlation of -0.65). Thus, Type A agent uses the bond market to smooth his consumption. He borrows when his income is low and lends when his income is high. In contrast, Type B agent uses the stock market to smooth his consumption as Figure 16 shows and borrows to maintain his stockholdings when his income is low.

We are particularly interested in quantifying the effect of the loss aversion term in the preferences of Type B agent on our results. To do so, we simulate a model (Model 5) where we keep all the properties of the model presented above with one exception: we assume that both agents have the standard preferences of Type A agent. The results for this simulated economy are presented below. We also simulate an economy with loss aversion where both agents have identical income processes, given by the income process of Type B agent and therefore, the only source of heterogeneity in this economy is heterogeneity in preferences.

3. Heterogenous Agents: Homogenous Standard Preferences, Heterogeneous Income Processes

The third row of Table 5 presents our results for a model with heterogeneous agents with identical standard preferences (Model 5). This model is essentially the model of Marcet and Singleton (1999) calibrated from microeconomic data and with different representative agents. The model generates a risk premium of 5.04% but as in the model with loss aversion, it fails to approximate the second moments of the empirical distributions of asset returns. Therefore, accounting for loss aversion increases the risk premium by about 20%. This is a significant contribution to explaining the equity premium given that the weight of the loss aversion term in the total utility of Type B agent is about 20%. Investors in our model derive utility from both consumption and fluctuations in financial wealth but the utility from consumption weighs more heavily in decision-making, and thus, investors care predominantly about consumption smoothing rather than stock market performance. The result is also noteworthy because it indicates that loss aversion is, indeed, preserved in the aggregate even though investors represent only 35% of the population.

A comparison of the correlation coefficients obtained under the two models shows that the introduction of loss aversion has a negligible impact on the co-movements of individual consumption processes with stock returns, individual incomes, and the aggregate income. However, several interesting patterns emerge. First, the volume of trade in both stocks and bonds is higher in the economy with loss aversion (see Figures 17, 18, 19, 20). Second, agents' consumption and savings behavior changes in the economy with loss aversion (Model 4). In the economy without loss aversion (Model 5) both agents smooth consumption through trading in the stock market. This is possible because the individual income processes are only weakly positively correlated. However, in the economy with loss aversion Type B agent uses the stock market to smooth his consumption while Type A agent uses the bond market. Interestingly, the average stock holdings of Type B agent fall in the economy with loss aversion (56% vs. 64% of the total). The result is driven by the asymmetric affect of gains and losses on the utility of Type B agent in Model 4. While on average, half of the time Type B agent has losses and half of the time he has gains, losses are more painful than gains and as a result, the average utility from fluctuations in financial wealth is negative. Thus, Type B agent changes the composition of his portfolio and invests less heavily in the risky asset. Therefore, individuals with preferences which account for loss aversion will not self-select themselves as investors.

Another interesting result is that the loss aversion property in the utility function of Type B agent leads to a decrease in the co-movements of consumption and the risky asset return, a result which is at odds with empirical data. However, this result is consistent with the preference specification. Type B agent in Model 4 cares not only about consumption but about fluctuations in his financial wealth as well. Thus, stocks are valued per se, not only as a means for buffering income shocks.

In conclusion, while the additional source of risk introduced through the preference specification of Type B agent can enable us to account for the empirical moments of asset returns, the model also produces results which are at odds with empirical data. Specifically, loss aversion induces stockholders to hold fewer stocks on average and generates a consumption process which is less correlated with stock returns than the consumption of non-stockholders.

4. Heterogenous Agents with Identical Income Processes and Loss Aversion

Finally, we solve a model (Model 6) which is identical with Model 4 except for the fact that agents have identical income processes, given by the income process of Type B agent. The purpose of this exercise is to isolate the effect of the preference heterogeneity on our results. Results are presented in the the fourth row of Table 5.

The model demonstrates that loss aversion is not well-preserved in the aggregate when the income processes are identical. Now, stockholders hold only 35% of the aggregate income, which corresponds to their share in the population. As a result, equity premium generated by the model is lower than the equity premium in the model with heterogeneous income processes (Model 4).

The model also demonstrates one of the mechanism in the preference specification of Type B agent, which drives up the risk premium. For Type B agent, the risk-free rate is the opportunity cost for holding stocks. His utility falls when the risk-free rate rises. This drives down the equilibrium risk-free rate. The higher the share of aggregate income held by Type B agent, the higher the impact of his preferences on the equilibrium asset returns. Thus, in this economy, the risk-free rate is higher compared to Model 4 economy and therefore, the equilibrium equity premium is lower.

VI Robustness

1. The Significance of the Prospect Utility Term in Type B Agent Preferences

On average, the share of the loss aversion term in the utility function of Type B agent in our model is 0.2. Therefore, it is of interest to explore the implications of changing the weight of the utility from gains and losses in the overall utility function. Table 6 presents the sensitivity of our results to changes in b_0, the term which controls for the importance of prior stock market outcomes in the utility function.

Our results show that the performance of our model in terms of its ability to match the first moments of the empirical distributions of asset returns increases with the increase in b_0. When $b_0 = 1.5$, the model generates a risk premium of 6.6% in real terms. This represents a 31-percent increase in the risk premium compared to a model without loss aversion. It is also notable that this increase in the equity premium is due to both a decrease in the risk-free rate and an increase in the return on the risky asset. When $b_0 = 0.5$, on average the loss aversion term weighs for 15% of the total utility of Type B agent while when $b_0 = 1.5$, the weight of the loss aversion term increases to 33%. This result indicates that accounting for loss aversion (and more precisely, for loss aversion and prior stock market performance) in preferences improves significantly the performance of the model and enables us to match closely the empirical distributions of asset returns.

2. Investor's Memory

Barberis, Huang, and Santos argue that loss aversion by itself is not able to account for the equity premium puzzle. In their model, accounting for prior stock-market performance is instrumental in matching the moments of the empirical distributions of asset returns. The higher the η, the longer the impact of prior stock market performance on the decision-making of Type B agent. A prior loss, which is remembered by the investor for a long period of time, pushes up the expected equity premium (and its volatility) and thus, the current stock price down in a complete market, representative-agent model.

Our results presented in Table 7 show that an increase in η decreases the rate of return on the risk-free asset and increases the investor's loss aversion. However, there is no clear pattern of a rise in the equity premium or asset volatilities. The reason is that Type B agent becomes more loss averse as η rises and thus, in this economy with heterogenous agents, he reallocates his portfolio by investing more heavily in bonds and less heavily in the more risky asset, stocks.

3. Disutility of a Loss After Prior Losses

Parameter k controls for the disutility of a loss when it occurs after prior losses. The higher the k, the more painful a loss is when it follows prior losses. Essentially, k governs Type B agent's loss aversion after prior losses and not surprisingly, loss aversion rises with k and so do the equity premium and its volatility. When the investor is more loss averse, he requires a higher expected equity premium to invest in the risky asset.

4. Borrowing Constraints

We test our results for robustness to the specification of the state-dependent borrowing constraint. There are two channels through which borrowing constraints affect consumption and savings decisions in any given period. The first one is intuitive: when the borrowing constraint is binding for an individual, his consumption is lower than what he would have consumed in the absence of the constraint. The price of bond is determined by the stochastic discount factor of the unconstrained agent. Compared to the unconstrained optimization, when the borrowing constraint is binding the constrained agent consumes less and the unconstrained agent more. This decreases the marginal utility of consumption for the unconstrained agent. His stochastic discount factor rises thus increasing the bond price, pushing down the return on the riskless asset and widening the equity premium. The second channel is emphasized by Zeldes (1989). When making consumption and savings decisions, an individual takes into consideration the probability with which borrowing constraints will be binding in the future. As long as there is some positive probability that the borrowing constraint will be binding in some future period, an individual consumes less today than if that probability were zero thus raising the demand for precautionary savings. Therefore, in any given period, even when the Euler equations are satisfied with an equality, borrowing constraints expected to be binding in the future affect the current period consumption and savings decisions.

Table 9 shows the equilibrium distributions of the variables of interest for different specifications of the state-dependent borrowing constraint. As expected, tightening the borrowing constraints increases the equity premium generated by our model by pushing down the rate of return on the risk-free asset as the risk-sharing opportunities shrink. However, the borrowing constraints in the benchmark model are rarely binding in equilibrium (in less than 5% of the cases for both agents) and as Table 9 shows, weakening the constraints from about 35% of current income in the benchmark model to 45% has a negligible effect on our results.

4. The Share of Non-Stockholders in the Population

Historically, the equity premium has decreased. This is evident in our data samples. The equity premium for the period from 1871 to 1997 is higher compared to the post-war period. One plausible explanation for this fall in the equity premium is the increased risk-sharing opportunities as more individuals enter into the stock market. To explore this possibility, we vary the share, π, of non-stockholders in the population. Our results presented in Table 10 suggest that the equity premium varies non-linearly with the share of non-stockholders in the population. Our model predicts that as the share of non-stockhders falls from 0.8 to 0.65, the equity premium falls from 9.39% to 5.96%. The risk-sharing possibilities are maximized when the population is equally split into stockholders and non-stockholders and the equity premium starts rising again when the share of stockholders exceeds the share of non-stockholders in the population.

VII Conclusion

We have calibrated a general equilibrium model with heterogeneous agents, incomplete markets, and portfolio constraints. While Type A agents are non-stockholders and have the standard preferences used in macroeconomics, Type B agents are stockholders who explicitly take into consideration prior stock market performance when making consumption and savings decisions. In equilibrium, consumers hold different portfolios and use both the stock and bond markets for consumption smoothing. Our results suggest that prior investment performance has a significant impact on the decision-making of Type B agents. Market clearing conditions and the fact that Type B agent holds a disproportionately large share of aggregate income imply that prior gains and losses in the stock market have spillover effects on the decision-making of Type A agents as well.

Our model generates a high average equity premium of about 6% while the risk-free rate is kept low at 2.1%. However, our model generates stock return volatilities which exceed the volatility of their empirical counterparts. Our model predicts that the equity premium falls when more investors enter into the stock market as risk-sharing opportunities increase. In line with historical data, the individual consumption has low correlation with stock returns. However, stockholders' consumption is less correlated with stock returns than the consumption of non-stockholders. This property, which is at odds with empirical data, stems from the preference specification of Type B agent. Because half of time an investor has losses, which are more painful than gains, and half of the time he has gains, on average an investor derives a disutility from participating in the stock market. Thus, an investor will reallocate his portfolio and invest more heavily in bonds if such an opportunity exists. Therefore, an individual who is loss averse, will not self-select as an investor unless ex ante there are more gains than losses. In addition, the fact that investors, who are wealthier, derive direct utility from fluctuations in their financial wealth decreases their incentive to smooth consumption. Thus, our model underpredicts the amount of consumption smoothing achieved by wealthier individuals. Of course, the reason also could be that we only allow for two assets in the economy. In reality, wealthier individuals have more financial instruments at their disposal to smooth consumption intertemporarily.

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Appendix 1: The Solution Algorithm

As our model does not have an analytical solution, we use the parameterized expectations algorithm (PEA) developed by Marcet (1988) and den Haan and Marcet (1990, 1994) to solve numerically for the stationary distribution of the endogenous variables in our model. There are several advantages of using the algorithm for solving our model: (1) The algorithm performs well when the state space is large and there are a number of stochastic shocks in the conditional expectations as its wide use in a variety of economic environments has shown; (2) Theoretically, we are still lacking understanding on the existence, uniqueness, and properties of equilibrium when markets are incomplete. The PEA enables us to solve the model based on the Euler equations even though we may not know theoretically the properties of the solution. The flip side of the coin though, is that the PEA can provide an arbitrarily close approximation to models with a unique stationary and ergodic distribution but it does not guarantee that the obtained solution is a global maximum. This can be a problem in models with multiple equilibria.

In what follows we first briefly describe the algorithm and then we discuss its application to our model. For a detailed discussion of the algorithm the reader should refer to Marcet; den Haan and Marcet; and Marcet and Lorenzoni (1998). Marcet and Singleton discuss the application of the algorithm to a heterogenous agent, incomplete markets model. Our discussion below borrows from these sources.

The equilibrium in dynamic general equilibrium models with uncertainty is usually described by a set of Euler equations, budget constraints and equilibrium conditions. Let x_t be a vector of n endogenous non-state variables, y_t a vector of m endogenous state variables, and u_t a vector of s exogenous processes that follow a first-order Markov process. For each t the equilibrium relations can be summarized in the following system:

$$0 = g(E_t\{\phi(x_{t+1}, y_{t+1})\}, x_t, y_t, x_{t-1}, y_{t-1}, u_t) \tag{35}$$

where $g : R^p \times R^n \times R^m \times R^n \times R^m \times R^s \longrightarrow R^q$ and $\phi : R^{n+m} \longrightarrow R^p$ are known functions and E_t denotes the conditional expectations operator. The PEA considers solutions such that

$$E\{\phi(x_{t+1}, y_{t+1})|u_t, x_{t-1}, y_{t-1}, u_{t-1}, x_{t-2}, y_{t-2}, \ldots\} = E\{\phi(x_{t+1}, y_{t+1})|y_{t-1}, u_t\} \tag{36}$$

where y_t is a finite-dimensional vector. The PEA computes a recursive solution to (35) where the conditional expectation is given by a time-invariant function Υ such that

$$\Upsilon(y_{t-1}, u_t) = E_t\{\phi(x_{t+1}, y_{t+1})\} = E\{\phi(x_{t+1}, y_{t+1})|y_{t-1}, u_t\} \tag{37}$$

The PEA consists of finding an approximation to Υ by finding ξ and a flexible function $\psi_t(\xi; y_{t-1}, u_t) : R^{m+s} \longrightarrow R^p$ such that for all t

$$0 = g(\psi_t(\xi; y_{t-1}(\xi), u_t), x_t(\xi), y_t(\xi), x_{t-1}(\xi), y_{t-1}(\xi), u_t) \tag{38}$$

where $\xi \in R^{w \times p}$ denotes a vector of parameters. The function is such that as $w \longrightarrow \infty$ we can approximate $E_t\{\phi(\cdot)\}$ and therefore, $\Upsilon(y_{t-1}, u_t)$ arbitrarily well. For example, we can choose a polynomial function for $\psi_t(\xi; \cdot)$ as it can approximate any function when the order of the polynomial increases. The algorithm entails 4 different steps:

1. A major assumption is that the system g in (35) is invertible with respect to its second and third arguments. Thus, the first step is to ensure that the system in (35) is invertible with respect to x_t and y_t, so that the endogenous variables can be uniquely determined from (35). Choose starting values for the endogenous state variables y_0 and exogenous processes u_0. Draw a series $\{u_t\}_{t=1}^T$ from the specified distribution of u with T sufficiently large.

2. Specify the initial values of ξ. For these values and the realizations of u drawn in the previous step, substitute $\psi_t(\xi; \cdot)$ for the conditional expectations in (35). Use (38) to compute recursively the law of motion for the endogenous variables $[x_t(\xi), y_t(\xi)] = f(\xi; y_{t-1}(\xi), u_t)$ and the series $\{x_t(\xi), y_t(\xi)\}_{t=1}^T$. A necessary condition for the implementation of the algorithm is to choose ξ in such a way that $\{x_t(\xi), y_t(\xi)\}_{t=1}^T$ is an ergodic process.

3. Find the mapping $G(\xi) : R^{w \times p} \longrightarrow R^{w \times p}$ such that

$$G(\xi) = \arg \min_{\xi \in R^{w \times p}} \frac{1}{T} \sum_{t=0}^{T} \|\phi(x_{t+1}(\xi), y_{t+1}(\xi)) - \psi_t(\xi'; y_{t-1}(\xi), u_t)\|^2 \tag{39}$$

   where ξ' is the new set of parameters that minimize the difference between the estimated expectations and their realizations. Typically, given ξ^i a non-linear least squares regression (NLS) is used to compute ξ^{i+1} for $i = 1, 2, ...$ (for more information on NLS please see Pindyck and Rubinfeld (1981, sec. 9.4.1).

4. Iterate until

$$\xi_f = G(\xi_f) \tag{40}$$

by repeating steps 2 and 3. In the literature, the following iterative scheme is used to update ξ until a fixed point ξ_f is found:

$$\xi^{i+1} = (1-\tau)\xi^i + \tau G(\xi^i) \quad \text{for} \quad i = 1, 2, ... \tag{41}$$

The approximate solution at the fixed point is given by a series for the endogenous variables $\{x_t(\xi_f), y_t(\xi_f)\}_{t=1}^{T}$, a law of motion for the endogenous variables, $f(\xi_f; \cdot)$, and an approximation to $\Upsilon(\cdot)$ given by $\psi_t(\xi_f; \cdot)$. The algorithm ensures that $\psi_t(\xi_f; \cdot)$ is the best predictor of $E_t\{\phi(x_{t+1}, y_{t+1})\}$ and thus, consistent with the rationality assumption, if agents use $\psi_t(\xi_f; \cdot)$ to predict $E_t\{\phi(\cdot)\}$ on average, they do not make systematic errors.

Appendix 2: Data Estimation

We use annual data on 903 households over the period 1968-1984 from the Panel Study of Income Dynamics (PSID) to calibrate the income processes of stockholders and non-stockholders. The PSID is a longitudinal study of a sample of the US population conducted annually since 1968 and biannually since 1997. The original 1968 sample consists of two independent samples: a sample drawn by the Survey Research Center (SRC sample) that includes about 3,000 households representative of the US population and a sample of about 2,000 households drawn from the Survey of Economic Opportunity respondents (SEO sample) which represents low-income families. As we are interested in a representative sample of the US population, we only consider the SRC sample in line with Lillard and Willis (1978) who suggest dropping the SEO sample because of endogenous selection problem.

The PSID follows both the original families as well as their split-offs. We use both individual- and family-level data to find the family labor income as a sum of the reported labor and transfer income of head and wife. Transfer income is included to measure idiosyncratic shocks net of the implicit insurance offered by transfer payments. Labor income includes the labor portion of income from all sources such as wages and salaries, bonuses, overtime, tips, commissions, professional practice or trade, and market gardening. Transfer income includes social security income, unemployment and workers compensation, child support, retirement income as well as other welfare transfers to the head and wife. The total family labor income is weighted by the number of family members where the adult family members (head and wife or head in the case of a single-parent household) are assigned a weight of 1 while children are assigned a weight of 0.5. The labor income per capita deflated by the CPI is used as a proxy for the individual labor income in our model.

Our sample covers the period from 1968 to 1984, the first year in which questions about stock ownership were asked in the survey thus enabling us to classify the respondents as stockholders or non-stockholders. The PSID survey is retrospective in the sense that it is administered at the beginning of the year and the reported income refers to the previous calendar year and is measured in previous year dollars. Thus, our sample refers to the period from 1967 to 1983.

There are several restrictions that we impose on the data. We include in our sample only families that completed the survey in all years from 1968 to 1984. We exclude missing observations, i.e.

families, which once in the survey, did not complete the survey in a given year. We also exclude families with zero reported income in a given year. We use data from the Wealth Supplement in 1984 to categorize families as stockholders or non-stockholders. We exclude those families who declined to answer the question on whether they hold stocks and thus, cannot be categorized as stockholders or non-stockholders. In our sample we have a total of 903 families of which 566 are non-stockholders and 337 are stockholders.

To estimate the law of motion for the exogenous income processes of Type A and B agents in Equation 26, we use the aggregation method suggested by Heaton and Lucas (1996). We use a time-series regression to estimate the individual income process for each stockholder and non-stockholder in our sample. We assume that the individual income processes of stockholders and non-stockholders mirror the income process of the respective representative agent. Thus, the individual income processes for non-stockholder n for $n = 1, ..., N$ and stockholder s for $s = 1, ..., S$ are given by:

$$\ln(y^A_{nt}) = a_{1n} + b_{1n} \ln(y^A_{nt-1}) + e^A_{nt} \tag{42}$$

$$\ln(y^B_{st}) = a_{2s} + b_{2s} \ln(y^B_{st-1}) + e^B_{st} \tag{43}$$

where $\{a_{1n}\}^N_{n=1}$, $\{a_{2s}\}^S_{i=1}$, $\{b_{1n}\}^N_{n=1}$, and $\{b_{2s}\}^S_{s=1}$ are parameters. Permanent differences in the individual labor incomes of non-stockholders and stockholders are captured by $\{a_{1n}\}^N_{n=1}$ and $\{a_{2s}\}^S_{s=1}$ respectively while $\{b_{1n}\}^N_{n=1}$ and $\{b_{2s}\}^S_{s=1}$ capture the persistence of idiosyncratic income shocks to the individual labor incomes.

The parameters in Equation 26 for Type i agent for $i = A, B$ are cross-sectional means of the corresponding parameter estimates for all individuals who fall in the category of non-stockholders and stockholders, respectively. Panel A of Table 1 reports the results as well as the cross-sectional mean of the residual variances.

The innovation to the labor income of stockholder s, e^B_{st}, has three components which are orthogonal to each other: an aggregate shock ε_{1t}, a group-specific shock ε^B_{2t}, and an idiosyncratic shock ε^B_{3st}. The innovation to the labor income of non-stockholder n is defined similarly. Thus,

$$e^A_{nt} = \varepsilon_{1t} + \varepsilon^A_{2t} + \varepsilon^A_{3nt} \tag{44}$$

$$e^B_{st} = \varepsilon_{1t} + \varepsilon^B_{2t} + \varepsilon^B_{3st} \tag{45}$$

where ε^A_{3nt} follows $\sim Niid(0, \sigma^2_{3An})$. Shocks to the individual labor income are correlated across individuals belonging to the same type as individual incomes are subject to the same aggregate and group-specific shocks, i.e.

$$E[e^A_{nt} e^A_{jt}] = \begin{cases} \sigma^2_1 + \sigma^2_{2A} & \text{for} \quad n \neq j \\ \\ \sigma^2_{An} & \text{for} \quad n = j \end{cases} \tag{46}$$

Similarly, ε^B_{3st} follows $\sim Niid(0, \sigma^2_{3Bs})$ and

$$E[e^B_{st} e^B_{jt}] = \begin{cases} \sigma^2_1 + \sigma^2_{2B} & \text{for} \quad s \neq j \\ \\ \sigma^2_{Bs} & \text{for} \quad s = j \end{cases} \tag{47}$$

for all t. Further, the aggregate shock in the economy affects the income processes of both stockholders and non-stockholders and therefore, innovations to the individual labor income of any stockholder s and any non-stocholder n are correlated. More formally, $E[e^A_{nt} e^B_{st}] = \sigma^2_1 - \frac{\pi}{1-\pi} \sigma^2_{2A}$.

To identify the distributions of the components of the innovations to the labor incomes of the representative stockholder and non-stockholder (Type A and B agents), we make two identifying assumptions, which essentially stem from the theoretical result that non-systematic risk, which is diversifiable in a complete market, is not priced: (1) if markets are complete, idiosyncratic income shocks will be diversified away within each group, i.e. $E[\varepsilon^i_{3jt}] = 0$; (2) if markets are complete, the group-specific shocks will be diversified away in the aggregate.

The cross-sectional means of the innovations to the income processes of non-stockholders and stockholders in our data are given by:

$$\frac{1}{N} \sum_{n=1}^{N} e^A_{nt} = \varepsilon_{1t} + \varepsilon^A_{2t} + \frac{1}{N} \sum_{n=1}^{N} \varepsilon^A_{3nt} \tag{48}$$

$$\frac{1}{S} \sum_{s=1}^{S} e^B_{st} = \varepsilon_{1t} + \varepsilon^B_{2t} + \frac{1}{S} \sum_{s=1}^{S} \varepsilon^B_{3st} \tag{49}$$

$$\tag{50}$$

Applying the first identifying assumption, we obtain:

$$\frac{1}{N} \sum_{n=1}^{N} e^A_{nt} \approx \varepsilon_{1t} + \varepsilon^A_{2t} \tag{51}$$

$$\frac{1}{S}\sum_{s=1}^{S} e_{st}^{B} \approx \varepsilon_{1t} + \varepsilon_{2t}^{B} \tag{52}$$

Therefore,

$$Var\left(\frac{1}{N}\sum_{n=1}^{N} e_{nt}^{A}\right) = \sigma_1^2 + \sigma_{2A}^2 \tag{53}$$

$$Var\left(\frac{1}{S}\sum_{s=1}^{S} e_{st}^{B}\right) = \sigma_1^2 + \sigma_{2B}^2 \tag{54}$$

Applying the second identifying assumption enables us to estimate the variance of the aggregate shock:

$$\pi\left(\frac{1}{N}\sum_{n=1}^{N} e_{nt}^{A}\right) + (1-\pi)\left(\frac{1}{S}\sum_{s=1}^{M} e_{st}^{B}\right) = \pi(\varepsilon_{1t} + \varepsilon_{2t}^{A}) + (1-\pi)(\varepsilon_{1t} + \varepsilon_{2t}^{B}) \tag{55}$$

$$\pi\left(\frac{1}{N}\sum_{n=1}^{N} e_{it}^{A}\right) + (1-\pi)\left(\frac{1}{S}\sum_{s=1}^{S} e_{st}^{B}\right) = \varepsilon_{1t} \tag{56}$$

Therefore,

$$Var\left[\pi\left(\frac{1}{N}\sum_{n=1}^{N} e_{st}^{A}\right) + (1-\pi)\left(\frac{1}{S}\sum_{s=1}^{S} e_{st}^{B}\right)\right] = \sigma_1^2 \tag{57}$$

Table 1: *Parameters of the first-order autoregressive models for the exogenous variables; standard errors are shown in parenthesis.*

Panel A. Income Processes of Type A and Type B Agents		
	Cross-sectional means	
Coefficient	Non-stockholders	Stockholders
ω_1^i	4.71	4.49
	(1.96)	(1.95)
ω_2^i	0.48	0.53
	(0.22)	(0.21)
σ_i^2	0.09	0.08
	(0.93)	(2.01)
Panel B. Aggregate Dividend Process		
Coefficient		
φ_1	0.41	
	(0.31)	
φ_2	0.93	
	(0.06)	
σ^2	0.003	

Panel A presents the cross-sectional means of the least squares estimators in the following regression equations: (1) $\ln(y_{nt}^A) = a_{1n} + b_{1n} \ln(y_{nt-1}^A) + e_{nt}^A$ and $\ln(y_{st}^B) = a_{2s} + b_{2s} \ln(y_{st-1}^B) + e_{st}^B$ where the subscripts $n = 1, .., N$ and $s = 1, .., S$ denote the individual and the superscripts A and B denote the type, non-stockholders and stockholders, respectively. Thus, $\omega_1^A = \frac{1}{N} \sum_{n=1}^{N} a_{1n}$ is the cross-sectional mean of the least squares estimators in the individual regressions for non-stockholders. In our sample we have 566 non-stockholders and 337 stockholders.
Panel B presents the least squares estimators for the aggregate dividend process.

Table 2: *Structural parameters*

Parameter	**Value**
ρ	0.96
γ	2
θ	0.65
b_0	0.7
k	1.4
λ	2.25
η	0.9
h	-1/3
m	0

Table 3: *Moments of asset returns implied by a representative agent model*

Sample Moments	**Postwar Sample***	**Full Sample***	**Model 1****	**Model 2****	**Model 3****
Risk-free rate					
Mean	0.0228	0.0296	0.0379	0.0093	0.0094
Standard deviation	0.0259	0.0683	0.0512	0.1709	0.1675
Stock return					
Mean	0.0783	0.0851	0.0463	0.0998	0.1082
Standard deviation	0.1516	0.1762	0.1023	0.3757	0.4279
Equity premium					
Mean	0.0681	0.0748	0.0084	0.0905	0.0988
Standard deviation	0.1515	0.1761	0.0878	0.3236	0.373
Price-dividend ratio					
Mean	29.15	22.69	24.75	26.72	33.07
Standard deviation	6.55	6.7	3.61	9.04	13.13
Sharpe ratio	0.45	0.43	0.10	0.28	0.27
Average loss aversion	2.25	2.25			2.31

* Source: Empirical values of the moments of asset returns are computed from annual data provided by Robert Shiller at Yale University on his website. The risk-free rate is the annualized real rate of return on Treasury bills and the stock return is the annual rate of return on the Real S&P 500 Stock Price Index. The Postwar Sample covers the period from 1947 to 1997 and the Full Sample is for the period from 1871 to 1997. The average loss aversion is set equal to the empirical value estimated by Kahneman and Tversky (1992) based on an experimental study. $R_{f,t+1} = 1/P_{f,t}$ and $R_{t+1} = (P_{t+1} + d_{t+1})/P_t$ are simple rates of return. Sharpe ratio $= E(R - R_f)/\sigma(R - R_f)$

**Model 1: a representative agent model with aggregate shocks and without loss aversion. Model 2: a representative agent model without loss aversion and idiosyncratic shocks. Model 3: a representative agent model with loss aversion and idiosyncratic shocks. The aggregate labor income in the economy in all three models is set equal to the labor income of Type B agent.

Table 4: *Sample moments of asset returns and consumption implied by the heterogeneous agent model with loss aversion*

Sample Moments	**Postwar Sample***	**Full Sample***	**Model 4**
Risk-free rate			
Mean	0.0228	0.0296	0.0211
Standard deviation	0.0259	0.0683	0.1332
Stock return			
Mean	0.0783	0.0851	0.0808
Standard deviation	0.1516	0.1762	0.3006
Equity premium			
Mean	0.0681	0.0748	0.0596
Standard deviation	0.1515	0.1761	0.2607
Price-dividend ratio			
Mean	29.15	22.69	26.21
Standard deviation	6.55	6.7	7.27
Sharpe ratio	0.45	0.43	0.23
Average loss aversion	2.25	2.25	2.26
$\rho_{C^A,R}$ **			0.44
$\rho_{C^B,R}$ **			0.42
$\rho_{C^A,y}$ **			0.93
$\rho_{C^B,y}$ **			0.90
ρ_{C^A,y^A} **			0.89
ρ_{C^B,y^B} **			0.79

* Source: see notes to Table 3.
** ρ is the correlation coefficient of the respective variables

Table 5: *Sample moments of asset returns and consumption implied by a model without loss aversion (Model 5) and a model with loss aversion and identical income processes (Model 6)*

Sample Moments	**Postwar Sample***	**Full Sample***	**Model 5**	**Model 6**
Risk-free rate				
Mean	0.0228	0.0296	0.0226	0.0225
Standard deviation	0.0259	0.0683	0.1337	0.1322
Stock return				
Mean	0.0783	0.0851	0.073	0.0802
Standard deviation	0.1515	0.1761	0.2795	0.2741
Equity premium				
Mean	0.0681	0.0748	0.0504	0.0577
Standard deviation	0.1515	0.1761	0.2416	0.2652
Price-dividend ratio				
Mean	29.15	22.69	27.81	26.18
Standard deviation	6.55	6.70	7.26	7.64
Sharpe ratio	0.45	0.43	0.21	0.23
$\rho_{C^A,R}$ **			0.45	0.42
$\rho_{C^B,R}$ **			0.45	0.41
$\rho_{C^A,y}$ **			0.93	0.96
$\rho_{C^B,y}$ **			0.91	0.85
ρ_{C^A,y^A} **			0.88	0.94
ρ_{C^B,y^B} **			0.78	0.75

* Source: see notes to Table 3.

** ρ is the correlation coefficient of the respective variables

Table 6: *Robustness:* b_0

Sample Moments	**Postwar Sample***	**Full Sample***	b_0 **0**	**0.5**	**1**	**1.5**
Risk-free rate						
Mean	0.0228	0.0296	0.0226	0.0219	0.0202	0.0187
Standard deviation	0.0259	0.0683	0.1337	0.0795	0.0823	0.0847
Stock return						
Mean	0.0783	0.0851	0.073	0.0795	0.0823	0.0847
Standard deviation	0.1515	0.1761	0.2795	0.2965	0.3069	0.3157
Equity premium						
Mean	0.0681	0.0748	0.0504	0.0576	0.0621	0.0660
Standard deviation	0.1515	0.1761	0.2795	0.2572	0.2674	0.2780
Price-dividend ratio						
Mean	29.15	22.69	27.81	26.33	26.33	26.50
Standard deviation	6.55	6.70	7.26	7.23	7.39	7.56
Sharpe ratio	0.45	0.43	0.21	0.22	0.23	0.24
Average loss aversion	2.25	2.25	-	2.26	2.26	2.28

Parameter b_0 controls for the importance of the prospect utility term in the overall utility of Type B agent. The higher the b_0, the higher the relative importance of prior stock market performance in investors' decision making. The Model with $b_0 = 0$ is Model 5, the model without loss aversion. We reproduce the results here for comparison.

* Source: see notes to Table 3.

Table 7: *Robustness: η*

Sample Moments	**Postwar Sample***	**Full Sample***	η **0.7**	**0.8**	**0.85**	**0.9**
Risk-free rate						
Mean	0.0228	0.0296	0.0224	0.0218	0.0213	0.0211
Standard deviation	0.0259	0.0683	0.1354	0.1348	0.1338	0.1332
Stock return						
Mean	0.0783	0.0851	0.0814	0.0819	0.0803	0.0808
Standard deviation	0.1515	0.1761	0.2999	0.3027	0.2977	0.3006
Equity premium						
Mean	0.0681	0.0748	0.059	0.0601	0.0590	0.0596
Standard deviation	0.1515	0.1761	0.2633	0.2658	0.2588	0.2607
Price-dividend ratio						
Mean	29.15	22.69	25.56	25.72	26.03	26.21
Standard deviation	6.55	6.70	6.95	7.06	7.14	7.27
Sharpe ratio	0.45	0.43	0.22	0.23	0.23	0.23
Average loss aversion	2.25	2.25	2.20	2.23	2.25	2.26

Parameter η controls for investor's memory. The higher the η, the longer prior gains and losses have an impact on Type B agent's decision-making.
*Source: see notes to Table 3.

Table 8: *Robustness: k*

Sample Moments	**Postwar Sample***	**Full Sample***	k **2**	**3**	**4**
Risk-free rate					
Mean	0.0228	0.0296	0.0208	0.0181	0.0198
Standard deviation	0.0259	0.0683	0.1334	0.1394	0.1371
Stock return					
Mean	0.0783	0.0851	0.0815	0.0832	0.087
Standard deviation	0.1515	0.1761	0.3018	0.3267	0.3437
Equity premium					
Mean	0.0681	0.0748	0.0608	0.065	0.0672
Standard deviation	0.1515	0.1761	0.2619	0.2992	0.3204
Price-dividend ratio					
Mean	29.15	22.69	25.94	27.99	27.56
Standard deviation	6.55	6.70	7.23	8.32	8.50
Sharpe ratio	0.45	0.43	0.23	0.22	0.21
Average loss aversion	2.25	2.25	2.35	2.60	2.84

Parameter k controls for the disutility of a loss when it comes after prior losses. The higher the k, the more painful a loss when there are prior losses, the more loss averse the investor is.
*Source: see notes to Table 3.

Table 9: *Robustness: Borrowing constraint*

Sample Moments	**Postwar Sample***	**Full Sample***	h **0.05**	**0.15**	**0.25**	**0.35**	**0.45**
Risk-free rate							
Mean	0.0228	0.0296	0.0153	0.0186	0.0196	0.0213	0.0219
Standard deviation	0.0259	0.0683	0.149	0.1347	0.1343	0.1332	0.1341
Stock return							
Mean	0.0783	0.0851	0.0798	0.0811	0.08	0.0807	0.0804
Standard deviation	0.1515	0.1761	0.3042	0.3022	0.2995	0.3005	0.3
Equity premium							
Mean	0.0681	0.0748	0.0645	0.0626	0.0604	0.0594	0.0585
Standard deviation	0.1515	0.1761	0.2784	0.262	0.2606	0.261	0.2622
Price-dividend ratio							
Mean	29.15	22.69	27.68	26.24	26.53	26.23	26.35
Standard deviation	6.55	6.70	8.00	7.33	7.31	7.27	7.3
Sharpe ratio	0.45	0.43	0.23	0.24	0.23	0.23	0.22
Average loss aversion	2.25	2.25	2.28	2.26	2.26	2.26	2.28

* Source: see notes to Table 3.

Table 10: *Robustness: The share of non-stockholders in the population*

Sample Moments	**Postwar Sample***	**Full Sample***	π **0.35**	**0.4**	**0.5**	**0.8**
Risk-free rate						
Mean	0.0228	0.0296	0.0180	0.0188	0.0204	0.0185
Standard deviation	0.0259	0.0683	0.1322	0.1294	0.1267	0.1521
Stock return						
Mean	0.0783	0.0851	0.0785	0.0775	0.0756	0.0939
Standard deviation	0.1515	0.1761	0.2992	0.2941	0.2845	0.3485
Equity premium						
Mean	0.0681	0.0748	0.0606	0.0587	0.0552	0.0754
Standard deviation	0.1515	0.1761	0.2574	0.2531	0.2449	0.2997
Price-dividend ratio						
Mean	29.15	22.69	26.92	26.78	26.62	26.18
Standard deviation	6.55	6.70	7.38	7.21	6.94	8.01
Sharpe ratio	0.45	0.43	0.24	0.23	0.23	0.25
Average loss aversion	2.25	2.25	2.29	2.28	2.27	2.29

π controls for the share of non-stockhoders in the population. The share of stockholders is (1 - π).
* Source: see notes to Table 3.

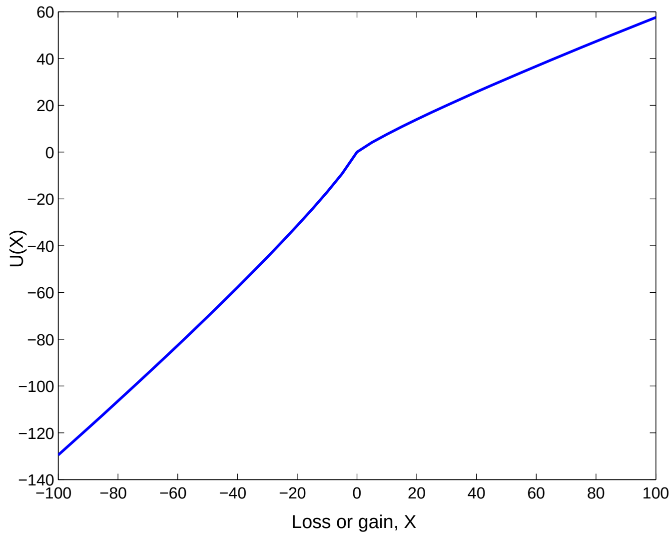

Figure 1: *Kahneman and Tversky's Value Function*

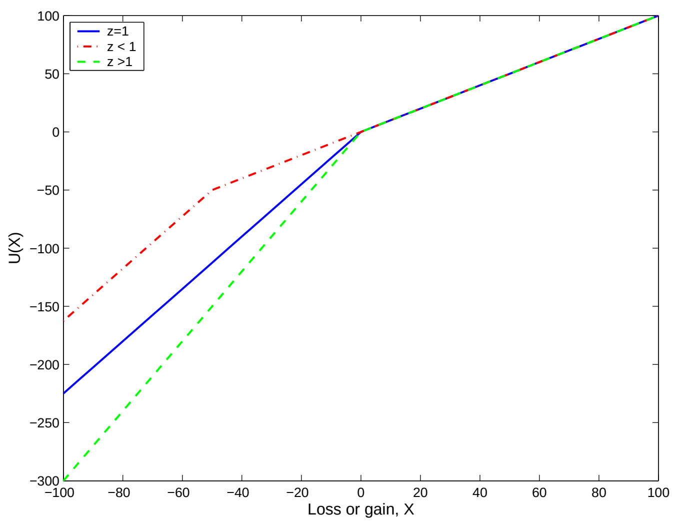

Figure 2: *Utility from gains and losses for different values of* z_t

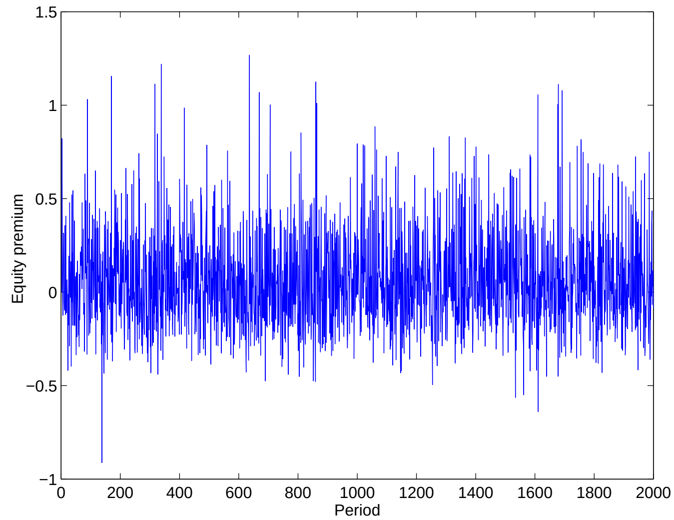

Figure 3: *The volatility of excess stock returns for t = 0:2000*

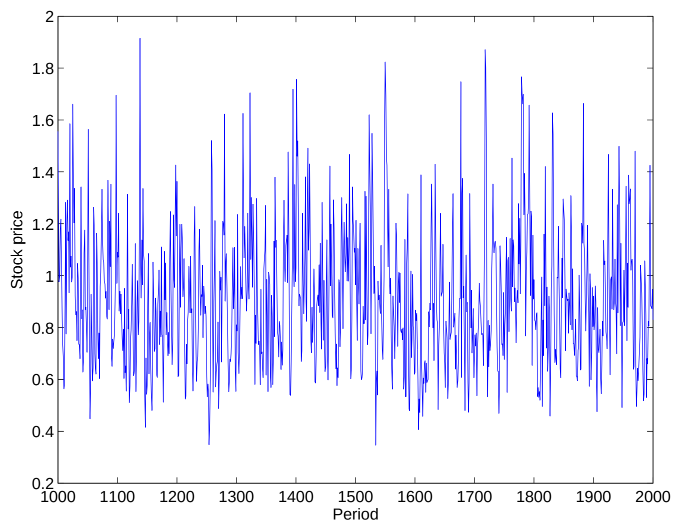

Figure 4: *Stock price for 1,000 periods*

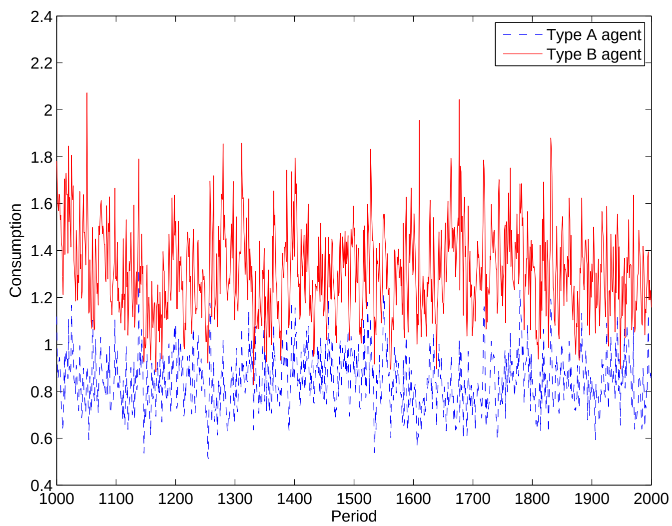

Figure 5: *Consumption for t = 1000:2000*

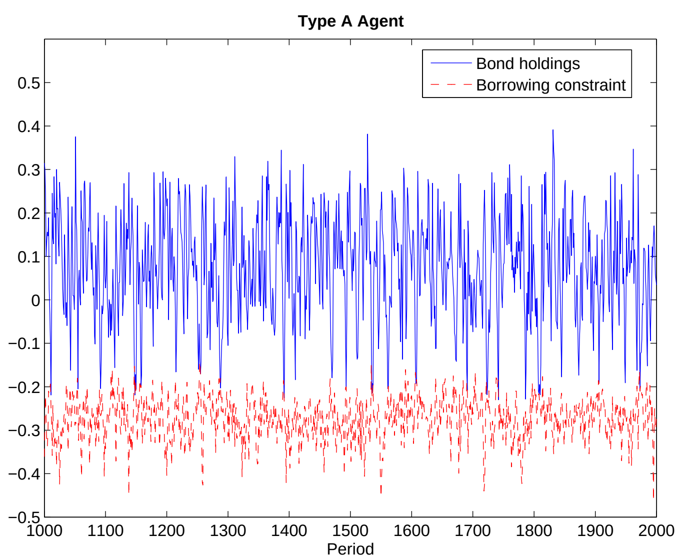

Figure 6: *Bond holdings and bond constraint for 1,000 periods, Type A agent*

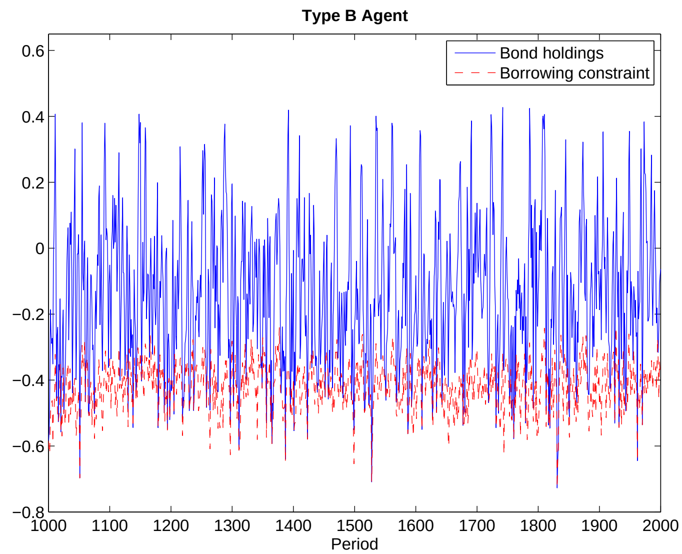

Figure 7: *Bond holdings and bond constraint for 1,000 periods, Type B agent*

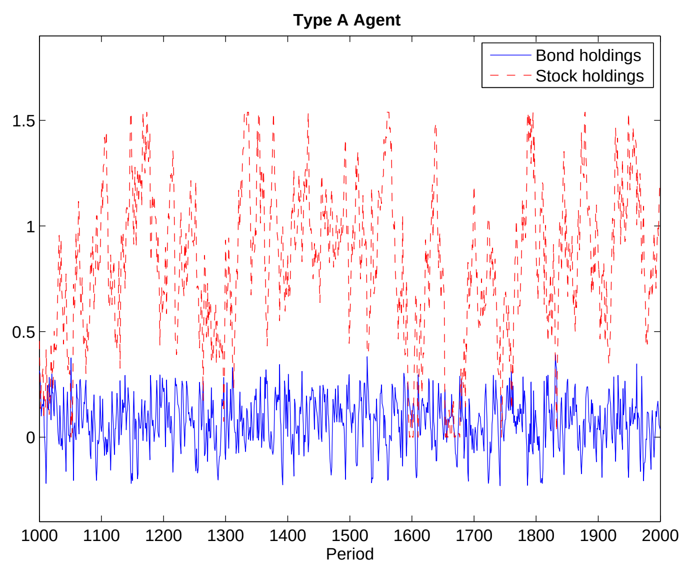

Figure 8: *Stock and bond holdings for 1,000 periods, Type A agent*

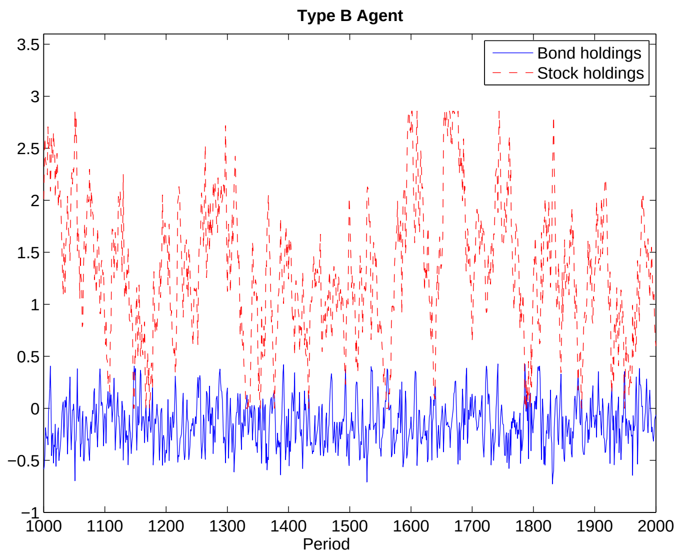

Figure 9: *Stock and bond holdings for 1,000 periods, Type B agent*

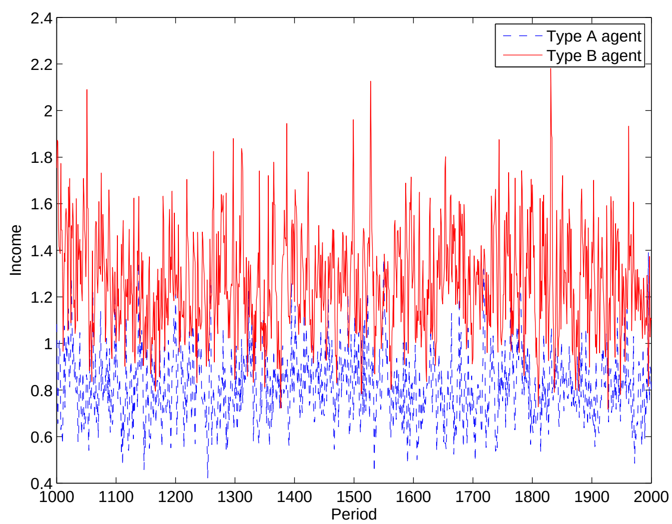

Figure 10: *Income for t = 1000:2000*

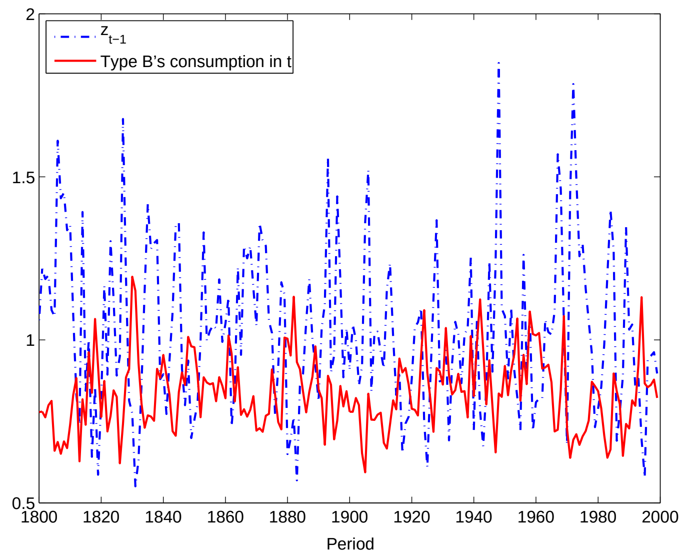

Figure 11: *Type B agent's consumption and z for t = 1800:2000*

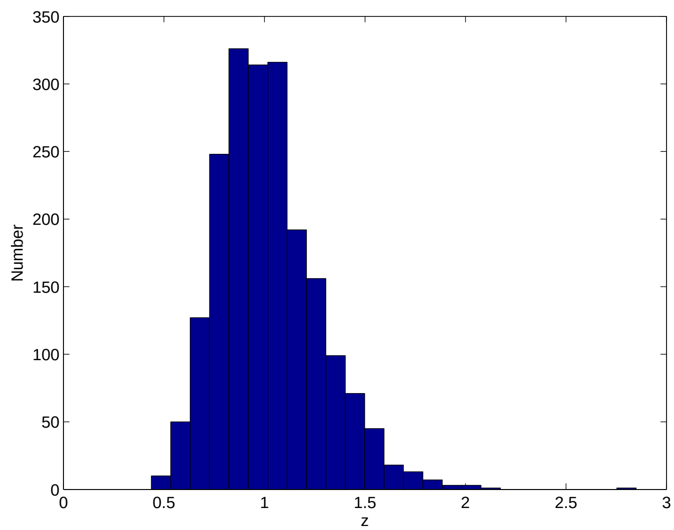

Figure 12: *Histogram of z*

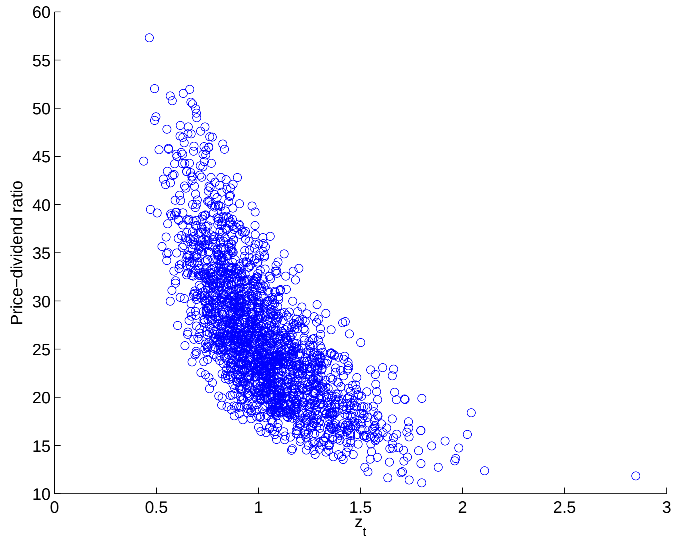

Figure 13: *Prior stock market performance and price-dividend ratio, scatter plot*

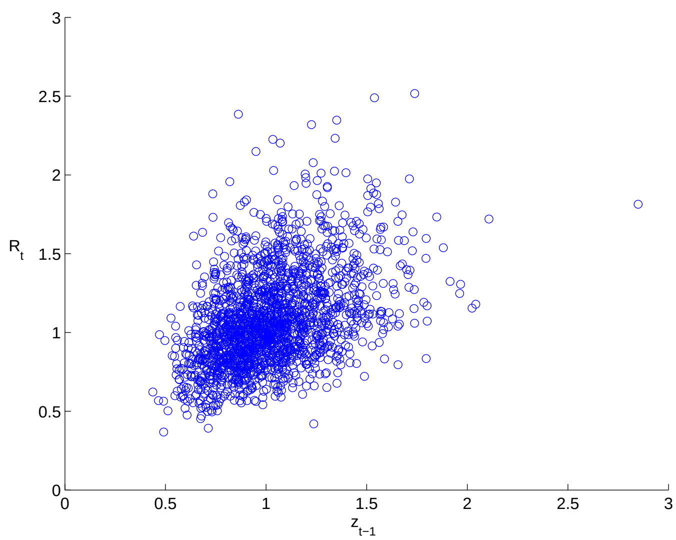

Figure 14: *Prior stock market performance and stock returns, scatter plot*

Figure 15: *Income, stock, and bond holdings for 200 periods, Type A agent*

Figure 16: *Income, stock, and bond holdings for 200 periods, Type B agent*

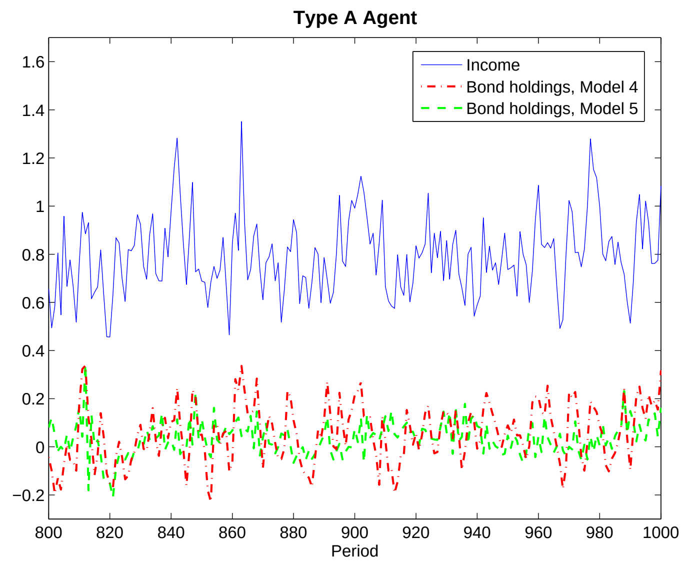

Figure 17: *Income and bond holdings generated by Model 4 (loss aversion) and Model 5 (no loss aversion), Type A agent*

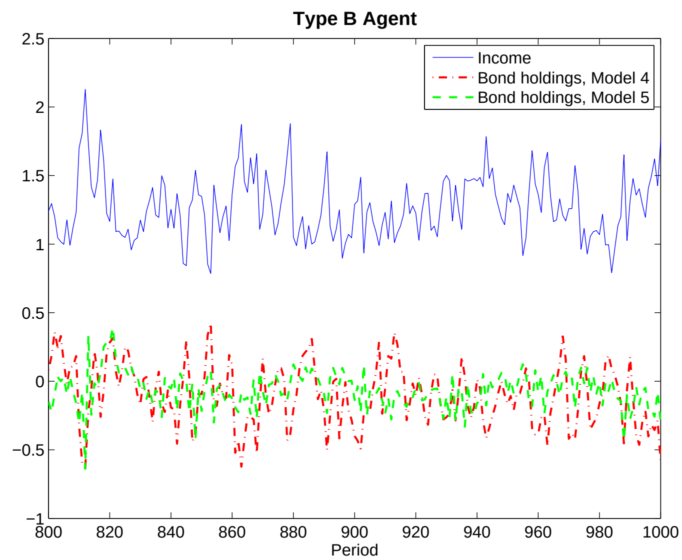

Figure 18: *Income and bond holdings generated by Model 4 (loss aversion) and Model 5 (no loss aversion), Type B agent*

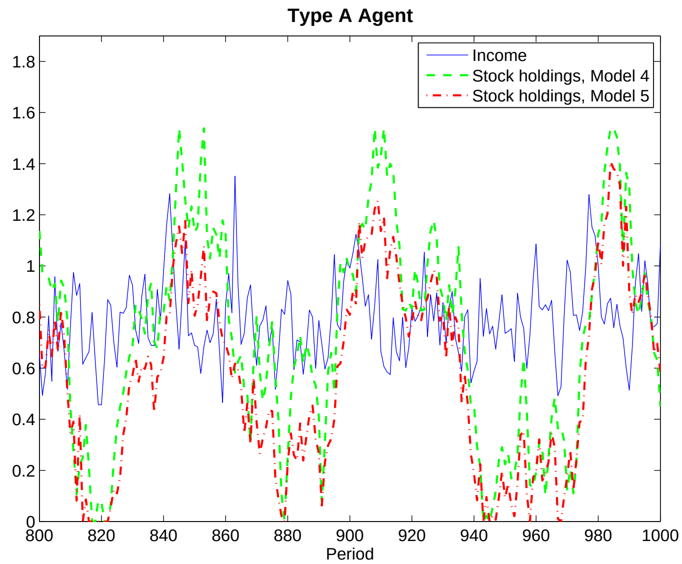

Figure 19: *Income and stock holdings generated by Model 4 (loss aversion) and Model 5 (no loss aversion), Type A agent*

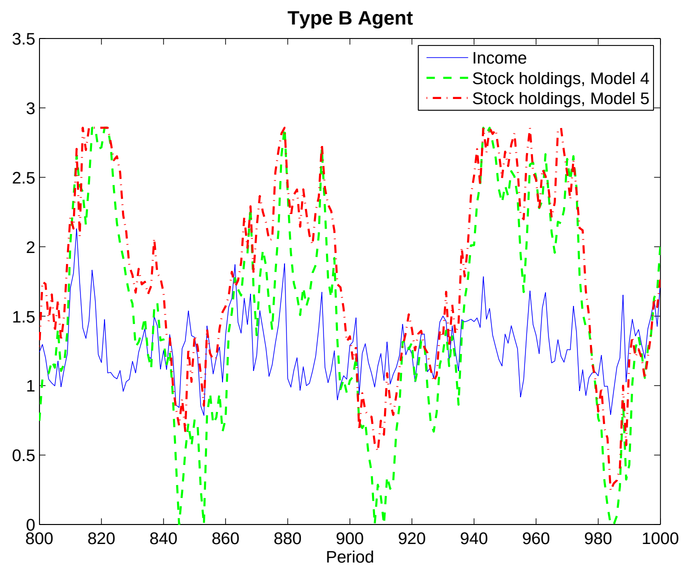

Figure 20: *Income and stock holdings generated by Model 4 (loss aversion) and Model 5 (no loss aversion), Type B agent*