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The Ownership of the Firm under A Property Rights Approach

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Abstract

The boundaries of the firm and the ownership of the firm have been two of the main themes of the economics of organization over the past several decades. In this paper, I develop a general multi-party framework that integrates the ownership of the firm into the property-rights approach to the firm. I consider the ownership of the firm as the ownership of the rights to terminate cooperation with any party while maintaining a contractual or employment relation with all the other related parties of the firm. The model in this paper allows for the separation of the ownership of the firm from the ownership of the alienable assets that partly constitute it. Such a general multi-party setup may provide new tools for the study of the problem of the firms boundaries as well as inspiration for further applications of the theory of property rights.

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1 Introduction

The boundaries of the firm and the ownership of the firm have been two of the main themes of the economics of organization over the past several decades. These two closely related topics reflect two different descriptions of firm structure. One can think of the issue of the boundary of the firm as a "satellite view" from above that demarcates inside parties from related outsiders. By contrast, one can consider the ownership of the firm as a "street view" of a firm describes its vertical control structure. I argue that these two related aspects should both be considered when we try to study the organization of a firm. Studies employing both perspectives simultaneously have been rare for at least two decades. Although theories of the boundaries of the firm have proliferated, the problem of firm ownership and the study of the interdependency between the boundaries and the control structure of the firm have been relatively few.

In this paper, I develop a general multi-party framework that integrates the ownership of the firm into the property-rights approach to the firm. I consider the ownership of the firm as the ownership of the rights to terminate cooperation with any party while maintaining a contractual or employment relation with all the other related parties of the firm. In other words, if a party could fire a real subset of the firm while keeping the rest, then it is the owner of the firm. This definition of the ownership of the firm is consistent with Alchian and Demsetz (1972). By contrast, the firing decisions in a two-party property-rights model is trivial because the two parties simply terminate cooperation with each other if the bargaining should fail. And in a multi-party model, no party takes active firing decisions, and the returns are not generally determined through bargaining with an owner of the firm(Hart and Moore, 1990).

The model in this paper allows for the separation of the ownership of the firm from the ownership of the alienable assets that partly constitute it. Ownership of the firm and ownership of alienable assets become independent but simultaneous choices within the firm's problem of organization. Such a general multi-party setup may provide new tools for the study of the problem of the firm's boundaries as well as inspiration for further applications of the theory of property rights.

Vertical integration has long been one of the main foci of the field of institutional economics (Coase, 1937). In this literature, the problem of vertical integration is identical at a general level to the problem of the firm's boundaries. The main insight is that the firm arises because it overcomes various frictions of market contracting under the price mechanism. Vertical integration replaces the market with the firm as an alternative coordination system. Transaction-cost economics (TCE) (Williamson, 1971, 1979, 1985; Klein et al., 1978) has analyzed the advantages versus disadvantages of the firm compared to the price mechanism. Both firms and markets generate transaction costs. In the case of the firm, those costs that arise from the shirking of employees with low-powered incentives and from resources expended to monitor employees (Holmstrom and Milgrom, 1991; Holmstrom and Tirole, 1991; Holmstrom and Milgrom, 1994; Holmstrom, 1999) and to communicate information (Hayek, 1945). In the case of the market, contracting costs arise from a number of sources, many of which can be traced to the incomplete nature of contracts. Organizing within a firm avoids costs like those of haggling over appropriable quasi-rents but incurs the costs of internal governance. In the end, then, the optimal boundary of the firm is determined by the trade-off between the cost of external coordination and the cost of internal coordination.

From the perspective of TCE, the problem of the ownership of the firm comes into view when we enquire more closely into the sources of transaction costs, especially internal governance costs. When principal-agent incentives and monitoring costs are considered the main sources of governance cost, the question of who employs whom becomes important. The cost of party a monitoring party b could very well be different from the cost of party b monitoring party a. Perhaps one party has less "shirking room" than the other, or perhaps one party has better monitoring skill than the other. To maximize social surplus, it is efficient to choose the owner of the firm who minimizes the overall transaction cost. As a result, given the boundary of the firm, the choice of the owner affects the governance cost of the firm, while the governance cost of the firm affects the integration decision. Under the TCE framework, then, the choices of ownership of the firm and the boundary of the firm are simultaneous (Hansmann, 1988). Intuitively, the choice of firm boundary is linked with the choice of firm ownership because (1) they are both determined by minimizing transaction costs and (2) the choice made along one dimension will affect transaction costs, which in turn will affect the choice along the other dimension.

One would expect that recent theoretical developments in the problem of the boundaries of the firm would aid the analysis of firm ownership choices. In fact, however, studies analyzing the ownership of the firm simultaneously with the boundaries of the firm have barely advanced beyond Hansmann (1988). I would argue that one of the major obstacles is that the development of TCE and its formal models does not offer an analytical structure in which one can conveniently compare internal governance costs à la Hansmann (1988) with the contracting costs of using the market.

The property-rights approach (PRA, also widely known as the Grossman-Hart-Moore framework) formally specifies the boundary problem of the firm (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995). According to Gibbons (2005), writers in this tradition "provide a unified account of the costs and benefits of integration (i.e., run the Coasean horserace on a level playing field)." The PRA generates a "level playing field" by replacing governance with a unified incentive scheme driven by the ownership of alienable assets. Notice that the ownership of alienable assets is different from the ownership of the firm, although the two are often related. What makes the PRA a convenient analytical framework is that it makes the ownership of the alienable assets determine *ex post* bargaining return, which in turn determines the *ex ante* investment incentive of each party. Therefore the inefficiencies from market contracting and firm coordination can both be modeled under the same framework with different ownership structures of alienable assets.

At the same time, however, the PRA approach creates a problem that hinders the unification of the ownership decision and the boundary decision. Because the PRA bypasses direct modeling of governance cost within the firm, it leaves governance costs outside of the analytical framework. And governance costs are the connection between firm boundaries and firm ownership. As a result, although the PRA provides an attractive multi-party analytical framework of the problem of the firm's boundaries (Hart and Moore, 1990), the model does not describe the other dimension, the internal control structure, under the multi-party case.

The attractiveness of the analytical framework of PRA has led researchers to apply it in a variety of ways to a variety of fields. Examples include Antràs (2003) in trade, and Caballero and Hammour (1998) in macro theory. These applications typical cope with the lack of firm ownership structure by avoiding any cases in which the ownership of the firm and the ownership of the alienable assets conflict. Such disagreement is easy to avoid under a two-party case. The ownership of the firm is clear when one party vertically integrates the other, and the ownership of the firm is trivial if two parties are outsourcing each other. Thus a typical assumption is to identify the ownership of the alienable asset with the ownership of firm control. For example, for two parties upstream u and downstream d, there is typically one alienable asset required for their joint production. If u owns the asset, it is interpreted as outsourcing, while if d owns the asset, it is considered as d integrating u. Even so, under PRA, if the researcher wants to discuss the model using terms like "employ" or "owner of the firm," she is restricted by the fact that firm ownership has to transfer together with alienable-asset ownership, which could be restrictive. Consider two restrictions that might appear in a typical application of the PRA.

The first is that the PRA cannot be applied directly to firms owned by human capital, no matter whether under a two-party case or a multi-party case. Because human capital is inalienable, when the provider of the physical capital is not the owner of the firm, the organizational interpretation in PRA – bundled ownership of physical asset and ownership of the firm – does not apply.[1] As a result, the tools of PRA do not obviously apply to examples

[1] For example Dow and Putterman (2000) provides a nice survey of literatures that try to explain why capital hires labor.

such as accounting firms and law firms.[2]

The second restriction, in my opinion, is even more severe than the first one. The method used in the two-party case to define integration versus outsourcing cannot be easily extended to the multi-party case, even if we restrict firm ownership to mean ownership of the alienable asset. The problem is that the identity of the owner of the firm is not clearly defined when the model considers more than two parties. Suppose we try to extend the typical two-party setting to a three-party firm, parties a, b and c, that allows for all possible organization structures, from totally integrated to totally non-integrated. Merely assuming the number of alienable assets to be less than three rules out some possible organization structures. Once we bring in three or more alienable assets for the three parties, the owner is not easily identifiable without further assumptions.

First of all, even assuming only one alienable asset will not allow for cases such as a integrates b while outsourcing c. When there are two alienable assets, then at least one party has to be integrated in all possible cases, which rules out the case of total non-integration.

Suppose, instead, we assume that there are three indivisible alienable assets; then we will have problem identifying the owner of the firm when one party is integrated. Without loss of generality, say a has no physical asset, b has two assets, and c has one asset. Then a is integrated, and b and c are contractors for each other. The question is, who does a work for, b or c? One might argue that b has more capital therefore b integrates a. However, one could equally argue the opposite interpretation by raising a counter example. If a is a waiter, b is a bank and c is a caterer, suppose c takes out a loan from b to finance the business. Does that mean it makes more sense for the bank b to hire waiter a to work for caterer c in order for him to pay back the loan to b? From this example, we see that without making further restrictive assumptions, it is generally difficult to apply the PRA to a firm related with multiple parties and keep the identity of the owner clear at the same time, even though the theoretical framework of PRA provides solid potential to extend to multiple parties (Hart

[2]Grossman and Hart talk about insurance agencies, but these have an alienable "client list".

and Moore, 1990).

Although it is difficult, there is great value in explicitly combining the multi-party property rights theory with the ownership of the firm. First of all, the earlier work of Hansmann (1988) shows the power of combining the two aspects of the firm. He analyzed the underlying reasons for different firm ownership structures, from coops to corporate firms. Studying the interaction of firm boundary and firm ownership in a more detailed framework could possibly provide new insights to research in both topics. Second, once the ownership of the firm is "integrated" into the PRA framework, the identity of firm ownership can be clearly identified and therefore separated from the ownership of alienable assets. This would extend the PRA to cases such as labor-owned firms. Last but not least, the clear identification of firm ownership under the PRA removes one major, if not the only, difficulty that prevents its application to multi-party firms.

In the remaining part of this section, I explain conceptually how to combine the identity of the owner of the firm with the PRA framework under a multi-party background.

I accommodate the multi-party framework following the structure of Hansmann (1988). Following the usual terminology, I consider the firm as "a nexus of contracts". And all the parties enter the nexus for joint production using their associated alienable and inalienable assets.

To simplify the analysis, I only consider the case in which only one party is the owner of the firm. Possible extensions are discussed further in the summary. As in the basic Hansmann structure, there is one party located in the center of a nexus, who is considered the owner of the nexus. The owner reaches out to each party with one contract to connect all the parties together. Such a network of different parties with contracts as connections forms the nexus as a firm, including its internal structure and related outside parties. The type of contract in the nexus is either an employment contract or an outsourcing contract, which identifies the boundary of the firm or the vertical-integration structure of the firm. To specify the types of contracts and their associated transaction cost, I follow the typical settings of the PRA. I determine the type of a contract by the alienable asset ownership. As for the ownership of the firm, as is mentioned, the party that takes the position of the center of the nexus is identified as the owner of the firm. Therefore, under a PRA-style bargaining structure for each contract in the nexus, the owner of the firm participates in each and every contract and bargains with every party. It is the owner's responsibility, or right, to bargain as representative of the nexus in addition to making specific investments for the joint production like other parties. To accommodate the bargaining structure of the PRA, the ownership of the firm is reflected by the arrangement that the firm owner represents the firm and bargains with each party. Therefore, suppose the bargaining fails between a waiter and his employer, the caterer; not only is the waiter fired from the caterer, he is also fired from the bank loan provided to the caterer. This insight is drawn from Alchian and Demsetz (1972): the key benefit of being the owner of a firm is that one could fire a party in the nexus without firing other related parties. By this construction, we model the role of the ownership of the firm under the PRA framework without introducing a governance structure, which leaves the analytical advantages of the Grossman-Hart-Moore framework intact.

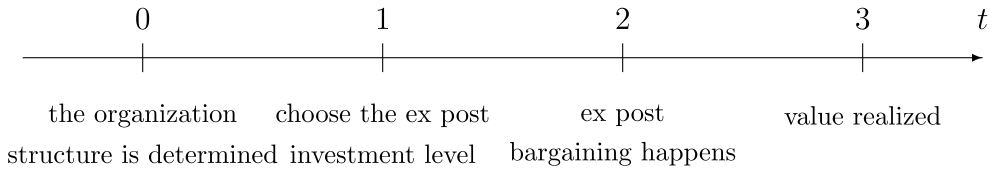
The rest of this paper proceeds as follows. Section II formalizes the structure of the model. Section III discusses the main results directly related to the ownership of the firm to highlight its characteristics. Section IV discusses properties of the model that provide insights about changes in firm ownership structure and capital ownership structure. Section V provides numerical examples to highlight some of the properties of the framework. And section VI summarizes the paper and discusses further possible applications of the model.

2 Model

The bargaining framework here results from a generalized Nash bargaining model, and it is consistent with the property rights literature, for example Schmitz (2006).

I follow the terminology of Hansmann (1988) and refer to the network that involves all the agents in the transaction relation as the nexus, and refer to different parties involved in the nexus as parties of the "firm". In this model, I only focus on the structure of one and only one firm. I consider the firm as a nexus of contracts, either employment contract or outsourcing market contract. I treat the parties linked with an employment contracts as the firm of interest, and I interpret other parties related with the firm through outsourcing contracts as individual external firms that are no longer separable into multiple sub-parties. More complicated extensions could be made based on the basic model by allowing multiple levels of nexus. Again, I define the owner of the nexus following the insights of Alchian and Demsetz (1972) that the key benefit of being the owner of a "firm" is that one could fire a party in the nexus without firing other related parties. More specifically, I assume that there is only one party who is the owner of the nexus, who is also interpreted as the owner of the firm in the model. The owner of the nexus bargains with each and every party in the nexus on behalf of other parties and splits the produced value. Notice that the owner of the nexus is not associated with the ownership of any physical asset, which allows us to treat the ownership of the firm and the ownership of the physical asset separately. Henceforth, I refer to the party who is the owner of the nexus as the owner, and any party who is not the owner of the nexus as the non-owner.

Following Hart and Moore (1990), we make the following assumptions. First, assume there is a set of parties $A = 1, 2, \ldots, N$ and a set of physical assets $K = k^1, k^2, \ldots, k^M$. We can think of k^i as the jigsaw for a woodworker, or the truck for a truck driver. Notice here, we assume agent i could make a contribution in production as long as this set of physical assets k^i is involved in production, irrespective of whether k^i belongs to i or not. The woodworker could produce as long as this set of tools is available to him, irrespective of whether he owns the tools or the factory provides them. Moreover, I define the control structure of physical asset as a mapping $\alpha : A \to K$. And I define the notation $k_i = \alpha(i) \in K$ for $i \in A$ as the physical assets that belongs to party i. Further assume that each party $i \in A$ is associated with an effort investment level n_i, which is noncontractible ex ante, observable by other parties, but not verifiable by any outside parties. Also, we assume party i can choose the level of n_i but not other features of the effort.



The timing of the model goes as followings: at $t = 0$, the organization structure is determined, which includes the owner of the nexus and the mapping of physical assets α such that $\alpha(i) \subseteq K, \forall i \in A$. At $t = 1$, agents make their choices of non-contractible, observable, but non-verifiable investment level $n_1, ..., n_N$ to maximize their future returns from the joint production. At $t = 2$ the bargaining happens and the shares of final value parties get are determined. At $t = 3$ the joint production is finished, and the value is realized and distributed by the bargaining results. I will analyze the game backwards from $t = 3$.

Define the value of joint production of N parties, $1, 2, \ldots, N$, as $F(K, \mathbf{n})$, where $K = k^1 \cup \ldots \cup k^N$ and $\mathbf{n}$ is a vector of associated effort investment levels $[n_1 n_2 \ldots n_N]'$.

Following Hart and Moore (1990), we assume the following properties of $F(K, \mathbf{n})$.

Assumption 1. *F is nondecreasing in $\boldsymbol{n}$. $F(K, \boldsymbol{n}) \geq F(K, \boldsymbol{n}')$ for $\boldsymbol{n}' < \boldsymbol{n}$.*

Assumption 2. *$F(K, \boldsymbol{n})$ is continuous in $\boldsymbol{n}$, twice differentiable w.r.t. $\boldsymbol{n}$, and strictly concave.*

Assumption 3. *F increasing in K. $F(K, \boldsymbol{n}) \geq F(K', \boldsymbol{n})$ for $K' \subseteq K$.*

Assumption 4. *(Alienable Asset and Inalienable Asset as Complements) $\partial F(K, \boldsymbol{n})/\partial n_i \geq \partial F(K', \boldsymbol{n})/\partial n_i$ for $K' \subseteq K$, for any i.*

At $t = 2$, we apply the results from the generalized Nash bargaining to split the value from joint production $F(K, \mathbf{n})$. We assume that each party has a threat point, which is to take the physical asset that belongs to him, k_i, and the effort he invested in $t = 1$, n_i, and walk away from the nexus to realize an alternative value somewhere else. Formally, any agent i could choose to opt out of the cooperation and realize value $g_i(k_i, n_i)$. Should this case happen, the rest of the nexus, the parties $1, \ldots, i-1, i+1, \ldots, N$ are left with the physical asset under their control $K_{-i} = k_1 \cup \ldots \cup k_{i-1} \cup k_{i+1} \cup \ldots \cup k_N$, and their invested efforts $\mathbf{n}_{-i}$. And they are able to realize alternative values which sum up to $G_{-i}(K_{-i}, \mathbf{n}_{-i})$ without i. This could happen by substituting i's position by an alternative, or simply produce without i, or even break up the entire nexus and pursue alternative values $g_k(k_k, n_k)$. Therefore, in the bargaining, any non-owner party i has the threat point $g(k_i, n_i)$, and the owner has the threat point $G_{-i}(K_{-i}, \mathbf{n}_{-i})$, to split the total value $F(K, \mathbf{n})$. Here we impose the following assumption.

Assumption 5. *(Existence of the Firm) The nexus generates nonnegative gains from trade, i.e.*

$$F(K, \boldsymbol{n}) \geq \sum_{i=1}^{N} g_i(k_i, n_i).$$

Therefore, we can formalize the earning for any non-owner party i from the generalized Nash bargaining as

$$B_i^n(\alpha, \mathbf{n}) = \rho_i[F(K, \mathbf{n}) - G_{-i}(K_{-i}, \mathbf{n}_{-i})] + (1 - \rho_i)g(k_i, n_i), \tag{1}$$

where we denote $k_i = \alpha(i) \in K$ as the subset of alienable assets controlled by party i, and $K_{-i} = \cup_{j \neq i}\alpha(j) = K \backslash k_i$ for all $i \in A$ as the subset of alienable assets controlled by parties other than i. Both k_i and K_{-i} are determined by the choice of mapping α. ρ_i is the bargaining power of party i in a generalized Nash bargaining problem. The superscript o denotes that the earning is for a non-owner, to differentiate from the return for i if she is the owner. Assume that the ex post investment level n incurs a cost $c(n)$, which has the following properties.

Assumption 6. *$c(n)$ is increasing in n.*

Assumption 7. *$c(n)$ is continuous in n, twice differentiable w.r.t. n, and strictly convex.*

Then we can describe any party i's net return from the nexus as

$$V_i^n(\alpha, \mathbf{n}) = B_i(\alpha, \mathbf{n}) - c(n_i). \tag{2}$$

The owner of the nexus j will bargain with each party $i \in A, i \neq j$, simultaneously. Therefore the payoff left for party j as the owner is given by the residual of all bargaining payments to non-owners:

$$B_j^o(\alpha, \mathbf{n}) = F(K, \mathbf{n}) - \sum_{\substack{i=1 \\ i \neq j}}^{N} B_i^n(\alpha, \mathbf{n}). \tag{3}$$

where the superscript o denotes the return for j as the owner of the nexus. And j's net return from the nexus is

$$V_j^o(\alpha, \mathbf{n}) = B_j^o(\alpha, \mathbf{n}) - c(n_j). \tag{4}$$

Therefore, for any party $i \in A$, the net return is a function of firm ownership, physical asset ownership, and effort investment levels

$$V_i(O, \alpha, \mathbf{n}) = \begin{cases} V_i^n(\alpha, \mathbf{n}) & \text{if } O \neq i \\ V_i^o(\alpha, \mathbf{n}) & \text{if } O = i \end{cases} \tag{5}$$

where O indicates the identity of the owner of the nexus, which is a singleton subset of A.

Notice that the net returns for the owner and all non-owner parties sum up to the social surplus

$$\begin{aligned} VS(O, \alpha, \mathbf{n}) &\stackrel{def}{=} F(K, \mathbf{n}) - \sum_{i=1}^{N} c(n_i) \\ &= \sum_{i=1}^{N} V_i(O, \alpha, \mathbf{n}). \end{aligned} \tag{6}$$

At $t = 1$, each party i looks forward to the bargaining returns given by (5), and chooses the non-contractible investment level n_i simultaneously to maximize his return, given the owner of the nexus O and the physical asset control structure α. We denote the equilibrium effort investment strategies by $\text{n}^*(O, \alpha)$. And the social surplus value for given possible combination of owner O and asset control structure is given by

$$VS^*(O, \alpha) = VS(O, \alpha, \mathbf{n}^*(O, \alpha)). \tag{7}$$

At $t = 0$, the most efficient organization structure is determined by choosing the optimal owner of the firm, $O \in A$, and the optimal boundary of the firm, mapping α, to maximize (7).[3] If, for non-owner party i, the capital ownership structure, mapping α, places some alienable asset k^i under i's control, i.e. $\alpha(i) \neq \emptyset$, then we say party i is an independent contractor, or we say that party i is non-integrated by the owner of the firm. If, however, for non-owner party i, the capital ownership structure, mapping α, arranges no alienable asset under i's control, i.e. $\alpha(i) = \emptyset$, then we say party i is an employee, or we say that party i is integrated by the owner of the firm.

3 General Results

3.1 The Theory of the Owner of the Firm

Consider the case in which a party k threatens to leave the nexus at $t = 2$. Should that really happen, the smallest return that any party i could get is bounded below by his alternative value g_i.[4] This is true because, if after k leaves, the nexus still generates positive gains from cooperation $G_{-k} - \sum_{\substack{i=1 \\ i \neq k}}^{N} g_i > 0$,[5] then i will stay to realize the value and split

[3]For N owners and N pieces of indivisible physical asset, the maximization problem is equivalent to finding the maximum element in a N by N^N matrix, where each element is associated with a combination of owner O and asset control structure α.

[4]Henceforth, I denote $g(k_i, n_i)$ by g_i

[5]Henceforth, I denote $G_{-k}(K_{-k}, \mathbf{n}_{-k})$ by G_{-k}

it with other parties except for k. However, if the inequality does not hold, i can always leave the nexus and realize his alternative value g_i. In the second case, we will actually have $G_{-k} - \sum_{\substack{i=1 \\ i \neq k}}^{N} g_i = 0$. Therefore, we come to the following insight.

Proposition 1. *The owner of the nexus has a generic threat point when she is bargaining with any party k. This is provided by the sum of alternative values for all parties other than k. Formally, we can write*

$$G_{-k} \geq \sum_{\substack{i=1 \\ i \neq k}}^{N} g_i.$$

Def 1. (Essential Party for the Nexus) We define the remaining gains for the nexus with respect to k as

$$RG_k = G_{-k} - \sum_{i \neq k} g_i.$$

And we say that a party k is *essential* to the nexus if the remaining gains with respect to k are zero.

Notice by proposition 1, the remaining gains for the nexus are always nonnegative. The definition says that, once the essential party k left, the rest of the nexus generates no extra return from the cooperation. This implies that party i has a critical role in the process of realizing the value of joint production. Notice that there could be multiple parties being essential. Intuitively, at least two aspects jointly determine the relative importance of a party in the joint production. They are the technology and substitutability. The essentialness in our model reflects similar mechanisms.

Let's consider the idea of a party's "essentialness" more carefully. We start by making a distinction between two factors in technology under the background of our model: technological vitalness and technological importance. Technological vitalness is associated with what would happen to the production of the rest of the nexus when one party leaves. It is a measure of the size of the "destruction" this particular party will bring to the rest of the nexus by leaving the nexus. When we say one party is technologically vital for a nexus, we mean that the rest of the nexus has no extra return from cooperation. For example, imagine a small restaurant with only one chef; this chef is technologically vital, because the restaurant will have no output when she leaves, and the best return for the waiter and the owner of the restaurant is to realize their individual outside options. However, technological importance is associated with the situation in which this party is producing in the nexus. It is a measure of her contribution when she increases her "intermediate output", which essentially refers to her marginal product. These two aspects are often closely related, but not necessarily in a perfect way. To see this point, consider the chef again. The value of the joint production certainly does not perfectly rely on the chef's input. It may not even be the most important input among all parties, such as in a fancy restaurant with excellent service that charges a high price for what is really ordinary food. In other words, one party could have low marginal product but be vital for the joint production. On the other hand, however, a party could also be non-vital but important. Take the example of the waiter, the caterer and the bank. The funds from the bank could have a very large marginal product by improving the equipment for the caterer and therefore the level of the services and, eventually, the value of the product. However, the bank may not necessarily be vital. Perhaps the caterer can still work with the waiter without the loan, albeit under poor conditions. To related the above discussion to the essentialness of a party, one should notice that it is actually the vitalness of the party, not her importance in the production, that affects her essentialness. Nonetheless, one might want to allocate resources, including firm and asset ownership, toward the important parties, because their investment incentives are more important in the production.

In this model, two aspects determines the essentialness of a party: technological vitalness and substitutability. One party has to be both technological vital for the nexus and non-substitutable outside the nexus.

Here we consider two extremes, one where a party is perfectly essential, and one where the party is perfectly non-essential.

For the first case, suppose i is essential. Then after i leaves, the rest of the nexus could not produce without i (vital), nor can they find a replacement for i (nonsubstitutable). Should this happen, the entire nexus breaks up, and the joint production is meaningless, i.e. the weak inequality in proposition 1 becomes equality. Then the alternative value that the rest of the nexus could realize becomes $G_{-i} = \sum_{j \neq i} g_j$, which equals the sum of outside alternative options for all parties except for i. As a consequence, plugging $G_{-i} = \sum_{j \neq i} g_j$ in equation (1), one will get $B_i^n = \rho_i[F - \sum_{j \neq i} g_j] + (1 - \rho_i)g_i = g_i + \rho_i[F - \sum_j g_j]$. This equation says that if one party is essential for the nexus, she will be able to extract her outside alternative return plus the 'fair' share of return to cooperation, where the share is determined by her bargaining power.

For the second case, however, suppose party i is perfectly non-essential. In other words, if i leave the nexus, the rest of the nexus could either (1) produce on their own without filling i's position and still realize value that equals the residual after paying for i (perfectly non-vital), or (2) produce by filling i's position by hiring an alternative i^a at a cost which equals the payment to i and still realize the same total value F (perfectly substitutable). In this case, formally we have $G_{-i} = F - B_i^n$, then after substitution into equation (1), we have i's return reduced to $B_i^n = \rho_i[F - (F - B_i^n)] + (1 - \rho_i)g_i$, which implies $B_i^n = g_i$. The interpretation is that if party i is perfectly non-essential, then she is only able to get the outside alternative value from the firm without any share of the return to joint production. It is interesting to notice, however, that a party could be perfectly non-essential even when she is very important technologically in the production process. As long as her position could be costlessly replaced by an outsider, she loses her leverage to bargain for the excess return to the cooperation. We will revisit this discussion in section 5.

We can consider an extreme example of a nexus in which everyone in the joint production is essential to the nexus. Suppose there are three parties involved in joint production of laptop computers. Suppose a produces the LCD monitor, b produces the software, and c produces the computer body and input devices. Without substitutes, any coalition of two of them will not gain extra value in addition to the sum of the retail prices of the separate parts. Instead, a nexus of three could produce a integrated ready-to-use computer which could gain a higher value than otherwise sold separately. In this simplified case, everyone in the nexus is essential.

Proposition 1 then leads to the following proposition which identifies the role of ownership of the nexus.

Proposition 2. *Given fixed ex post investment level* $\boldsymbol{n}$ *and asset control structure* α*, for any party* j *in the firm, the earning as a owner,* $B_j^o(\alpha, \boldsymbol{n})$*, is bounded below by the earning as a nonowner,* $B_j^n(\alpha, \boldsymbol{n})$*. And the return to the ownership of the nexus is given by*

$$B_j^o - B_j^n = \sum_i \rho_i[G_{-i}(\alpha, \boldsymbol{n}_{-i}) - \sum_{k \neq i} g_k(\alpha, n_k)] = \sum_i \rho_i RG_i(\alpha, \boldsymbol{n}_{-i}). \tag{8}$$

The proposition states that the return to the ownership of the nexus comes from extracting returns from bargaining with non-essential parties in the nexus.[6] Suppose a party i is non-essential because of a competitive environment, which increases the substitutability of party i's position, then the intuition here is in line with the classical result that competition extracts surplus and reallocate it toward other parts of the economy. From the proposition, we can draw the following results.

Corollary 1. *(Source of Return to the Firm Ownership) The return to the ownership of the nexus is zero if and only if all parties in the nexus are essential.*

To see this, set the right hand side of equation (8) equal to zero. Corollary 1 implies that in a nexus where every party is essential, the allocation of ownership of the nexus is irrelevant. This does not mean, however, that there should not be an owner in the nexus. The choice of owner in this particular case is irrelevant, because it offers no incentive to

[6] We try to avoid using the term bargaining power in discussion due to its ambiguity. One could refer the bargaining power to strictly the Nash bargaining power, or refer it in a more general sense to the combination of Nash bargaining power and outside threat points. Here, in a general sense, the owner of the firm could use a third non-owner as bargaining leverage to increase her bargaining power when she is bargaining with any non-essential party.

any party at all; thus choosing any party in the nexus to be the owner will have equivalent consequences. In this case, only the firm boundary matters for an optimal organization choice. The other way to interpret the result is that as long as there exists a non-essential party in the nexus, either employed or outsourced, there is a return to the owner of the nexus.[7]

Corollary 2. *(Anonymous Return to the Firm Ownership) The return to the ownership is invariant with respect to the identity of the owner given fixed ex post investment level* $\boldsymbol{n}$ *and asset control structure* α*.*

This conclusion comes from noticing that the identity of the owner j and her characteristics does not explicitly enter the return function for firm ownership. In other words, the right hand side of equation (8) is not a function of j. This corollary means the return to the ownership of the nexus is solely determined by the intrinsic properties of the nexus. More specifically, the return is determined by how essential all the parties are in the nexus.

4 Other Results

In order to further analyze the properties of the bargaining framework, we impose the following assumption for alternative production functions G_{-i} and g_i for any $i \in A$.

Assumption 8. *The alternative values* $G_{-i}(\alpha, \boldsymbol{n}_{-i})$ *and* $g_i(k_i, n_i)$ *are production functions that satisfy assumptions 1 through 4.*

We can show the following proposition that is in line with the results from the Grossman-Hart-Moore framework.

Proposition 3. *(General Existence of Transaction Costs) For any owner allocation* $O \in A$ *and asset control structure* α*, the equilibrium choices of ex post investment levels are*

[7]The case we raise here in the corollary is certainly an extreme. However, we consider this case more as a benchmark that highlights the role of firm ownership in the model and demonstrates the source of its extra return.

suboptimal. i.e.

$$\boldsymbol{n}^* < \boldsymbol{n}^s \stackrel{def}{=} \arg\max_{\boldsymbol{n}\in\mathbb{R}^n} VS(\boldsymbol{n}).$$

In addition to the previous assumptions, we impose the following.

Assumption 9. *The parties are complementary in subsets of the nexus, more specifically, $\partial G_{-i}/\partial n_j \geq \partial g_j/\partial n_j$ for parties $i \neq j$.*

This assumption requires that each party have a higher marginal product when they are producing with other parties. By proposition 1, we have $G_{-i} > g_j$ for $j \neq i$, and considering G as a joint production function of certain form, the complimentary assumption is not a very unrealistic one.

In the next proposition, we use a slight variation of notation for $v_i(O, k_i, K\backslash k_i) \stackrel{def}{=} v_i(O, \alpha(i), \cup_{j\neq i}\alpha(j))$. In $v_i(O, k_i, K\backslash k_i)$, the first argument denotes the owner of the firm, the second argument denotes the capital at i's control, while the third argument denotes the capital that is not under i's control.

Proposition 4. *(An Isolated Tradeoff Effect in Asset Allocation) Suppose there is a piece of physical asset that is not owned previously by any party in the nexus. If now, given the owner of the firm unchanged, we assign this asset to any party in the nexus, it will increase the equilibrium ex ante investment levels of all parties, and the equilibrium realized final value. Formally, suppose there is a set of physical asset $k \notin K$, then for any given owner allocation $O \in A$ and asset allocations k_i and $K\backslash k_i$ associated with control structure α, we have $v_i^*(O, k_i, K\backslash k_i) \leq v_i^*(O, k_i \cup k, K\backslash k_i)$, for the party who gets the asset k, and $v_i^*(O, k_i, K\backslash k_i) \leq v_i^*(O, k_i, K \cup k\backslash k_i)$ for parties who do not get the asset. Moreover, in equilibrium $F^*(O, K) \leq F^*(O, K \cup k)$.*

Proposition 4 does not have a direct interpretation. But since the tradeoffs in the asset allocation leave too much ambiguity for analysis, we use this proposition to isolate the force that causes the tradeoffs when we try to reallocate one piece of alienable asset to another party in the nexus. One can think of the reallocation of an alienable asset as two separated steps. First, we take the asset away from the nexus entirely. According to the proposition, this leads to decrease in the *ex ante* investment incentives for all the parties in the nexus, not just the one who loses the asset, and a corresponding drop in the final value of the product. Second, we take the asset and give it back to another party, which will boost up the *ex ante* investment incentives for all the parties and also the final value. Therefore, given the ownership of the nexus unchanged, a reallocation of one piece of alienable asset will possibly increase or decrease any party's *ex ante* investment incentive, and the result depends on which force is stronger.

It is interesting to discuss a special case in which there is only one party, i, who is nonessential in the nexus.

Proposition 5. *(A Party Who is Never the Optimal Owner) Suppose in a nexus there is only one nonessential party, i, then with additional assumption 9, it is never optimal to allocate the ownership of nexus to the only non-essential party i.*

Proof. When there is only one non-essential party, by Proposition 2, the return to the ownership of the nexus is given by $G_{-i}(\alpha, \mathbf{n}_{-i}) - \sum_{k \neq i} g_k(\alpha, n_k)$, which is not a function of n_i. Thus the first order conditions for i with respect to n_i will be the same no matter whether he is the owner or now. This implies that assigning the ownership to i will not change i's incentive to invest in n_i.

By assumption 9, the return to the ownership of the nexus has a positive partial derivative with respect to any n_j for $j \neq i$. Therefore, reallocating the nexus ownership to any j from i will increase party j's marginal return by $\sum_{k \neq j} \frac{\partial G_{-k}}{\partial n_j} - \frac{\partial g_j}{\partial n_j}$, thus increase, albeit not strictly, the equilibrium n_j. By contrast, n_k for any $k \neq j$ will be nondecreasing due to the complimentarity assumption of production function F. Therefore, the social surplus $SV^*(O = j, \alpha) \geq SV^*(O = i, \alpha)$ for any $j \neq i$. □

5 Examples

In this section, we consider two calculated examples with specified functions. And I will demonstrate the changes in the optimal organizations, including both optimal firm ownership and firm boundary allocations, as the technology changes. In the following examples, we change the technological importance of one party by increasing the power of her input in the joint production function. At the same time, we keep essentialness unchanged. Surprisingly, in one case it is never optimal to assign the ownership of the firm to one party no matter how technologically important she is in production. And in another case, we will see that even when a party is non-essential, in other words, the party is easily substitutable by outside parties, or not technologically vital, it is still optimal to reallocate the firm ownership to her from an essential party when she is significantly important in production technology.

Consider a nexus with only three parties. In addition, assume the physical capital to be homogeneous. Each piece of capital is mapped to the real line by a counting measure.

In order to correspond to what is shown in proposition 5, in this example, we make i the only non-essential party.

One can think of a case in which there is no substitute for any subset of the nexus elsewhere. Then the best alternative for any party p, for $p = i, j, k$, is a discounted self-production after leaving the nexus, i.e. $g_p(k_p, n_p) = F(k_p, n_p)/\lambda$. We parametrize $\lambda = 10$ so that proposition 1 is satisfied. And for the rest of the nexus, after party i left, there is no substitute for i's position, i.e. $G_{-i}(K \backslash k_i, \mathbf{n}_{-i}) = max\{F(K \backslash k_i, \mathbf{n}_{-i}), \sum_{\substack{j=1 \\ j \neq i}}^{N} g_j\}$. In this case, assumption 9 is satisfied. We use production function $F(k_i \cup k_j \cup k_k, n_i, n_j, n_k) = [(k_i+1)(n_i+1)]_i^{\beta}[(k_j+1)(n_j+1)]_j^{\beta}[(k_k+1)(n_k+1)]_k^{\beta}$. And we assume when i left the nexus, $n_j = n_k = 0, k_j = k_k = \emptyset$ for i, and $n_i = 0, k_i = \emptyset$ for j and k. Moreover, we set j and k to be essential by setting $G_j = g_i + g_k$ and $G_j = g_i + g_k$.

To simplify the analysis, we set $\rho_k = \rho_j$, thus j and k are symmetric. We further simplify the optimal organization problem to a joint allocation of ownership of the nexus O, and one single piece of alienable asset. Moreover, we consider k as always integrated by the owner of the nexus. We are left with four alternatives.

	$k_i = 1, k_j = 0, k_k = 0$	$k_i = 0, k_j = 1, k_k = 0$
$O = i$	i integrating j and k	i outsourcing j and integrating k
$O = j$	j outsourcing i and integrating k	j integrating i and k

We can carry out a simple experiment by changing the value of β_i while keeping $\beta_j = \beta_k = (1 - \beta_i)/2$. And we can solve for the equilibrium investment levels n_i, n_j, n_k for each β_i, under the four alternative organization choices, to find the corresponding social surplus level SV.

Figure 5 shows the result of the experiment. We can see from the upper-left graph that as i becomes more "technologically" important in the joint production, it becomes optimal to allocate the physical asset to i. However, it is never optimal to allocate the ownership of the nexus to party i no matter how technologically important she is. When one considers the firm as a nexus of contracts, as in Hansmann (1988), this example highlights a case in which it is optimal to separate the ownership of a firm from the ownership of physical asset, which is not possible within a classical PRA model.

5.1 Separation of ownership of the nexus and ownership of the physical asset

In reality, the case that is shown in the previous example could be rare. However, we are still able to show that even when there is more than one non-essential party in the nexus, it still might be optimal to separate the ownership of the nexus from the ownership of the physical asset. We conduct a different experiment based on the previous one by making party k symmetric to i. Then there are two non-essential parties in the nexus and only party j is essential. We consider the same four alternative choices of organization structures and we again consider k as being integrated by the owner of the nexus.

Figure 5.1 shows the result of the experiment.[8] The interpretation of the result includes several aspects.

First, when β_i is around value 0.33, there is a small window in which allocating the ownership to j while assigning the asset to i is optimal among all four alternatives.[9] This small window shows the optimality of separating the firm ownership from the asset ownership in a less extreme case than is demonstrated in the previous example.

Second, we can consider the essential party j as a non-substitutable and technologically vital party for the nexus. Once j leaves, by our setup, i and k only get to realize their individual outside alternative values. And we can consider i, equivalently for k in this example, as a substitutable or technologically non-vital party. When i is not technologically important, it is optimal for the essential party j to integrate i because j is both essential and technologically important. However, as i becomes more important in production, it becomes more efficient to make i an independent contractor, while j still owns the firm, employs k and outsources i. When i becomes even more important, the optimal organization is to turn the firm ownership over to i, and make i outsource both j and k, even though j is non-substitutable and technologically vital. One possible example of this is the joint production of engineers, managers and ancillary parties. The engineers, especially once specialized, are very hard to substitute for and vital in production. However, there is a thick market for managers. But, as the example shows, the substitutability of the manager does not necessarily exclude her from being the owner of the firm, if her input is significantly important in the production. This observation may provide a possible explanation for manager owned firms hiring expert engineers.

[8] Some readers might have noticed the upper bound of β_i in this example only goes to 0.5. This is because we set i and k being symmetric, which implies $\beta_k = \beta_i$, and thus the upper bound of β_i only goes up to 0.5 to guarantee a concave production function.

[9] I have conducted multiple different experiments with different specification of functional forms and parameters, and this result remains robust.

6 Summary

In this paper, I develop a model that enriches the property rights approach by adding the ownership structure of the firm under a multi-party case. The ownership of the firm in this model is constructed to be independent of the ownership of alienable assets, something not allowed in the standard property rights theory. The model also makes it possible to study the interactions between the ownership of the firm and the boundary of the firm. We hope this development will allow for applications of the property rights theory to a greater variety of organizations and a broader set of topics.

There are several restrictions to the model in this paper. First of all, the firm ownership defined in the model is trivial when the firm only involves two parties, so it does not provide further control structure for the property rights approach under a two-party case. In the setup, the only fact that differentiate the owner of the firm from other parties is that she gains extra bargaining power by using other parties as leverage. In other words, the source of the extra bargaining power comes from the extra loss for the non-owner when he is also fired from all the other parties, in addition to the firm owner, when the bargaining fails. Such extra leverage is lost, as in proposition 2 and corollary 1, when she is bargaining against an essential party, because such threat of "loss" from being fired from all the other parties is not a credible one. The definition of the firm also becomes trivial when there are only two parties in the firm and no third party exists who could possibly provide the extra bargaining power for the owner.

Second, in this basic model, I only allow for one party to be the owner of the firm. More accurately, I only allow for one homogeneous group of agents to be the owner of the firm. We could imagine extending the model to a multiple owner case in which the firm is owned by multiple groups of heterogeneous agents. Each party would control a subset of the nexus and bargain with each other owner and also with parties in her own subset. We can consider such subsets as different groups in a large firm, or as separate firms in a long production chain, or a combination of both. The meanings are flexible and thus requires more specific background to be appropriately defined. In this case, the optimal choice problem over the boundaries and owners becomes an optimal choice of a much more complex organization. And such models may provide more insights about business groups, or unions, or even a more general vertical market organization structure.

Third, in the tradeoff between the generality of the model and insightfulness of the results, I picked the former one. Thus I sacrificed the ability to provide a more intuitive and more specific explanations for choosing the firm ownership and the firm boundary in exchange for a more general setup where the characteristics of different parties are not specified. I expect to learn more about the tradeoffs of the firm ownership changes among parties under a more detailed model. Possible applications include topics such as human-capital versus physical-capital owned firms, and how ownership and firm boundary change while the market for some party develops, or how the firm boundary varies once the firm ownership changes in an exogenous manner.

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Appendix

Proof for Proposition 2

Proof. Party j's return as a non-owner is

$$\begin{aligned} B_j^n &= \rho_j[F - G_{-j}] + (1-\rho_j)g_j \\ &= \rho_j[F - \sum_{i\neq j} g_i - RG_j] + (1-\rho_j)g_j \\ &= \rho_j[F - \sum_{i} g_i - RG_j] + g_j \end{aligned}$$

Therefore party j's return as the owner is

$$\begin{aligned} B_j^o &= F - \sum_{i\neq j} B_i^n \\ &= F - \sum_{i\neq j} \rho_i[F - \sum_k g_k - RG_i] - \sum_{i\neq j} g_i \\ &= F - \sum_i g_i - \sum_{i\neq j} \rho_i[F - \sum_k g_k] + \sum_{i\neq j} \rho_i RG_i + g_j \\ &= (1 - \sum_{i\neq j} \rho_i)[F - \sum_i g_i] + \sum_{i\neq j} \rho_i RG_i + g_j \\ &= \rho_j[F - \sum_i g_i] + \sum_{i\neq j} \rho_i RG_i + g_j \end{aligned}$$

Therefore

$$\begin{aligned} B_j^o - B_j^n &= \sum_i \rho_i RG_i \\ &= \sum_i \rho_i[G_{-i} - \sum_{k\neq i} g_k] \geq 0 \end{aligned}$$

And the last inequality holds by proposition 1. $\square$

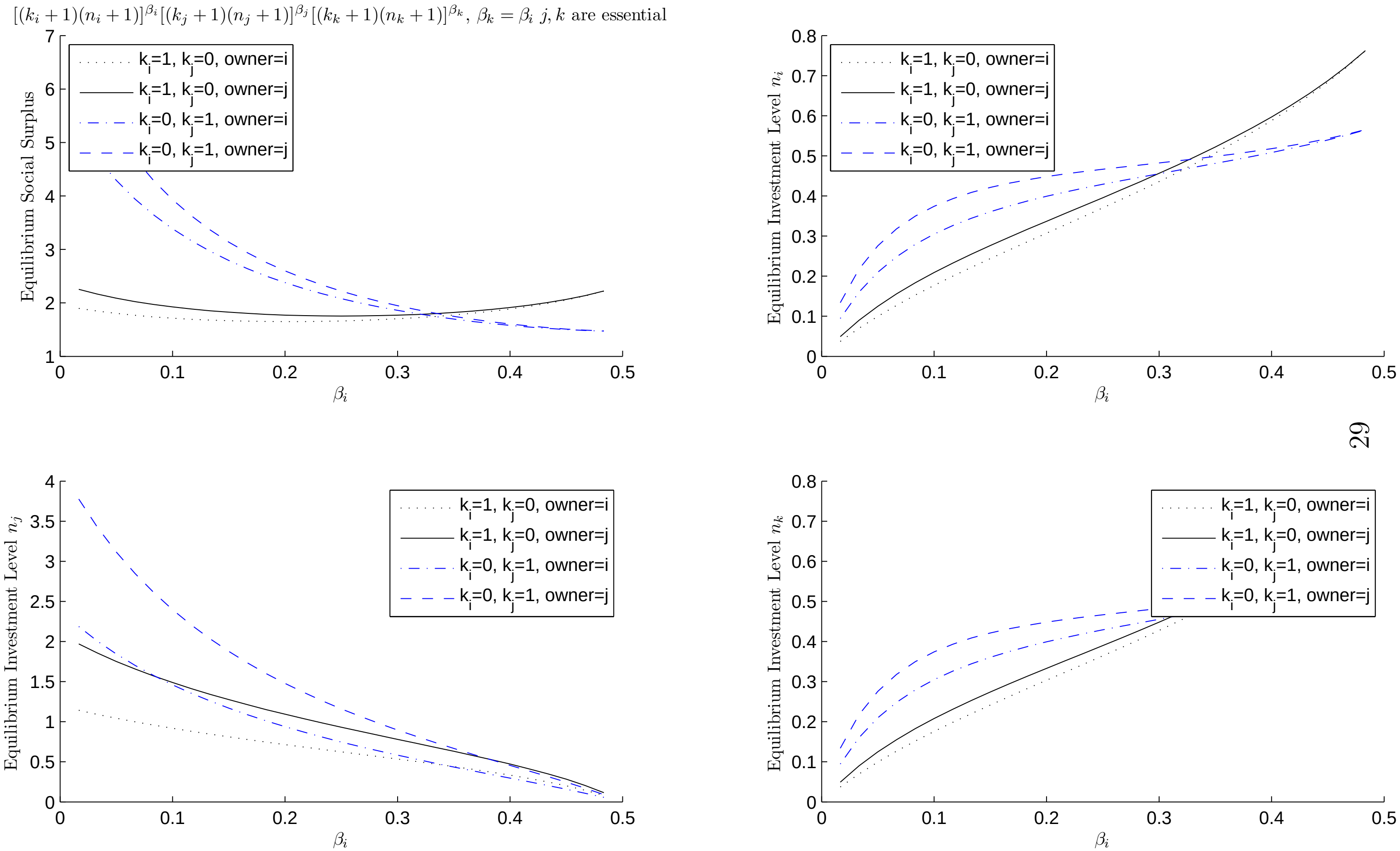

Figure 1: Only i is non-essential

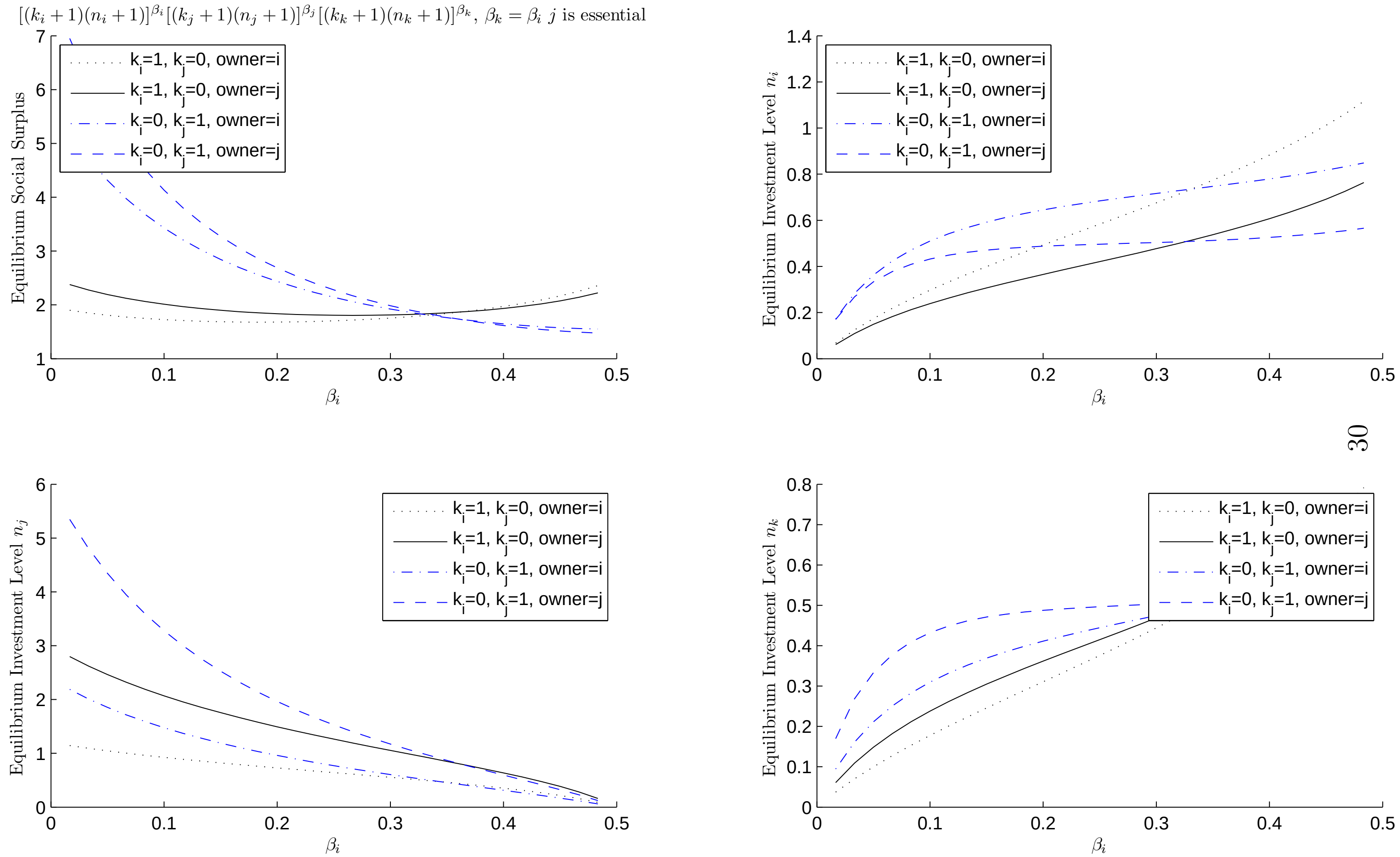

Figure 2: Only j is essential