University of Connecticut

Department of Economics Working Paper Series

”Ripple Effects” and Forecasting Home Prices in Los Angeles, Las Vegas, and Phoenix

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Working Paper 2009-05

January 2009

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Abstract

We examine the time-series relationship between housing prices in Los Angeles, Las Vegas, and Phoenix. First, temporal Granger causality tests reveal that Los Angeles housing prices cause housing prices in Las Vegas (directly) and Phoenix (indirectly). In addition, Las Vegas housing prices cause housing prices in Phoenix. Los Angeles housing prices prove exogenous in a temporal sense and Phoenix housing prices do not cause prices in the other two markets. Second, we calculate out-of-sample forecasts in each market, using various vector autoregessive (VAR) and vector error-correction (VEC) models, as well as Bayesian, spatial, and causality versions of these models with various priors. Different specifications provide superior forecasts in the different cities. Finally, we consider the ability of theses time-series models to provide accurate out-of-sample predictions of turning points in housing prices that occurred in 2006:Q4. Recursive forecasts, where the sample is updated each quarter, provide reasonably good forecasts of turning points.

Journal of Economic Literature Classification: C32, R31

Keywords: Ripple effect, housing prices, forecasting

1. Introduction

This paper considers the dynamics of housing prices and the ability of different pure time-series models to forecast housing prices in three Southwestern Metropolitan Statistical Areas (MSAs) – Los Angeles, Las Vegas, and Phoenix. Recent popular wisdom argues that residents of Southern California sell their local homes, cash out significant equities, and move (retire) to Las Vegas and Phoenix, where they significantly upgrade the quality of their homes. In fact, other Mountain Southwest MSAs may also respond to home prices in Los Angeles (and San Francisco). Recently, the Brookings Institution (2008) released a report on the rapid growth in the Mountain Southwest, identifying five megapolitan areas – Las Vegas, Phoenix, Denver, Salt Lake City and Albuquerque.

Housing experts on the UK economy identified a "ripple" effect of housing prices that begins in the Southeast UK and proceeds toward the Northwest. Meen (1999) describes four different theories that may explain the ripple effect – migration, equity conversion, spatial arbitrage, and exogenous shocks with different timing of spatial effects. A ripple effect does not yet receive much support in the US economy. For example, most analysis relates to a given geographic housing market, such as a metropolitan area (Tirtirglou 1992; and Clapp and Tirtirglou 1994). More recent evidence across census regions also exists, which may reflect the fourth of Meen's explanations (Pollakowski and Ray, 1997; Meen 2002).

This paper first tests for cointegration between real house prices in the three MSAs, using the Johansen technique (1991). Given that we find one cointegrating relationship between the real house prices, the block exogeneity tests on the vector error correction (VEC) model reveal that housing prices in Los Angeles temporally cause prices in Las Vegas directly and Phoenix indirectly, and that housing prices in Las Vegas temporally cause prices in Phoenix directly, but that Las Vegas and Phoenix housing prices do not temporally cause prices in Los Angeles.

We next compare the out-of-sample forecasting performance of various time-series models – vector autoregressive (VAR), vector error-correction (VEC), and various Bayesian time-series models. For the Bayesian models, we estimate Bayesian VAR (BVAR) and VEC (BVEC) models as well as BVAR and BVEC models that include spatial and causality priors (LeSage 2004). A BVEC model performs the best across all three cities, although the forecasting performances in the individual cities do differ. That is, none of the cities perform the best in this BVEC model that performs the best across all three cities.

We organize the rest of the paper as follows. Section 2 examines the relevant literature. Section 3 specifies the various time-series models estimated in Section 4. Section 5 concludes.

2. Literature Review

The literature review considers three different areas. First, we discuss housing dynamics and the various theories offered to explain those dynamics. Next, we describe the implications of housing dynamics on the time-series properties of housing prices. Finally, we consider the differences between dynamic structural models and time-series models in forecasting ability.

Housing Dynamics: Observations and Theory

We begin with the Law of One Price (LOOP), which states that a homogeneous good that sells in two different markets should sell for the same price, ignoring transaction and transportation costs. At the fundamental level, the operation of LOOP requires that the good is transportable between markets. Clearly, housing fails on at least two important fronts – housing is not homogeneous and is not transportable between markets.

Housing economists address the issue of a non-homogeneous good by appealing to the characteristics of housing. Hedonic models allow the researcher to compare housing prices based on the characteristics imbedded into the sales, such as number of bedrooms and baths and so on. Typically, the geographic reach of the housing market reflects the commuting shed for the metropolitan area. That is, houses compete with each other within the same metropolitan area. Tirtirglou (1992) and Clapp and Tirtirglou (1994) provided some of the earliest tests of whether the housing market exhibited efficiency in a spatial market in Hartford, Connecticut.

Does the fact that we cannot transport houses from one metropolitan market to another necessarily mean that the markets do not exhibit some linkage? Borrowing from trade theory, we know that labor and capital frequently do not move between countries. Nonetheless, Samuelson (1948) shows that factor prices equalize, if goods and services flow freely between countries. That is, other flows between countries act as surrogates and cause the prices of labor and capital to equalize even though capital and labor do not move between countries. Since housing cannot flow between markets, migration of home buyers to purchase owner-occupied and non-owner occupied homes between metropolitan areas can link the housing markets. Moreover, home builders can shift their operations between metropolitan areas in response to differential returns on home building activity.

Meen (1999) offers four different explanations of the "ripple" effect in the UK housing markets. As noted above, a tendency exists in the UK for housing price innovations in the Southeast part of the UK to transmit across geography to the Northwest. The basic theoretical model to explain the housing-consumption decision relies on a life-cycle model of household behavior (Meen, 1990). The life-cycle model assumes market efficiency, which clearly does not hold exactly in the housing market. Thus, the theoretical model reflects a long-run equilibrium situation and practical implementation of the theory requires significant amounts of lagged (stock) adjustment effects. His explanations fall into the following categories: migration, equity conversion, spatial arbitrage, and exogenous shocks with different timing of spatial effects.

Migration. The migration explanation requires that households move from one metropolitan area to another to take advantage of regional house price differences. This explanation does not offer a strong rationale in the UK, because regional migration flows prove weak at best. Migration patterns between Los Angeles (Southern California) and Las Vegas or Phoenix does exhibit the magnitude and direction of movement that could link Las Vegas and Phoenix prices to those in Los Angeles. But, what factors cause the migration in the first place? Migration may reflect several factors – lower housing prices in Las Vegas and Phoenix, significantly higher congestion in Los Angeles, faster economic growth may provide more valuable work possibilities in Las Vegas and Phoenix, and so on.

Equity Conversion. A further explanation for migration may reflect the extra run up in housing prices in Los Angeles. Longer-term residents of Southern California may accumulate significant wealth in their home equity. In order to cash out that wealth, residents of Southern California must sell their home and move to a lower cost region where they can buy a similar quality house for a lower price and pocket the residual equity. Of course, the movement of home owners because of equity conversion inflates prices at the margin in the new residential areas where they drop anchor.

Spatial Arbitrage. Rather than households moving to link the housing prices in different regional markets, investors could use spatial arbitrage to acquire properties in lower priced regions, where higher anticipated return on housing investment exist. In this case, financial capital moves between regions to link housing prices, rather than the migration of households. Pollakowski and

Ray (1997) find limited evidence of a spatial arbitrage (diffusion) effect across metropolitan regions in the US.

Spatial Patterns of House Price Determinants. This argument really represents a non-theory of regional house price movements, or spurious correlation. That is, regional housing markets are independent. Nonetheless, if the determinants of housing prices in different regions experience a correlated movement, then housing prices will also exhibit the same correlated movement.

Meen (1999) considers the possible explanations for a "ripple" effect in the UK. He relies on the life-cycle model of consumer choice. But, this leaves out an important factor in the housing market, the supply side. To discuss the role of the supply side, we can think of housing prices as including two components – construction (replacement) costs and land value.

As we noted above, even though we cannot transport housing between regions, other factors can flow to link housing prices, such as migration of households or financial capital. Another possibility relates to factors of production. That is, if the demand for housing rises in one region, that will draw resources, including construction labor, from other regions. As a result, construction costs in both regions will rise. It rises first in the market where the demand for housing rises to attract more construction workers. And as a consequence, as the supply of construction workers in the other region falls, their wages will rise. The equalizing of construction costs tends to equilibrate housing prices across regions.

Just as we cannot transport housing between regions, we cannot transport land as well. Thus, if a region faces a fixed, or extremely inelastic, supply of land, then that regions housing prices and land values will rise. That is, since housing prices include construction (replacement) costs and land prices, higher land prices will drive up housing prices even though construction (replacement) costs may equilibrate between regions. All three metropolitan areas in this paper face land restrictions that respond in this manner. That is, all three regions experienced a housing "bubble" in recent years that deflated recently. See Figure 1.

In sum we argue that the housing "bubbles" in Los Angeles, Las Vegas, and Phoenix reflect, in large measure, run ups and then crashes in land values. While other factors such as construction costs also played a role, lands values dominated the movement in home prices.

Time-Series Implications for Housing Prices

To the extent that housing prices follow a ripple effect between different geographic regions, then we should observe Granger temporal causality between regions. That is, price movements in one region should temporally precede price movements in another region. We perform temporal causality tests using a vector autoregressive (VAR) specification. On the other hand, if housing prices are I(1) series, exhibiting non-stationarity, then a long-run relationship between the housing prices may exist, especially if the ripple effect holds. As such, then the housing price series may exhibit cointegration and require the tests for Granger temporal causality to occur within a vector error-correction model (VEC).

Dynamic Structural Versus Time-Series Models

Two different approaches exist to modeling dynamic adjustment – dynamic structural and time-series models. Zellner and Palm (1974) demonstrate that the two approaches are theoretically equivalent. That is, any dynamic structural model implicitly generates a series of univariate time-series models for each endogenous variable. The dynamic structural model, however, imposes restrictions on the parameters in the reduced-form time-series specification.

Dynamic structural models prove most effective in performing policy analysis, albeit subject to the Lucas critique. Time-series models prove most effective at forecasting. That is, in both cases errors creep in whenever the researcher makes a decision about the specification.

Clearly, more researcher decisions relate to a dynamic structural model than a univariate time-series model, suggesting that fewer errors enter the time-series model and allowing the model to produce better forecasts.

The "atheoretical" VAR and VEC models do not impose any exogeneity assumptions on the included variables. That is, lagged values of each variable may provide valuable information in forecasting each endogenous variable. VAR and VEC models, however, prove subject to over-parameterization, since the number of parameters to estimate increases dramatically with additional variables or additional lags in the system. Bayesian VAR or VEC models economize on the number of parameters estimated by using a small number of hyper-parameters in the specification.

3. VAR, VEC, BVAR, BVEC, SBVAR, and SBVEC Specification and Estimation[1]

We can write an unrestricted VAR model (Sims, 1980) as follows:

$$y_t = A_0 + A(L)\,y_t + \varepsilon_t \;\; (1),^2 \qquad (1)$$

where *y* equals a ($n \times 1$) vector of variables to forecast; *A(L)* equals an ($n \times n$) polynomial matrix in the backshift operator *L* with lag length *p*, and ε equals an ($n \times 1$) vector of error terms. In our case, we assume that $\varepsilon \sim N(0, \sigma^2 I_n)$, where I_n equals an ($n \times n$) identity matrix.

Additional restrictions on the standard VAR model lead to a VEC model, designed for use with cointegrated non-stationary series. While allowing for short-run adjustment dynamics, the VEC model builds into the specification the cointegration relations so that it restricts the long-run behavior of the endogenous variables to converge to their long-run relationships. The

[1] The discussion in this section relies heavily on LeSage (1999), Gupta and Sichei (2006), and Gupta (2006).

[2] *A(L)* = $A_1 L + A_2 L^2 + ... + A_p L^p$; and A_0 equals an ($n \times 1$) vector of constant terms.

cointegration term, known as the error correction term, gradually corrects through a series of partial short-run adjustments.

More explicitly, assume that the n time series variables in y_t are integrated[3] of order one, (i.e., *I(1)*).[4] The error-correction counterpart of the VAR model in equation (1) converts into a VEC model as follows.[5]

$$\Delta y_t = \pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + \varepsilon_t \tag{2}$$

where $\pi = -[I - \sum_{i=1}^{p} A_i]$ and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$.

VAR models typically use equal lag lengths for all variables in the model, which implies that the researcher must estimate many parameters, some of which may prove statistically insignificant. This over-parameterization problem can result in multicollinearity and a loss of degrees of freedom, leading to inefficient estimates, and possibly large out-of-sample forecasting errors. Often, researchers simply exclude lags with statistically insignificant coefficients. Alternatively, researchers use near VAR models, which specify unequal lag lengths for the variables and equations.

Litterman (1981), Doan *et al.,* (1984), Todd (1984), Litterman (1986), and Spencer (1993), use a Bayesian VAR (BVAR) model to overcome the over-parameterization problem. Instead of eliminating lags, the Bayesian method imposes restrictions on the coefficients across different lag lengths, assuming that the coefficients of longer lags may prove nearer zero than the coefficients on shorter lags. If, however, stronger effects come from longer lags, the data can

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[3] A series is integrated of order *q*, if it requires *q* differences to transform it into a zero-mean, purely non-deterministic stationary process.

[4] See LeSage (1990) and references cited therein for further details regarding the non-stationarity of most macroeconomic time series.

[5] See Dickey *et al.* (1991) and Johansen (1995) for further technical details.

override this initial assumption. Researchers impose the restrictions by specifying normal prior distributions with zero means and small standard deviations for most coefficients, where the standard deviation decreases as the lag length increases. The first own-lag coefficient in each equation proves the exception with a mean of unity. Finally, Litterman (1981) uses a diffuse prior for the constant. Researchers popularly refer to this as the "Minnesota prior," due to its development at the University of Minnesota and the Federal Reserve Bank at Minneapolis. In our analysis, we implement a Bayesian variant of the Classical VEC model based on the Minnesota prior.

Formally, as discussed above, the means and variances of the Minnesota prior take the following form:

$$\beta_i \sim N(1, \sigma^2_{\beta_i}) \text{ and } \beta_j \sim N(0, \sigma^2_{\beta_j}) \qquad (3)$$

where β_i denotes the coefficients associated with the lagged dependent variables in each equation of the VAR model (i.e., the first own-lag coefficient), while β_j represents any other coefficient. In sum, the prior specification reduces to a random-walk with drift model for each variable, if we set all variances to zero. The prior variances, $\sigma^2_{\beta_i}$ and $\sigma^2_{\beta_j}$, specify uncertainty about the prior means $\overline{\beta}_i$ = 1, and $\overline{\beta}_j$ = 0, respectively.

Doan *et al.*, (1984) suggest a formula to generate standard deviations as a function of a small numbers of hyper-parameters: *w*, *d*, and a weighting matrix *f*(*i*, *j*) to address the over-parameterization in the VAR model. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few hyper-parameters. The specification of the standard deviation of the distribution of the prior imposed on variable *j* in equation *i* at lag *m*, for all *i*, *j* and *m*, equals $S_1(i, j, m)$, defined as follows:

$$S_1(i, j, m) = [w \times g(m) \times f(i, j)] \frac{\hat{\sigma}_i}{\hat{\sigma}_j}, \tag{4}$$

where $f(i, j) = 1$, if $i = j$ and k_{ij} otherwise, with ($0 \le k_{ij} \le 1$), and $g(m) = m^{-d}$, with $d > 0$. Note that $\hat{\sigma}_i$ equals the estimated standard error of the univariate autoregression for variable i. The ratio $\hat{\sigma}_i / \hat{\sigma}_j$ scales the variables to account for differences in the units of measurement and, hence, causes specification of the prior without consideration of the magnitudes of the variables. The term w indicates the overall tightness and equals the standard deviation on the first own lag, with the prior getting tighter as we reduce the value. The parameter $g(m)$ measures the tightness on lag m with respect to lag 1, and equals a harmonic shape with decay factor d, which tightens the prior on increasing lags. The parameter $f(i, j)$ represents the tightness of variable j in equation i relative to variable i, and by increasing the interaction (i.e., the value of k_{ij}), we loosen the prior.[6]

The overall tightness (w) and the lag decay (d) hyper-parameters equal 0.1 and 1.0, respectively, in the standard Minnesota prior, while k_{ij} = 0.5, implying a weighting matrix (F) of the following form for our three city example of Los Angeles, Las Vegas, and Phoenix:

$$F = \begin{bmatrix} 1.0 & 0.5 & 0.5 \\ 0.5 & 1.0 & 0.5 \\ 0.5 & 0.5 & 1.0 \end{bmatrix}. \tag{5}$$

Since researchers believe that the lagged dependant variable in each equation prove most important, F imposes $\overline{\beta}_i = 1$ loosely. The β_j coefficients, however, that associate with less-important variables receive a coefficient in the weighting matrix (F) that imposes the prior means

[6] For an illustration, see Dua and Ray (1995).

of zero more tightly. Given that the Minnesota prior treats all variables in the VAR, except for the first own-lag of the dependent variable, in an identical manner, several attempts exist that try to alter this fact. Usually, this boils down to increasing the value for the overall tightness (*w*) hyper-parameter from 0.10 to 0.20, so that the larger value of *w* allows more influence from other variables in the model. In addition, Dua and Ray (1995) propose a prior with less restrictions on the other variables in the VAR model, specifically with $w = 0.30$ and $d = 0.50$.

Alternatively, LeSage and Pan (1995) suggest constructing spatial BVAR (SBVAR) and BVEC (SBVEC) models. They propose the weight matrix based on the first-order spatial contiguity (FOSC) prior, which simply implies a non-symmetric F matrix that gives more importance to variables from neighboring states/cities than those from non-neighboring states/cities. They propose using unity both for the diagonal elements of the weight matrix, as in the Minnesota prior, as well as for place(s) that correspond to variable(s) from state(s)/city(ies) with which the specific state in consideration shares common border(s). For the elements in the F matrix that correspond to variable(s) from state(s)/city(ies) that are not immediate neighbor(s), Lesage and Pan (1995) adopt a weight of 0.1. In sum, some of the 0.5 weights in the specification shown in (4) become 1.0 for neighbors and 0.1 for non-neighbors.

In our specific example of Los Angeles, Las Vegas, and Phoenix, we could argue that each city neighbors the other cities or does not neighbor the other cities. Thus, the coefficients of 0.5 either change to 1.0 or to 0.1. If we assume that the cities all neighbor each other, then the F matrix becomes the following:

$$F = \begin{bmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \end{bmatrix}. \qquad (6)$$

We also propose new specifications called causality BVAR (CBVAR) and BVEC

(CBVEC) models, where the weight matrix depends on tests for Granger temporal causality –- the temporal causality (TC) prior. This modification of the LeSage and Pan (1995) first-order spatial-contiguity (FOSC) prior considers some neighbors as more important than other neighbors. In fact, non-neighbors may exert more influence than neighbors. If one city's home prices temporally cause another city's home prices, then we code the weight matrix for that off-diagonal entry at 1.0. If no temporal causality exists, then we code the off-diagonal entry as 0.1. We hypothesize a hypothetical *F* matrix under a temporal causality prior as follows:

$$F = \begin{bmatrix} 1.0 & 0.1 & 0.1 \\ 1.0 & 1.0 & 0.1 \\ 1.0 & 1.0 & 1.0 \end{bmatrix}. \tag{7}$$

In this specification, the first city's (Los Angeles) home prices temporally cause home prices in Las Vegas and Phoenix. Then the second city's (Las Vegas) home prices temporally cause the third city's (Phoenix) home prices.

More recently, LeSage and Krivelyova (1999) develop an alternative approach to remedy the equal treatment nature of the Minnesota prior, called the "random-walk averaging" (RWA) prior. As noted above, most attempts to adjust the Minnesota prior focus mainly on alternative specifications of the prior variances. The RWA prior requires that both the prior mean and variance incorporate the distinction between important variables, neighbors and non-neighbors, for each equation in the VAR model. Now the neighbors receive a weight on 1.0 and non-neighbors receive a weight of 0.0.

Consider the weight matrix *F* for the VAR model consisting of house prices of the three metropolitan areas. The weight matrix contains values of unity in each position (i.e., the home price in each city proves important), while no city receives a zero values, since all cities are neighbors. In addition, we continue with 1.0 down the main diagonal of the *F* matrix, to emphasize the importance of the autoregressive influences from the lagged values of the dependant variable (house price of a specific metropolitan area).[7] In sum, the weight matrix *F* in our application remains as shown in equation (6).

We then standardize the weight matrix in equation (6) so that each row sums to unity. Formally, we write the standardized *F* matrix, called *C*, as follows:

$$C = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \\ 0.33 & 0.33 & 0.33 \end{bmatrix}. \tag{8}$$

We can interpret the *C* matrix as generating a pseudo random-walk process with drift, where the random-walk component averages across the important variables in each equation *i* of the VAR. Formally,

$$y_{it} = \delta_i + \sum_{j=1}^{3} C_{ij} y_{jt-1} + u_{it}\text{ , } i = 1, 2, \text{ and } 3. \tag{9}$$

Expanding equation (9), we observe that by multiplying y_{jt-1},containing the house prices of the three metropolitan areas at *t-1*, with *C* produces a set of explanatory variables for the VAR equal to the mean of observations from the important variables (neighboring house prices) in each equation *i* at *t-1*.[8] This also suggests that the prior mean for the coefficients on the first own-lag of the important variables equals $1/c_i$, where c_i (=3) equals the number of important variables in a specific equation *i* of the VAR model.[9]

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[7] Using 1.0 on the main diagonal of the *F* matrix for the RWA prior, however, does not always prove obvious. LeSage and Krivelyova (1999) provide the exposition for when the autoregressive influences do not influence importantly certain variables.

[8] Just as with the constant in the Minnesota Prior, δ is also estimated based on a diffuse prior.

[9] As in the Minnesota prior, the RWA prior uses a prior mean of zero for the coefficients on all lags, except for the first own lags. The RWA approach of specifying prior means requires that the researcher scale the variables to similar magnitudes, since otherwise it does not make intuitive sense to say that the value of a variable at *t* equals the average of values from the important variables at *t-1*. This issue does not affect our analysis, since our variables are

In sum, the prior variances for the parameters under the RWA prior, as proposed by LeSage and Krivelyova (1999), retaining the distinction between important and unimportant variables, require the following ideas:

(i) Assign a smaller prior variance to parameters associated with unimportant variables, imposing the zero prior means with more certainty;

(ii) Assign a small prior variance to the first own-lag of the important variables so that the prior means force averaging over the first own-lags of such variables;

(iii) Impose the prior variance of parameters associated with unimportant variables at lags greater than one such that it becomes smaller as the lag length increases, imposing decay in the influence of the unimportant variables over time;

(iv) Assign larger prior variances on lags other than the first own-lag of the important variables important variables, allowing those lags to exert some influence on the dependant variable; and

(v) Finally, impose decreasing prior variances on the coefficients of lags, other than the first own-lag of the important variables.

Thus, in the specification of the RWA, as in the Minnesota prior, longer lag influences decay irrespective of whether we classify the variable as important or unimportant.

Given *(i)* to *(v)*, we adopt a flexible form, where the RWA prior standard deviations $S_2(i, j, m)$ for a variable *j* in equation *i* at lag length *m* equal the following:

$$
\begin{aligned}
&S_2(i, j, m) \sim N(1/c_i, \sigma_c); \quad j \in C; \quad m = 1; \quad i, j = 1, \dots, n; \\
&S_2(i, j, m) \sim N(0, \eta\, \sigma_c/m); \quad j \in C; \quad m = 2, \dots, p; \quad i, j = 1, \dots, n; \text{ and} \\
&S_2(i, j, m) \sim N(0, \rho\, \sigma_c/m); \quad j \neg\in C; \quad m = 1, \dots, p; \quad i, j = 1, \dots, n;
\end{aligned} \tag{10}
$$

all scaled in the same fashion.

where $0 < \sigma_c < 1$, $\eta > 1$, $0 < \rho \leq 1$, and c_i equals the number of important variables in equation *i*. For the important variables in equation *i* (i.e., $j \in C$), the prior mean for the lag length of 1 equals the average of the number of important variables in equation *i*, and equals zero for the unimportant variables (i.e., $j \neg \in C$). With $0 < \sigma_c < 1$, the prior standard deviation for the first own lag imposes a tight prior mean to reflect averaging over important variables. For important variables at lags greater than one, the variance decreases as *m* increases, but the restriction that $\eta > 1$ allows for the loose imposition of the zero prior means on the coefficients of these variables. We use $\rho \, {}^{\sigma_c}\!/_{m}$ for lags on unimportant variables, with prior means of zero, to indicate that the variance decreases as *m* increases. In addition, since $0 < \rho \leq 1$, we impose the zero means on the unimportant variables with more certainty. In our model, however, we do not include any unimportant variables.

We also propose a weighted random-walk averaging (WRWA) prior. That is, we extend the specification of LeSage and Krivelyova (1999) by assuming that the first own-lagged value proves more important than the other important variables (neighbors).[10] We impose the condition that the first own-lagged variable proves twice as important as the other important variables.

$$
\begin{aligned}
&S_3(i, j, m) \sim N\left\{ {}^{2}\!/_{(c_i + 1)}, \sigma_c \right\}; \quad j \in C; \quad m = 1; \quad j = i \quad i, j = 1, \ldots, n; \\
&S_3(i, j, m) \sim N\left\{ {}^{1}\!/_{(c_i + 1)}, \sigma_c \right\}; \quad j \in C; \quad m = 1; \quad j \neq i \quad i, j = 1, \ldots, n; \\
&S_3(i, j, m) \sim N\left\{ 0, \eta \, {}^{\sigma_c}\!/_{m} \right\}; \qquad j \in C; \quad m = 2, \ldots, p; \quad i, j = 1, \ldots, n; \text{ and} \\
&S_3(i, j, m) \sim N\left\{ 0, \rho \, {}^{\sigma_c}\!/_{m} \right\}; \qquad j \neg \in C; \quad m = 1, \ldots, p; \quad i, j = 1, \ldots, n.
\end{aligned}
\tag{11}
$$

[10] Kuethe and Pede (2008) specify a similar prior, where they assume that the coefficient of the own-lagged term equals one and the sum of the lags of the other important variables, not including the own-lagged term, sums to one as well. Thus, their weighting scheme doubles the weight as compared to our scheme as well as requiring the own-lagged term to retain the coefficient of one, which reflects the essence of the random-walk averaging (RWA) prior.

Thus, in our three-variable system, c_i equals 3 and the prior means for the first own lag equals one half (i.e., $2/(c_i+1)$ = $2/(3+1)$) and the first lags of the other two important variables in each equation equal one fourth (i.e., $1/(c_i+1)$ = $1/(3+1)$). We employ the following values for the hyperparameters: $\sigma_c = 0.1, \eta = 8,$ and $\rho = 0.5$.[11]

We estimate the BVAR, BVEC, SBVAR, SBVEC, CBVAR, and CBVEC models, based on the FOSC, TC, RWA, and WRWA priors, using Theil's (1971) mixed estimation technique. Specifically, we denote a single equation of the VAR model as: $y_1 = X\beta + \varepsilon_1$, with $Var(\varepsilon_1) = \sigma^2 I$. Then, we can write the stochastic prior restrictions for this single equation as follows:

$$\begin{bmatrix} r_{111} \\ r_{112} \\ . \\ . \\ . \\ r_{nnp} \end{bmatrix} = \begin{bmatrix} \sigma/\sigma_{111} & 0 & . & . & . & 0 \\ 0 & \sigma/\sigma_{112} & 0 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & . & . & . & . & 0 \\ 0 & 0 & . & . & 0 & \sigma/\sigma_{nnp} \end{bmatrix} \begin{bmatrix} \beta_{111} \\ \beta_{112} \\ . \\ . \\ . \\ \beta_{nnp} \end{bmatrix} + \begin{bmatrix} u_{111} \\ u_{112} \\ . \\ . \\ . \\ u_{nnp} \end{bmatrix} \tag{12}$$

Note that $Var(u) = \sigma^2 I$, and the prior means r_{ijm} and the prior variance σ_{ijm}[12] take the forms shown in equations (3) and (4) for the Minnesota prior; in equations (3), (4) and (6) for the FOSC prior; in equations (2), (3), and (7) for the TC prior, in equation (10) for the RWA prior, and in equation (11) for the WRWA prior. With equation (12) written as follows:

$$r = \Sigma\beta + u , \tag{13}$$

we derive the estimates for a typical equation as follows:

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[11] LeSage (1999) suggested ranges for the values for these hyperparameters.

[12] Note σ_{ijm} in equation (12) is a generic term used to describe *$S_k(i, j, m)$, $k=1, 2, 3$.*

$$\hat{\beta} = (X'X + \Sigma'\Sigma)^{-1}(X'y_1 + \Sigma'r) \tag{14}$$

Essentially then, the method involves supplementing the data with prior information on the distribution of the coefficients. The number of observations and degrees of freedom increase artificially by one for each restriction imposed on the parameter estimates. Thus, the loss of degrees of freedom from over-parameterization in the classical VAR or VEC models does not emerge as a concern in the BVAR, BVEC, SBVAR, SBVEC, CBVAR, and CBVEC models.

4. Model Estimation and Results

This section reports our econometric findings. First, we determine whether cointegration exists between the variables in our model. Second, we select the optimal model for forecasting each market's housing price, using the minimum root mean square error (RMSE) for one- to four-quarter-ahead out-of-sample forecasts. Finally, we examine the ability of the optimal forecasting models to detect turning points in our-of-sample forecasts.

Evidence on Cointegration

The first step in our analysis tests for Granger temporal causality between the three housing price series. Temporal causality tests emerge from VAR or VEC models. We first consider various lag-length selection criteria for the VAR specification, including the sequential modified likelihood ratio (LR) test statistic (each test at the 5-percent level), the final prediction error (FPE), the Akaike information criterion (AIC), the Schwarz information criterion (SIC), and the Hannan-Quinn information criterion (HQIC). All criteria choose four lags, except the Schwarz information criterion that chooses two lags. Table 1 reports the results.

We next run the Johansen test for cointegration with four lags, we determine that the VAR model is not stable. Thus, we adopt the SIC and estimate with two lags, where we find that the VAR model is stable. Cointegration tests – the trace statistic and maximum eigen-value test –

both indicate one cointegrating vector. Table 2 tabulates the findings.

Running the VEC specification and using the block exogeneity test, we discover that housing prices in Los Angeles temporally cause housing prices in Las Vegas and that housing prices in Las Vegas temporally cause housing prices in Phoenix.[13] Further, housing prices in Las Vegas or Phoenix do not temporally cause housing prices in Los Angeles. In addition, housing prices in Los Angeles do not directly cause housing prices in Phoenix, but will exhibit an effect through Las Vegas and Las Vegas's effect on Phoenix housing prices. Finally, housing prices in Las Vegas do not cause housing prices in Los Angeles. Table 3 reports the findings. We did not expect to find that housing prices in Los Angeles only directly cause housing prices in Las Vegas and that only Las Vegas's housing prices directly cause housing prices in Phoenix. This result contradicted our prior beliefs, since we expected Los Angeles housing prices to cause Phoenix housing prices directly.

One- to Four-Quarter-Ahead Forecast Accuracy

Given the specification of priors in Section 2, we estimate numerous Bayesian, spatial, causality, and random-walk VAR and VEC models based on the FOSC, TC, RWA, and WRWR priors for Los Angeles, Las Vegas, and Phoenix over the period 1978:Q1 to 1995:Q4 using quarterly data. We then compute out-of-sample one- through four-quarters-ahead forecasts for the period of 1996:Q1 to 2005:Q4, and compare the forecast accuracy relative to the forecasts generated by an unrestricted VAR and VEC models.[14] Note that the choice of the in-sample period, especially, the starting date depends on data availability. The starting point of the out-of-sample period follows Rapach and Strauss (2007, 2008), who observe marked differences in housing price

[13] Since the VEC specification constitutes the first differenced form of the three endogenous variables, and the optimal lag length used for the VAR is 2, we estimate all VEC models with 1 lag.

[14] Note that the initial estimation period does not include the dramatic run up in home prices at the end of the out-of-sample forecast period.

growth across U.S. regions since the mid-1990s. Finally, we choose the end-point of the horizon as 2005:Q4, since we also use our alternative models to predict the turning point(s) in the real housing prices of these three MSAs and, hence, stop prior to the date where the turning point actually occurred. In our case, all three real house prices peaked in 2006Q4. The models include house prices for the above mentioned three metropolitan areas. The nominal housing price data for the three MSAs come from the Freddie Mac. Using matched transactions on the same property over time to account for quality changes, the Conventional Mortgage Home Price Index (CMHPI) of the Freddie Mac provides a means of measuring typical price inflation for houses within the U.S. The Freddie Mac data consist of both purchase and refinance-appraisal transactions, and include over 33 million homes. We deflate the MSA-level nominal CMHPI housing price by the personal consumption expenditure (PCE) deflator from the Bureau of Economic Analysis (BEA) to generate our real housing price series. As Hamilton (1994, p. 362) notes, we seasonally adjust the data, since the Minnesota-type priors do not perform will with seasonal data.

Each equation of the various VAR (VEC) models includes 7 (5) parameters with the constant, given that we estimate the models with 2 (1) lag(s) of each variable.[15] We estimate the three-variable models for a given prior for the period 1978:Q1 to 1995:Q4, and then forecast from 1996:Q1 through to 2005:Q4. Since we use two lags, the initial six quarters from 1978:Q1 to 1979:Q2 feed the lags. We re-estimate the models each quarter over the out-of-sample forecast horizon in order to update the estimate of the coefficients, before producing the 4-quarters-ahead forecasts. We implemented this iterative estimation and the 4-quarters-ahead forecast procedure

[15] We initially chose 4 lags based on the unanimity of the sequential modified LR test statistic, the final prediction error (FPE), Akaike information criterion (AIC), and the Hannan-Quinn information criterion (HQIC). The Schwarz information criterion (SIC) provided the exception of 2 lags. The VAR model using 4 lags, however, proved unstable. Thus, we opted for the 2 lags indicated by the SIC, which generated a stable VAR.

for 40 quarters, with the first forecast beginning in 1996:Q1. This produced a total of 40 one-quarter-ahead forecasts, …, up to 40 four-quarters-ahead forecasts.[16] We calculate the root mean squared errors (RMSE)[17] for the 40 one-, two-, three-, and four-quarters-ahead forecasts for the three home prices of the models. We then examine the average of the RMSE statistic for one-, two-, three-, and four-quarters ahead forecasts over 1996:Q1 to 2005:Q4. We follow the same steps to generate forecasts from the Bayesian, spatial, random-walk, and causality versions of VAR and VEC models based on the FOSC, TC, RWA, and WRWA priors.

For the BVAR models, we start with a value of $w = 0.1$ and $d = 1.0$, and then increase the value to $w = 0.2$ to account for more influences from variables other than the first own lags of the dependant variables of the model. In addition, as in Dua and Ray (1995), Gupta and Sichei (2006), and Gupta (2006), we also estimate a BVAR model with $w = 0.3$ and $d = 0.5$. We also introduce $d = 2$ to increase the tightness on lag m. Finally, we specify $\sigma_c=0.1$, $\eta=8$, $\theta=0.5$ for the random-walk models with the two different specifications for causality and spatial priors. We select the model that produces the lowest average RMSE values as the 'optimal' specification for a specific metropolitan area.

Table 4, 5, and 6 report the findings for Los Angeles, Las Vegas, and Phoenix. Table 4 reports the findings for Los Angeles. The last column looks at the average of RMSEs across the one-, two-, three-, and four-quarter-ahead forecast RMSEs. The spatial BVEC model with $w=0.1$, and $d=2.0$ provides the lowest average RMSE, which we identify as the optimal specification. This specification also minimizes the RMSE for the two-quarter-ahead forecasts as

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[16] For this, we used the algorithm in the Econometric Toolbox of MATLAB, version R2006a.

[17] Note that if A_{t+n} denotes the actual value of a specific variable in period $t + n$ and ${}_tF_{t+n}$ equals the forecast made in period t for $t + n$, the RMSE statistic equals the following: $\sqrt{\left[\sum_1^N \left({}_tF_{t+n} - A_{t+n} \right)^2 \Big/ N \right]}$ where N equals the number of forecasts.

well. The BVAR model with $w=0.2$, and $d=1.0$ provides the optimal specification for the one-quarter-ahead forecast, while the spatial RBVEC and causality RBVEC models with the first priors prove optimal for the three- and four-quarter-ahead-forecast horizon.

Table 5 reports the findings for Las Vegas. The VAR specification provides the lowest average RMSE, as well as the lowest RMSE for the three- and four-quarter-ahead forecast horizon. The spatial BVEC model with $w=0.1$, and $d=1.0$ provides the optimal specification for the one-quarter-ahead forecast, while the causality RBVEC models with the second prior proves optimal for the two-ahead-forecast horizon.

Table 6 reports the findings for Phoenix. The spatial RBVAR model with the second prior provides the lowest average RMSE, as well as the lowest RMSE for the two- and three-quarter-ahead forecast horizon. The causality RBVAR model with the first prior provides the optimal specification for the one-quarter-ahead forecast, while the VAR model proves optimal for the three-quarter-ahead forecast horizon.

In sum, different specifications yield the lowest RMSE in different cities.[18] No common pattern emerges. Comparing the forecasting performance across cites, however, we see that Los Angeles experiences the lowest RMSE for the one-, two-, and three-quarter-ahead forecast horizon, while Las Vegas experiences the lowest RMSEs for the four-quarter-ahead forecast horizon and for the average across all four forecast horizons.

Forecasting Turning Points

Figure 1 illustrates that each housing market experienced a marked reversal of real housing prices after the peak in fourth quarter of 2006. We exposed our optimal forecast models to the

[18] We also considered the specifications that produce the lowest average RMSE across all three cities (not reported, results available on request). The BVEC specification with $w=0.1$, and $d=2.0$ provides the optimal specification for the two- and four-quarter-ahead forecast horizon as well as for the average across all four horizons. The VEC specification proves the optimal model for the one-quarter-ahead forecast horizon, while the RBVEC specification with the second prior proves optimal for the three-quarter-ahead forecast horizon.

acid test – predicting turning points. We estimated the optimal models from Tables 4, 5, and 6 using data through the fourth quarter of 2005 and then forecasted prices from the first quarter of 2006 through the end of the sample period in the first quarter of 2008. The results of this forecasting experiment appear in Tables 7, 8, and 9. Table 7 reports the forecasting results for Los Angeles, where we used the spatial BVEC model with $w=0.1$, and $d=2.0$. Table 8 reports the forecasting results for Las Vegas, where we used the VAR specification. Finally, Table 9 reports the forecasting results for Phoenix, where we used the spatial RBVAR model with the second prior.

Next, we re-estimated the optimal forecasting models through the first quarter of 2006 and forecast the housing price in the second quarter of 2006 through the end of the sample. We continued to update the estimated model by adding data one quarter at a time and then forecasting out of sample. The recursive forecast results appear in Tables 10, 11, and 12.

Tables 7, 8, and 9 report the ten-quarter-ahead forecasts of housing prices using the VAR and VEC models as well as the optimal BVAR and BVEC models for each city chosen from Tables 4, 5, and 6. With actual data that ends one-year ahead of the actual turning points for home prices in each city, none of the forecasting models forecasts a turning point in home prices. All forecasting models, however, use data that lies on the still rising portions of the "bubble" curves that we see in Figure 1. That is, it proves difficult to forecast a turning point when recent history shows a continuing rise in home prices. The recursive forecasts allow the forecaster to update the data set with new information, which we shall consider in due course.

We use the best performing models from Tables 4, 5, and 6 in the findings reported in Tables 7, 8, and 9 – Los Angeles (BVEC), Las Vegas (VAR), and Phoenix (BVAR). We bold the forecast values in Tables 7, 8, and 9. For Los Angeles and Phoenix, the overall optimal forecast model does the best of keeping the forecast price from rising too high. In other words, the deviations for the actual price are minimized when we use the optimal BVEC model to forecast Los Angeles prices and when we use the optimal BVAR model to forecast Phoenix prices. The Las Vegas numbers provide a different picture. Both the VEC and the optimal BVEC models produce smaller forecast errors from one to ten-quarters ahead than the VAR model. The optimal BVEC shows the best performance. The performance of the VAR model in Las Vegas provides a different outcome, especially at longer out-of-sample forecast horizons. We return to this point below.

Tables 10, 11, and 12 report the recursive forecasts. Once again, we employ the optimal models from Tables 4, 5, and 6 to generate the recursive forecasts – Los Angeles (BVEC), Las Vegas (VAR), and Phoenix (BVAR). The diagonal forecasts report the one-quarter-ahead forecast as we re-estimate the models by adding one quarter at a time. These one-step-ahead forecasts do reasonably well. In fact, the forecast prices peak in Las Vegas and Phoenix in the second quarter of 2006, two quarters before the actual series peak. This probably reflects the fact that in both Las Vegas and Phoenix, the forecasts begin to exceed the actual values by enough to cause the forecasts to attempt to close that overestimation gap. Less of a gap appears in Los Angeles and its forecasts do not peak until the fourth quarter of 2006, when the actual series itself peaks.

The Los Angeles forecasts also prove interesting in that exactly when the price index falls in Los Angeles (i.e., 2007:Q1), the pattern of forecasts into the future fall monotonically (See Table 10, Forecast 5). Until this point, the future forecasts monotonically increased (See Table 10, column 4). Las Vegas and Phoenix do not experience the same type of forecasting precision.

In Las Vegas, we observe this downward movement in future forecasts with data through 2007:Q4 (See Table 11, Forecast 9). Phoenix never experiences this phenomenon.

Given the anomalies in the forecasts for Las Vegas, we re-ran the recursive forecasts, using the regular BVEC model with *w=0.1*, and *d=2*.0. Table 13 reports the findings. The BVEC model performs better than the VAR model before the turning point in 2006:Q4 and that performance improves at longer forecasting horizons. After the turning point in home prices, then the VAR model general produces better forecasts than the BVEC model.

5. Conclusion

The bloom is off the rose of the housing boom. Housing prices rose dramatically in Los Angeles, Las Vegas, and Phoenix in the early 2000s, peaking in real terms in 2006:Q4. This paper considers the time-series relationships between the housing prices in these three MSAs, using Freddie Mac data from 1978:Q1 to 2008:Q1. First, we test for Granger temporal causality. Second, we generate out-of-sample forecasts using VAR, VEC and Bayesian, spatial, and causality VAR and VEC models with various priors. Finally, we explore the ability of these models to forecast turning points in housing prices that occurred in 2006:Q4.

Los Angeles housing prices directly cause Las Vegas housing prices and indirectly cause Phoenix housing prices through their effect on Las Vegas housing prices. That is, Las Vegas housing prices directly cause Phoenix housing prices. Las Vegas housing prices do not cause Los Angeles housing prices and Phoenix housing prices do not cause housing prices in Las Vegas or Los Angeles. As a result, Los Angeles housing prices prove temporally exogenous.

Different time-series models prove better at forecasting housing prices in the different MSAs. For Los Angeles, a spatial BVECs model provides the best forecasts. For Las Vegas, the

VAR specification provides the best forecasts. Finally, for Phoenix, a spatial RBVAR model provides the best forecasts.

Forecasting turning points in housing prices proves a difficult task. When we estimate our model using data before the turning points in 2006:Q1, forecasts continue to predict a rising trend in housing prices and do not signal any turning point. When we update the data for the estimated model as new data become available, then we do forecast turning points with some degree of accuracy. The one-step-ahead forecasts do reasonably well. The forecast prices actually peak in Las Vegas and Phoenix in the second quarter of 2006, two quarters before the actual series peak. That is, in both Las Vegas and Phoenix, the forecasts begin to exceed the actual values sufficiently to cause the forecasts to attempt to close that overestimation. Less of a gap appears in Los Angeles and its forecasts do not peak until the fourth quarter of 2006, when the actual series itself peaks.

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Table 1: Lag-Length Selection Tests

Lag	LogL	LR	FPE	AIC	SIC	HQIC
0	445.6276	NA	9.11e-08	-7.697871	-7.626264	-7.668806
1	1090.430	1244.749	1.44e-12	-18.75530	-18.46887	-18.63904
2	1174.314	157.5557	3.91e-13	-20.05763	-19.55638*	-19.85418
3	1184.930	19.38667	3.80e-13	-20.08574	-19.36967	-19.79509
4	1203.038	32.12151*	3.25e-13*	-20.24414*	-19.31325	-19.86629*
5	1209.859	11.74336	3.38e-13	-20.20624	-19.06053	-19.74120
6	1213.612	6.267124	3.72e-13	-20.11500	-18.75447	-19.56276
7	1218.061	7.195283	4.05e-13	-20.03584	-18.46049	-19.39642
8	1228.529	16.38470	3.97e-13	-20.06137	-18.27120	-19.33475

Note: The star indicates lag order selected by the criterion. The criterion include the sequential modified likelihood ratio (LR) test statistic (each test at 5% level), the final prediction error (FPE), the Akaike information criterion (AIC), the Schwarz information criterion (SIC), and the Hannan-Quinn information criterion (HQIC).

Table 2: Johansen Cointegration Tests

Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(s)	**Eigenvalue**	**Trace Statistic**	**0.05 Critical Value**	**Prob.****
None *	0.191438	36.95976	29.79707	0.0063
At most 1	0.064056	11.46000	15.49471	0.1847
At most 2	0.028875	3.516001	3.841466	0.0608
Unrestricted Cointegration Rank Test (Maximum Eigenvalue)				
Hypothesized No. of CE(s)	**Eigenvalue**	**Max-Eigen Statistic**	**0.05 Critical Value**	**Prob.****
None *	0.191438	25.49976	21.13162	0.0114
At most 1	0.064056	7.943997	14.26460	0.3844
At most 2	0.028875	3.516001	3.841466	0.0608

Note: The trace and maximum eigen-value tests both indicate one cointegrating vector at the 5-percent level.

* denotes rejection of the hypothesis at the 0.05 level
** MacKinnon-Haug-Michelis (1999) p-values

Table 3: Granger Temporal Causality Tests

Dependent variable: D(lnP_{LA})			
Excluded	**χ^2**	**df**	**Prob.**
D(***lnP_{LV}***)	1.910253	1	0.1669
D(***lnP_{PH}***)	0.023862	1	0.8772
All	1.922211	2	0.3825
Dependent variable: D(lnP_{LV})			
Excluded	**χ^2**	**df**	**Prob.**
D(***lnP_{LA}***)	8.442305	1	0.0037
D(***lnP_{PH}***)	0.009186	1	0.9236
All	10.88809	2	0.0043
Dependent variable: D(lnP_{PH})			
Excluded	**χ^2**	**df**	**Prob.**
D(***lnP_{LA}***)	0.708430	1	0.4000
D(***lnP_{LV}***)	10.99597	1	0.0009
All	20.42951	2	0.0000

Note: *D* equals the first difference operator, *ln* stands for the natural logarithm, and P_{LA}, P_{LV}, and P_{PH} equal the real home price indexes in Los Angeles, Las Vegas, and Phoenix, respectively. χ^2 equals the chi-squared statistic, *df* equals the number of degrees of freedom, and Prob. equals the probability of insignificance.

Table 4: Forecast Results for Los Angeles

Parameterization	Models	RMSEs 1	2	3	4	Average
	VAR	0.002356	0.088082	0.220955	0.258075	0.142367
	VEC	0.021484	0.031859	0.092055	0.081208	0.056652
w=0.3, d=0.5	**BVAR**	0.001749	0.086686	0.218564	0.254763	0.14044
	BVEC	0.021475	0.031655	0.092104	0.073687	0.05473
	Causality BVAR	0.001095	0.080932	0.208305	0.239763	0.132524
	Spatial BVAR	0.001963	0.087144	0.219394	0.255942	0.141111
	Causality BVEC	0.021525	0.032473	0.095811	0.080522	0.057583
	Spatial BVEC	0.021464	0.031604	0.091767	0.0732	0.054509
w=0.2, d=1	**BVAR**	**0.000174**	0.08311	0.212534	0.246499	0.135579
	BVEC	0.021557	0.030879	0.092068	0.074064	0.054642
	Causality BVAR	0.003021	0.077191	0.20125	0.229086	0.127637
	Spatial BVAR	0.000913	0.084657	0.215304	0.250409	0.137821
	Causality BVEC	0.021784	0.031897	0.099015	0.087188	0.059971
	Spatial BVEC	0.021426	0.030864	0.09109	0.07262	0.054
w=0.1, d=1	**BVAR**	0.004474	0.072855	0.195783	0.224005	0.124279
	BVEC	0.021574	0.028754	0.091612	0.074864	0.054201
	Causality BVAR	0.001705	0.080587	0.20612	0.235084	0.130874
	Spatial BVAR	0.00289	0.075835	0.201064	0.231422	0.127803
	Causality BVEC	0.02041	0.033229	0.104824	0.097478	0.063986
	Spatial BVEC	0.021267	0.02838	0.088716	0.070581	0.052236
w=0.2, d=2	**BVAR**	0.00402	0.073906	0.197566	0.226481	0.125493
	BVEC	0.021977	0.028326	0.091127	0.073915	0.053836
	Causality BVAR	0.00652	0.069507	0.18848	0.211431	0.118984
	Spatial BVAR	0.002215	0.077429	0.20369	0.235004	0.129585
	Causality BVEC	0.022319	0.029651	0.09841	0.087776	0.059539
	Spatial BVEC	0.021374	0.02862	0.089376	0.071166	0.052634
w=0.1, d=2	**BVAR**	0.013584	0.054179	0.168069	0.189412	0.106311
	BVEC	0.022129	0.023876	0.088433	0.072849	0.051822
	Causality BVAR	0.009299	0.063384	0.179251	0.199808	0.112936
	Spatial BVAR	0.011731	0.056881	0.172534	0.195458	0.109151
	Causality BVEC	0.019994	0.030473	0.102467	0.095455	0.062097
	Spatial BVEC	0.021195	**0.023058**	0.084782	0.067271	**0.049077**
σ_c=0.1, η=8, θ=0.5	**RBVAR Causality1**	0.064297	0.114092	0.279637	0.358194	0.204055
	RBVAR Causality2	0.064297	0.114675	0.280637	0.359171	0.204695
	RBVAR Spatial1	0.064297	0.114675	0.280637	0.359171	0.204695
	RBVAR Spatial2	0.1283	0.054725	0.208176	0.278513	0.167428
	RBVEC Causality1	0.045523	0.461615	0.1023	**0.060789**	0.167557
	RBVEC Causality2	0.045523	0.462689	0.100334	0.060934	0.16737
	RBVEC Spatial1	0.137511	0.190315	**0.004198**	0.170303	0.125582
	RBVEC Spatial2	0.121492	0.263264	0.020017	0.150712	0.138871

Note: VAR and VEC refer to vector autoregressive and vector error-correction models. BVAR and BVEC refer to Bayesian VAR and VEC models. The text discusses the various priors and parameterizations. RMSE means root mean square error. The entries measure the average RMSE across all forecasts at each horizon – one-, two-, three-, and four-quarter-ahead forecasts. The column Average computes the average RMSE across the one-, two-, three-, and four-quarter-ahead forecast RMSEs.

Table 5: Forecast Results for Las Vegas

Parameterization	Models	RMSEs 1	2	3	4	Average
	VAR	0.078444	0.04269	**0.029767**	**0.041256**	**0.048039**
	VEC	0.006589	0.136091	0.148878	0.146749	0.109577
w=0.3, d=0.5	**BVAR**	0.079121	0.043777	0.03139	0.043041	0.049332
	BVEC	0.00685	0.132051	0.144884	0.119994	0.100945
	Causality BVAR	0.084955	0.05189	0.043126	0.056006	0.058994
	Spatial BVAR	0.078615	0.04304	0.030257	0.041755	0.048417
	Causality BVEC	0.007094	0.129858	0.139978	0.119237	0.099042
	Spatial BVEC	0.006443	0.135484	0.14826	0.123195	0.103346
w=0.2, d=1	**BVAR**	0.080869	0.046702	0.035636	0.047759	0.052742
	BVEC	0.008045	0.120528	0.133617	0.109972	0.093041
	Causality BVAR	0.088254	0.056899	0.050216	0.063556	0.064731
	Spatial BVAR	0.079221	0.044233	0.031877	0.043461	0.049698
	Causality BVEC	0.007731	0.122886	0.131449	0.115325	0.094347
	Spatial BVEC	0.006405	0.132535	0.145469	0.121116	0.101381
w=0.1, d=1	**BVAR**	0.084759	0.053288	0.045101	0.058026	0.060293
	BVEC	0.01044	0.094044	0.108185	0.088073	0.075185
	Causality BVAR	0.086554	0.055231	0.048263	0.060856	0.062726
	Spatial BVAR	0.081062	0.047949	0.036904	0.048647	0.053641
	Causality BVEC	0.007743	0.11545	0.127988	0.116988	0.092042
	Spatial BVEC	**0.005808**	0.12413	0.137804	0.116374	0.096029
w=0.2, d=2	**BVAR**	0.085178	0.054051	0.045931	0.059175	0.061084
	BVEC	0.01158	0.091931	0.105437	0.083712	0.073165
	Causality BVAR	0.09097	0.061821	0.056548	0.070289	0.069907
	Spatial BVAR	0.081347	0.048284	0.037249	0.049193	0.054018
	Causality BVEC	0.008918	0.112864	0.121747	0.107675	0.087801
	Spatial BVEC	0.007147	0.120896	0.134603	0.111457	0.093526
w=0.1, d=2	**BVAR**	0.090525	0.064068	0.059548	0.073853	0.071999
	BVEC	0.015346	0.05478	0.070029	0.052046	0.04805
	Causality BVAR	0.088148	0.059505	0.053176	0.065699	0.066632
	Spatial BVAR	0.086208	0.057752	0.049651	0.062084	0.063924
	Causality BVEC	0.011154	0.095126	0.112308	0.102702	0.080322
	Spatial BVEC	0.007735	0.095709	0.111545	0.092854	0.076961
σ_c=0.1, η=8, θ=0.5	**RBVAR Causality1**	0.065611	0.199103	0.298367	0.335746	0.224707
	RBVAR Causality2	0.056531	0.198103	0.300921	0.341953	0.224377
	RBVAR Spatial1	0.064987	0.223822	0.323479	0.37275	0.246259
	RBVAR Spatial2	0.066808	0.229459	0.336238	0.38459	0.254274
	RBVEC Causality1	0.059485	0.078669	0.116412	0.164565	0.104782
	RBVEC Causality2	0.056374	**0.02368**	0.125339	0.176229	0.095406
	RBVEC Spatial1	0.059444	0.124821	0.094246	0.131503	0.102504
	RBVEC Spatial2	0.062531	0.116138	0.101992	0.142064	0.105681

Note: See Table 4.

Table 6: Forecast Results for Phoenix

Parameterization	Models	RMSEs 1	2	3	4	Average
	VAR	0.074832	0.100296	**0.095718**	0.126134	0.099245
	VEC	0.073432	0.129337	0.138516	0.163259	0.126136
w=0.3, d=0.5	**BVAR**	0.075224	0.100462	0.095971	0.126325	0.099495
	BVEC	0.073843	0.129165	0.138505	0.206787	0.137075
	Causality BVAR	0.079603	0.108269	0.104137	0.134589	0.10665
	Spatial BVAR	0.075068	0.100542	0.09608	0.126446	0.099534
	Causality BVEC	0.074959	0.126639	0.134927	0.200671	0.134299
	Spatial BVEC	0.073723	0.128984	0.138321	0.206597	0.136906
w=0.2, d=1	**BVAR**	0.075877	0.10041	0.096023	0.126148	0.099615
	BVEC	0.074428	0.129873	0.139126	0.207078	0.137626
	Causality BVAR	0.084352	0.115085	0.111587	0.141669	0.113173
	Spatial BVAR	0.075558	0.10104	0.096828	0.127053	0.10012
	Causality BVEC	0.077399	0.128404	0.134175	0.197235	0.134303
	Spatial BVEC	0.074122	0.128898	0.13835	0.206199	0.136892
w=0.1, d=1	**BVAR**	0.104364	0.101909	0.097972	0.127298	0.107886
	BVEC	0.125753	0.130867	0.139875	0.206002	0.150624
	Causality BVAR	0.08697	0.116768	0.114216	0.144162	0.115529
	Spatial BVAR	0.077482	0.102932	0.099572	0.129275	0.102315
	Causality BVEC	0.083161	0.123299	0.124886	0.179133	0.12762
	Spatial BVEC	0.076081	0.127809	0.13774	0.203571	0.1363
w=0.2, d=2	**BVAR**	0.077517	0.100184	0.095874	0.125234	0.099702
	BVEC	0.074754	0.13344	0.141858	0.209966	0.140004
	Causality BVAR	0.088677	0.120144	0.117114	0.145979	0.117979
	Spatial BVAR	0.07672	0.102174	0.098539	0.128334	0.101442
	Causality BVEC	0.078368	0.132964	0.137335	0.200364	0.137258
	Spatial BVEC	0.074313	0.130621	0.139723	0.207425	0.138021
w=0.1, d=2	**BVAR**	0.085407	0.10511	0.100817	0.127966	0.104825
	BVEC	0.07733	0.136097	0.143635	0.209612	0.141669
	Causality BVAR	0.091197	0.118606	0.115586	0.14285	0.11706
	Spatial BVAR	0.081054	0.106144	0.103833	0.132269	0.105825
	Causality BVEC	0.083672	0.125198	0.12621	0.179663	0.128686
	Spatial BVEC	0.076408	0.132238	0.140814	0.20636	0.138955
σ_c=0.1, η=8, θ=0.5	**RBVAR Causality1**	**0.004513**	0.084711	0.158477	0.130565	0.094566
	RBVAR Causality2	0.004749	0.072686	0.145066	0.114462	0.084241
	RBVAR Spatial1	0.0059	0.041866	0.114944	0.068773	0.057871
	RBVAR Spatial2	0.014564	**0.038689**	0.104193	**0.060713**	**0.05454**
	RBVEC Causality1	0.028227	0.101913	0.208121	0.145082	0.120835
	RBVEC Causality2	0.035707	0.121426	0.208969	0.148627	0.128682
	RBVEC Spatial1	0.013979	0.039875	0.185631	0.119491	0.089744
	RBVEC Spatial2	0.022113	0.073755	0.189474	0.125316	0.102665

Note: See Table 4.

Table 7: Forecast of the Real Housing Price Index: Los Angeles

Quarters	Actuals	VAR	VEC	Optimal BVAR	Optimal BVEC
2005:Q4	377.5166	377.5166	377.5166	377.5166	**377.5166**
2006:Q1	390.9894	402.3522	402.0597	403.0200	**402.1154**
2006:Q2	398.2135	433.2707	416.7950	434.7338	**416.9335**
2006:Q3	403.5069	470.9669	433.1485	473.3162	**433.3638**
2006:Q4	405.6439*	516.7845	451.3739	520.0366	**450.5531**
2007:Q1	401.5254	572.5970	471.7691	576.7335	**469.5853**
2007:Q2	398.3245	640.9297	494.6857	645.9269	**489.6644**
2007:Q3	391.1110	725.1947	520.5412	731.0308	**511.8802**
2007:Q4	376.7706	830.0309	549.8346	836.6817	**535.5034**
2008:Q1	350.6978	961.8065	583.1659	969.2328	**561.6401**
2008:Q2	321.4719	1129.3723	621.2610	1137.4966	**589.6411**

Note: One- to ten-quarter-ahead real housing price index forecasts. The star identifies the turning point. Bold numbers reflect the best forecasts. The Actual column gives the actual data. The Optimal models come from the best performing model in Table 4.

Table 8: Forecast of the Real Housing Price Index: Las Vegas

Quarters	Actuals	VAR	VEC	Optimal BVAR	Optimal BVEC
2005:Q4	282.0519	**282.0519**	282.0519	282.0519	282.0519
2006:Q1	289.5116	**297.5824**	297.3371	297.5839	297.4738
2006:Q2	289.6797	**315.4873**	312.8668	315.4916	314.3471
2006:Q3	290.6893	**336.1310**	330.0674	336.1392	333.1445
2006:Q4	292.2187*	**360.1764**	349.1903	360.1890	353.1478
2007:Q1	288.1104	**388.3283**	370.5339	388.3458	375.4483
2007:Q2	281.6813	**421.4800**	394.4539	421.5029	399.3427
2007:Q3	274.4277	**460.7782**	421.3763	460.8075	426.0163
2007:Q4	263.3262	**507.7027**	451.8145	507.7394	454.7897
2008:Q1	243.3095	**564.1813**	486.3898	564.2266	486.9704
2008:Q2	217.7319	**632.7524**	525.8589	632.8080	521.9175

Note: See Table 7. The Optimal models come from the best performing model in Table 5.

Table 9: Forecast of the Real Housing Price Index: Phoenix

Quarters	Actuals	VAR	VEC	Optimal BVAR	Optimal BVEC
2005:Q4	266.3477	266.3477	266.3477	**266.3477**	266.3477
2006:Q1	275.8913	284.9617	287.0331	**276.9951**	297.5065
2006:Q2	280.0896	307.1025	303.6691	**309.8393**	278.9715
2006:Q3	281.7792	332.7481	322.5838	**330.5377**	287.0832
2006:Q4	283.9403*	362.6791	344.1586	**347.4027**	314.0445
2007:Q1	280.4600	397.9571	368.8553	**350.4730**	362.5610
2007:Q2	276.5298	439.9076	397.2370	**386.6116**	345.4533
2007:Q3	271.3676	490.2433	429.9940	**402.5804**	351.8249
2007:Q4	263.5439	551.2117	467.9784	**412.7865**	394.2349
2008:Q1	251.8645	625.7996	512.2481	**405.3843**	479.3154
2008:Q2	235.7252	718.0279	564.1266	**441.0331**	471.3939

Note: See Table 7. The Optimal models come from the best performing model in Table 6.

Table 10: Recursive Forecasts of the Real Housing Price Index: Los Angeles

Quarter	Actual	Diagonal	Forecast 1	Forecast 2	Forecast 3	Forecast 4	Forecast 5	Forecast 6	Forecast 7	Forecast 8	Forecast 9	Forecast 10
2005:Q4	377.5166	377.5166										
2006:Q1	390.9894	402.1154	402.1154									
2006:Q2	398.2135	412.5039	416.9335	412.5039								
2006:Q3	403.5069	412.7654	433.3638	426.6052	412.7654							
2006:Q4	405.6439*	414.3912	450.5531	441.9711	422.6916	414.3912*						
2007:Q1	401.5254	407.5248	469.5853	457.9749	433.2754	421.7162	407.5248†					
2007:Q2	398.3245	398.7056	489.6644	475.4030	444.1630	429.3809	407.2615	398.7056				
2007:Q3	391.1110	395.6097	511.8802	493.6764	455.7652	437.2031	406.6742	397.7281	395.6097			
2007:Q4	376.7706	386.2894	535.5034	513.5718	467.7519	445.3857	406.0646	397.1972	395.1479	386.2894		
2008:Q1	350.6978	366.6303	561.6401	534.5652	480.5196	453.7590	405.4656	396.7122	395.1598	385.6374	366.6303	
2008:Q2	321.4719	331.3388	589.6411	557.4264	493.7649	462.5154	404.8825	396.2724	395.2414	385.8262	365.1201	331.3388

Note: The Actual column gives the actual data. The Diagonal column gives the one-quarter-ahead forecast for Forecast 1, 2, …,, and 10. Forecast 1 estimates the model through 2005:Q4 and then forecasts one-, two-, ..,, and ten-quarters ahead. Forecast 2 estimates the model through 2006:Q1 and then forecasts one-, two-, …, and nine-quarters ahead, and so on. Finally, Forecast 10 estimates the model through 2008:Q1 and then forecasts one-quarter ahead.

Table 11: Las Vegas

Quarter	Actual	Diagonal	Forecast 1	Forecast 2	Forecast 3	Forecast 4	Forecast 5	Forecast 6	Forecast 7	Forecast 8	Forecast 9	Forecast 10
2005:Q4	282.0519	282.0519										
2006:Q1	289.5116	297.5824	297.5824									
2006:Q2	289.6797	305.7904	315.4873	305.7904*								
2006:Q3	290.6893	303.0592	336.1310	322.8715	303.0592							
2006:Q4	292.2187*	301.1435	360.1764	342.2867	316.4940	301.1435						
2007:Q1	288.1104	300.2653	388.3283	364.6742	331.4168	311.8731	300.2653					
2007:Q2	281.6813	293.0282	421.4800	390.5373	348.2562	323.7356	308.7708	293.0282				
2007:Q3	274.4277	284.6283	460.7782	420.5474	367.2575	336.9079	318.1989	298.1557	284.6283			
2007:Q4	263.3262	274.7753	507.7027	455.5541	388.7349	351.5190	328.5949	303.9814	287.8426	274.7753		
2008:Q1	243.3095	260.2166	564.1813	496.6275	413.0761	367.7257	340.0161	310.4200	291.7451	275.6238	260.2166†	
2008:Q2	217.7319	234.3917	632.7524	545.1260	440.7519	385.7180	352.5374	317.4574	296.1465	277.1921	257.7795	234.3917

Note: See Table 10.

Table 12: Phoenix

Quarter	Actual	Diagonal	Forecast 1	Forecast 2	Forecast 3	Forecast 4	Forecast 5	Forecast 6	Forecast 7	Forecast 8	Forecast 9	Forecast 10
2005:Q4	266.3477	266.3477										
2006:Q1	275.8913	276.9951	276.9951									
2006:Q2	280.0896	310.1822	309.8393	310.1822*								
2006:Q3	281.7792	306.8161	330.5377	331.8392	306.8161							
2006:Q4	283.9403*	304.7925	347.4027	350.1096	322.1683	304.7925						
2007:Q1	280.4600	295.0095	350.4730	352.4698	329.0192	310.9655	295.0095					
2007:Q2	276.5298	286.0993	386.6116	392.6300	327.3146	313.5200	298.8613	286.0993				
2007:Q3	271.3676	281.1712	402.5804	411.2223	356.3408	312.4512	301.4895	289.8576	281.1712			
2007:Q4	263.5439	276.2977	412.7865	424.2500	369.7165	337.8164	304.2295	294.787	286.534	276.2977		
2008:Q1	251.8645	263.4873	405.3843	415.8885	372.5957	342.7264	317.2396	293.1513	286.3704	277.3012	263.4873	
2008:Q2	235.7252	249.695	441.0331	458.0949	365.6043	343.6502	321.8623	301.1827	285.026	277.5153	265.2239	249.695

Note: See Table 10.

Table 13: Las Vegas

Quarter	Actual	Diagonal	Forecast 1	Forecast 2	Forecast 3	Forecast 4	Forecast 5	Forecast 6	Forecast 7	Forecast 8	Forecast 9	Forecast 10
2005Q4	282.0519	282.0519										
2006Q1	289.5116	297.4404	297.4404									
2006Q2	289.6797	306.2669	314.3403	306.2669								
2006Q3	290.6893	302.2734	333.1195	322.8494	302.2734							
2006Q4	292.2187*	299.5712	353.1832	341.0358	313.7618	299.5712						
2007Q1	288.1104	295.1889	375.4914	360.4126	325.9868	307.4681	295.1889					
2007Q2	281.6813	289.5343	399.3006	381.6805	338.7661	315.5731	297.2350	289.5343				
2007Q3	274.4277	281.7085	425.9906	404.3896	352.3709	323.9664	299.0198	290.4023	281.7085			
2007Q4	263.3262	273.9050	454.7786	429.4375	366.5220	332.6596	300.6769	291.6084	281.9681	273.9050		
2008Q1	243.3095	260.8555	486.9675	456.2470	381.5926	341.6646	302.3431	292.8869	282.6832	273.4639	260.8555	
2008Q2	217.7319	237.1920	521.9153	485.8475	397.3745	350.9942	304.0885	294.1711	283.4654	273.8420	259.2985	237.1920

Note: See Table 10.

Figure 1: Housing Price Indexes: Las Vegas, Los Angeles, and Phoenix