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Abstract

This paper empirically examines how the Fed responds to stock prices and inflation movements, using the forward-looking Taylor rule augmented with the stock price gap. The typical linear policy reaction function has a substantial change after 1991, but lacks the robustness in that the estimation result is sensitive to a minor change of the sample period. To alleviate the problem, we allow for temporary and permanent variations of the reaction coefficients by introducing nonlinearity and a structural break. The time variation of the inflation coefficient shows that the Fed is more aggressive in periods of inflationary pressure. However, unlike the linear model case, we find little evidence of a significant change in the Fed's active response to inflationary pressure after the structural break at around 1990. We also find a positive response to the stock price change after the break. However, its time-variation does not lead the stock price bubble, which is counter to the view that the Fed has preemptively reacted to a stock price bubble.

Key words: Monetary policy rule, nonlinear model, stock market, structural break, and time varying coefficient.

JEL classification: E31, E44, E52

1. Introduction

Modeling how monetary policy changes in response to economic circumstances has long been an interesting topic in macroeconomics for various reasons. The policy reaction function: helps to forecast changes in the federal funds target rate, plays an important role in evaluating the Fed's monetary policy, and provides a better understanding of the various macroeconomic issues as an important element in macroeconomic models. Consequently, a robust estimation of the Fed's reaction function is of central importance in empirical macroeconomic analysis.

This paper has two objectives. The first is to examine the Fed's response to stock prices, as well as expected inflation, using the forward-looking interest rate rule with an addition of the stock price variable. The role of stock prices in the setting of monetary policy has been an important issue, both in policy making and academic research. The current research focuses on two questions; 1) Should the Fed respond to the stock price change? and 2) Has the Fed responded to the stock price change? This paper does not attempt to answer the first question. Instead, we focus on the second question, that is, to examine whether and how the federal funds target rate has moved in accordance with the movement of stock prices. Existing works suggest non-unified answers to the question. **?** and **?** find evidences of monetary policy response to stock price movements, while **?**, and **?** show that the policy response to a stock price change is not evident. **?** and **?** also have contradictory findings in their studies of the relationship in the European Central Bank.

Most of the existing works assume that the Fed's reaction process is stable and linear. However, there may be a structural break in the policy response to inflation, even in the last two decades, as shown in **?**, and **?**. **?** find that the explanatory power of stable models of US inflation has diminished in recent decades. Moreover, the estimation results of existing models highly depend on the choice of the sample period, which is another sign of possible instability in the reaction function.

This leads us to our second objective in this paper: to construct a more robust model of monetary policy response, by explicitly considering the time variation of the Fed's response and the structural break. **?** consider a structural break in the reaction function at 1991:II in that, since then, inflation has never exceeded 4%. However, their estimation provides a counterintuitive result that the Fed's response to inflation is not clear after the break. Moreover, we find that the estimated inflation coefficient in the linear forward-looking rule shows a substantial change to a minor shift in the breakpoint, which implies that the rigorous choice of the breakpoint and a more robust model is required.

To tackle this problem, we introduce a nonlinearity based on the 'Series method', along with a single structural break, which allows us to carry over the original GMM representation. Applying the series method allows for the temporary time variation, and the smooth change between two regimes as well as the asymmetric responses between inflationary and deflationary pressures, which is able to capture the nonlinearity due to asymmetric preference (See **?**, **?**, and **?** for details.) Consequently, together with the structural break which captures a permanent change, our method covers a wide range of possible time variations of the response function both in transitory and permanent manner. Our nonlinear model captures different nonlinear dynamics from the Treshold model or Smooth Transition Model of **?**, and **?** in that they focus on identifying recurrent regimes. We test whether both the nonlinearity and structural breaks are present in the reaction function. The test result strongly supports the nonlinearity in the Fed's response to inflation pressure. The structural break test detects the existence of a break at 1991:I, even in the nonlinear model.

The time varying dynamics of the expected inflation coefficient supports the asymmetric response of the Fed: The coefficient shows the Fed's aggressive monetary policy under high inflationary pressure and moderate response when the inflation is low. However, unlike the linear model case such as in **?**, we find little evidence of a significant change in the Fed's behavior after the structural break. The estimated coefficient still supports active responses to future inflation increases, even after the break, while it stays low in the other period of stable inflation. The result also coincides with the finding of **?**, that the inflation model is better explained by time varying coefficients since 1990s.

The stock price coefficient remains positive for most of the time after the break. However, the positive response does not necessarily imply that the Fed has preemptively responded to possible stock market bubbles. Note that it is generally expected that the Fed has two responses to movements in stock price: one is the standard policy that responds to stock price when it conveys information about future path of inflation and output. The other is so called the bubble policy that preemptively reacts to reduce the possible stock price bubble, which is more contentious. If the Fed actively reacts the possible bubble, it is expected that the Fed's response should be more active in the early periods of bubbles. However, the time varying pattern of the stock price coefficient reveals that it remains low until the late periods of bubbles, and hikes at the end of them.

The remainder of this paper is organized as follows. Section 2 derives and specifies the forward-looking interest rate rule including a stock price gap variable. Section 3 discusses linear estimation results and the instability issue. Section 4 constructs a nonlinear model based on the Series method and discusses its estimation results and the movement of time-varying coefficients. Section 5 explores the test of structural change for the nonlinear framework, and Section 6 concludes.

2. The Fed's policy reaction function: a forward-looking rule with a stock price gap

This section outlines the theoretical framework of the forward-looking monetary policy reaction function. Conventional interest rate rules explain the policy interest rate as a function of current inflation (**?**) or past inflation, in addition to output gap. **?** provide a theoretical basis for a forward-looking policy rule and modify the reaction function, based on expected inflation and also account for fine tuning of policy by introducing the speed of adjustment to the change in economic conditions. **?**, **?**, and **?** add a stock price gap to the forward-looking Taylor rule, which is used in this paper. A simple forward looking rule by **?**, with a stock price gap is expressed as

$$R_t^* = R^* + \alpha \left[E(\pi_{t+k}|\Omega_t) - \pi^* \right] + \beta E(y_{t+q}|\Omega_t) + \gamma s_t \tag{1}$$

where R_t^* is the target rate for the nominal interest rate (e.g. Federal Funds Rate), R^* is the desired nominal rate when both inflation and output are at their target levels, π_{t+k} is the k-step ahead inflation, Ω_t is the information set available at time t when the interest rate is set, π^* is the inflation target, y_{t+q} is the q-step ahead output gap, and s_t is the stock price gap at time t. We assume that, unlike the inflation and the output gaps, the Fed responds to the current stock price gap, in that the Fed is more likely to respond to stock price after it watches the current movement, rather than responding on the basis of its expected future value. This can be also understood in that, while monetary policy can affect output and inflation with a certain degree of lag, e.g. 4 quarters, it can affect the stock market simultaneously, as **?** indicate. In order to better understand the intuition of Equation (1), we introduce the following two identities that represent short and long-run Fisher equations, respectively.

$$r_t^* \equiv R_t^* - E(\pi_{t+k}|\Omega_t) \quad \text{and}$$
$$r^* \equiv R^* - \pi^* \tag{2}$$

where r^* is the long-run equilibrium real interest rate, which is assumed constant and independent of monetary policy. After substituting Equations (2) into (1), we obtain the ex ante implied real interest rate rule.

$$r_t^* = r^* + (\alpha - 1)\left[E(\pi_{t+k}|\Omega_t) - \pi^*\right] + \beta E(y_{t+q}|\Omega_t) + \gamma s_t. \tag{3}$$

The basic intuition of Equation (3) is that interest rate rules characterized by $\alpha > 1$ imply active or aggressive responses to the expected inflationary pressure, while those with $\alpha \leq 1$ are likely to accommodate inflation shocks. A similar logic can be applied to the output gap and stock price gap coefficients, i.e., active or aggressive response to the output gap (stock price gap) if $\beta(\gamma)$ is positive, with accommodative responses, otherwise. Since the policy rule represented by Equation (1) ignores the Fed's tendency to smooth changes in interest rates, we consider more realistic adjustment of interest rate to the target rate R_t^* as follows,

$$R_t = \rho R_{t-1} + (1 - \rho) R_t^* \tag{4}$$

where R_t is the actual interest rate (i.e., actual Federal Funds Rate). Equation (4) implies that, in each period, the Fed adjusts its interest rate with a fraction $(1 - \rho)$ of its current target level. The gap between current interest rate and current target level can be represented by some linear combination of the one-period lagged realized actual interest rate and its current target level. Finally, after substituting Equation

(1) into (4), and using the long-run Fisher equation, we obtain the testable forward-looking interest rate rule:

$$R_t = \rho R_{t-1} + (1-\rho)\left[\theta + \alpha\pi_{t+k} + \beta y_{t+q} + \gamma s_t\right] + e_t \tag{5}$$

where $\theta = r^* - (\alpha - 1)\pi^*$, and $e_t = (1-\rho)\left[\alpha(E(\pi_{t+k}|\Omega_t) - \pi_t) + \beta(E(y_{t+q}) - y_{t+q})\right]$. Note that e_t is a linear combination of forecasting errors and, thus, orthogonal to any variables in the information set Ω_t. Let Z_t denote a vector of instruments known when R_t is set (i.e., $Z_t \in \Omega_t$), then we have the orthogonal condition to estimate parameters $(\rho, \theta, \alpha, \beta, \gamma)$ using GMM as follows,

$$E(e_t Z_t) = E\{\left[R_t - \rho R_{t-1} - (1-\rho)(\theta + \alpha\pi_{t+k} + \beta y_{t+q} + \gamma s_t)\right] Z_t\} = 0. \tag{6}$$

3. **Linear estimation results and the break-point issue**

This section estimates the linear policy reaction function of Equation (5), and examines its validity. We use quarterly data spanning from 1979:I to 2011:I, but we concentrate on data before 2008:II to eliminate instability in the reaction function caused by the recent Great Recession, in which the federal fund rate hits the lower bound. Table 7 briefly describes the data. The annualized growth rate of the GDP deflator is used to measure the inflation. The output gap is measured by the percentage deviation of GDP from its potential level. We use two alternative measures of potential GDP: the CBO's estimate of the potential GDP and the Hodrick-Prescott(HP) filtered trend GDP. The forecast horizon parameters k and q are set at 4 and 1, respectively, which follows **?** that these horizons are roughly in line with the conventional wisdom regarding the lag with which monetary policy affects either variable. We also consider $q = 2$, and find no nontrivial change in the overall pattern of the estimated coefficients.

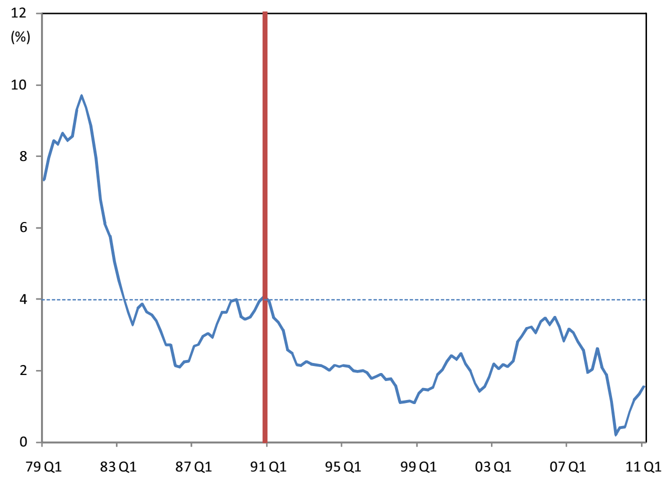


FIGURE 1. **US Inflation Movement**

We use the price-earning ratio (PER) to measure the stock price gap following **?** and **?**. The stock market is considered to be appropriately valued if the PER equals the inverse of some appropriately risk adjusted return. A higher than average PER is generally regarded as an overvaluation (positive gap) and vice versa. The PER is used in the form of a two-year growth rate because it seems more plausible that the Fed would respond to lower-frequency changes in a stock price index or a PER than to higher-frequency changes. In the estimation, we consider both the current value and a one quarter lag of the PER. As **?** point out, the current stock price variable depends on the current monetary policy, which causes a simultaneous bias problem. Using the lagged PER avoids the endogeneity problem. In addition, since the instruments set contains the lagged PER, the Fed's response to the stock price as an instrument for forming the forecasts of π_t and y_t are reflected in their coefficients, and it is more evident that the estimated γ significantly from zero indicates the direct response to the stock price. When the current PER is used, we follow **?** and use the current to two lags of GDP growth rates as instruments for the PER.

In the GMM estimation, we use the conventional set of instruments (**?**, and **?**), which includes lags of the macroeconomic variables that would possibly comprise the information set Ω_t, as listed in Table 7. This set of large macroeconomic variables often faces a criticism that it has a risk of weak instruments, and there might be a discrepancy between the set and the Fed's actual information set. For this reason, we also use the set of the Fed's Green book forecasts of Inflation and growth as instruments following **?**. But we don't find significant differences in the given sample periods, and do not present the result in this paper. Another reason to present only the conventional instruments result is because the available data set of the Green book forecasts does not entirely cover the high inflation period around 2006, which is crucial to examine the post-break behavior of the Fed.

The starting year 1979 is chosen to focus on the post-Volcker period, as in **?**. In late 1979, Paul Volcker began his term as Fed Chairman. After a period of turmoil and damage to the real economy associated with high inflation, he sent a strong and explicit signal to the market that the Fed would curb inflation as a first priority. This fundamental shift in monetary policy leads us to adopt 1979 as a natural starting point of the analysis.

We also split the sample into high inflation and low inflation periods, following **?**. They choose 1991:II as the breakpoint because, as Figure 1 shows, inflation has not exceeded 4% since then. Later in this section, we review the validity of the breakpoint and also check if the estimation is robust to the choice of the breakpoint.

Due to the overlapping observations structure, the error term e_t is expected to be serially correlated. Therefore, the heteroscedasticity and serial correlation consistent (HAC) estimator is required in estimating GMM. We use the Bartlett kernel with Newey-West's fixed bandwidth selection.

Table 1. **Estimation result of the linear interest rate rule**

Periods		CBO's GDP gap			H-P filtered GDP gap		
		α	ρ	γ	α	ρ	γ
1979:IV - 2011:I	s_t	2.243**	0.890**	0.056**	2.237**	0.879**	0.075**
		(0.367)	(0.020)	(0.020)	(0.448)	(0.027)	(0.019)
	s_{t-1}	2.388**	0.867**	0.050**	2.595**	0.876**	0.054**
		(0.339)	(0.027)	(0.018)	(0.433)	(0.030)	(0.015)
1979:IV - 2008:II	s_t	3.090**	0.897**	0.068**	2.236**	0.884**	0.076**
		(0.503)	(0.019)	(0.025)	(0.493)	(0.025)	(0.022)
	s_{t-1}	2.895**	0.874**	0.068**	2.610**	0.874**	0.058
		(0.446)	(0.025)	(0.020)	(0.473)	(0.028)	(0.057)
1979:IV - 1991:I	s_t	2.257**	0.875**	-0.013	1.746**	0.764**	0.025
		(0.506)	(0.033)	(0.027)	(0.331)	(0.039)	(0.023)
	s_{t-1}	1.726*	0.754**	-0.025	1.641	0.746**	0.003
		(0.412)	(0.041)	(0.016)	(0.424)	(0.060)	(0.026)
1991:II - 2008:II	s_t	7.291	0.950**	0.165(*)	6.686	0.960**	0.183(*)
		(5.115)	(0.030)	(0.097)	(5.828)	(0.025)	(0.106)
	s_{t-1}	5.253	0.937**	0.114*	3.556	0.936**	0.119**
		(3.287)	(0.030)	(0.054)	(2.496)	(0.027)	(0.041)

Notes: a) Standard errors appear in parentheses.
b) (*), *, ** in α columns reject $H_0 : \alpha < 1$ at 10%, 5%, 1% levels, respectively.
c) (*), *, ** in the other columns indicate statistical significance at 10%, 5%, 1% levels, respectively.

The estimation results are shown in Table 1. The stock price coefficient highly depends on the sample period. It is positive and significant in the full sample and the low inflation sub-period, while it is negative and statistically close to zero in the high inflation period. In the low inflation period, the coefficient ranges from 0.014 to 0.183, depending on the choice of the output gap and the lag of the PER.

The inflation coefficient is significantly greater than one in all sample periods, which indicates that the Fed reacts aggressively to the expected inflation pressure. The result is robust to the change of output gap and the PER in the full sample and in high inflation sub-periods. However, it varies widely after 1991:II, and is highly dependent on the variable choices, but in most cases is statistically insignificant. **?** report similar findings and suggest that the lack of significance may mean that price-setters tend not to choose price time paths with period-on-period price increases if they think U.S. monetary policy aggressively combats inflation. However, this argument is insufficient because, although the coefficient is statistically insignificant, the magnitude itself is quite large in an economic sense. Considering the variation of the inflation coefficient and the magnitude of the stock price coefficient, we suspect that the model is unstable in that period and are motivated to construct a more robust model with the form of Equation (5) unchanged.

We also check the robustness of the estimation, by examining how the results vary with respect to a minor change of the break point, which is shifted back and forth by one quarter around the chosen breakpoint up to one year. Table 2 shows that small variations of the breakpoint leads to large deviations in the estimated coefficients in the low-inflation sub-period. There are numerous possible explanations of this variation. One is that the Fed needs time to smoothly adjust its new optimal level after the shock, which leads to gradual coefficient changes in periods following the shock. But this explanation is insufficient in that α varies more as the sample moves further from the break point. Another explanation is that there are too many moderate shocks in the economy, so that the coefficients change almost every period, which is generally regarded as a time varying coefficient process. A third possible explanation is that the policy reaction is better explained by a nonlinear relationship, rather than the linear one.

For these reasons, a nonlinear model with a power of capturing time-varying coefficients would be an alternative to overcome the problem. Recent works have been

Table 2. **Estimated α and γ with different break points**

	1990:II	1990:III	1990:IV	1991:I	1991:II	1991:III	1991:IV	1992:I	1992:II
α	2.91^{*}	3.24^{**}	3.24^{**}	$3.27^{(*)}$	7.29	25.54	26.83	546.0	n.a.
	(1.04)	(0.48)	(0.71)	(1.47)	(5.11)	(43.20)	(45.41)	(n.a.)	(n.a.)
γ	0.09^{**}	0.09^{**}	0.09^{**}	0.07^{**}	$0.16^{(*)}$	0.48	0.50	9.54	20.03
	(0.02)	(0.02)	(0.06)	(0.02)	(0.10)	(0.48)	(0.76)	(275.3)	(n.a.)

Notes: a) Standard errors appear in parentheses.
b) $^{(*)}$, *, ** in α row reject $H_0 : \alpha < 1$ at 10%, 5%, 1% levels, respectively.
c) $^{(*)}$, *, ** in γ row indicate statistical significant at 10%, 5%, and 1% levels, respectively.
d) n.a. indicates that the estimated speed of adjustment $(1-\rho)$ is zero at three decimal places
e) CBO's GDP gap and the current PER are used

devoted to incorporate nonlinearity into the Taylor rule (**??**). These attempts are mostly aimed at allowing for the asymmetry between different regimes such as inflationary and deflationary periods, and the Threshold Regression (TR) model or the Smooth Transition Regression (STR) model are widely used to capture the asymmetry. One practical problem of using TR in our model is that it is often difficult to capture both the structural break and the threshold in a single equation because, in practice, regime switching is often not distinctive to structural breaks in finite samples. Moreover, if there is a permanent shift in threshold variables, like the inflation shift-down in our case, it is reasonable to consider that the accompanying threshold parameter also shifts. In this circumstance, the attempt to capture the asymmetric response often fails because the shift makes it difficult to identify, or dominates the regimes.

This paper keeps the possible existence of a structural break. In addition, we propose the so called 'Series method' combined with the single structural break for several reasons. First, as shown in the next section, this process coincides with time varying coefficients in the linear relationship with a specific form of the time variation. Second, it can capture the smooth adjustment of coefficients to a shock by allowing for time variation of the linear coefficients. Third, it is easily associated with a structural break. Note that a structural break captures a permanent shift in the coefficient, while the nonlinearity in the Series method, as shown in the next section, explains the temporary fluctuations. Considering both nonlinearity and a structural break is expected to cover a wide range of unstable policy reaction functions. We also estimate the reaction function without the stock price gap; results are shown in Tables 6 and 9 in the appendix. The robustness problem since 1991:II still exists in this reaction function.

4. **Estimation of the nonlinear model**

This section extends the policy reaction function of Equation (5), by allowing for the nonlinearity which is captured by the Series method. This method allows polynomial terms in the inflation and stock price gap coefficients, and generates nonlinearity without harming the structure of the original GMM specification in (6). In the Series method, the inflation and stock variable coefficients can be approximated by a function of chosen explanatory variables, denoted by δ_t. The potential candidates include: the level value of inflation, the inflation deviation from its target level, the level value of the interest rate, and various combinations. Equation (7) illustrates a nonlinear specification expressed by the coefficient functions.

$$E[e_t Z_t] = E\left[\{R_t - \rho R_{t-1} - (1-\rho)(\theta + \alpha(\delta_t)\pi_{t+k} + \beta y_{t+k} + \gamma(\delta_t)s_t\} Z_t\right] = 0. \tag{7}$$

Each coefficient function is approximated by polynomial terms and can be written:

$$\begin{aligned} \alpha(\delta_t) &\approx \alpha_0 + \alpha_1\delta_t + \alpha_2\delta_t^2 + \ldots + \alpha_{k_1}\delta_t^{k_1} \\ \gamma(\delta_t) &\approx \gamma_0 + \gamma_1\delta_t + \gamma_2\delta_t^2 + \ldots + \gamma_{k_2}\delta_t^{k_2}. \end{aligned} \tag{8}$$

Note that the linear model is a special case, among a variety of possible nonlinear models, in which the power parameters of the polynomial terms are jointly zero, i.e., $k_1 = k_2 = 0$. We use the deviation of the federal funds rate, from its Hodrick-Prescott filtered trend, as the only element of δ_t because other candidate variables, such as the inflation and the output gap, statistically fail to generate the nonlinearity. We select the series term k by two criteria, Mallow's C_L and generalized cross-validation, which are usual in this kind of Series method, that is, to select k_1 and k_2 to minimize

$$\text{Mallow's } C_L = \hat{\sigma}^2 \left[1 + \frac{2(k_1 + k_2)}{T}\right]$$

$$\text{Generalized cross validation} = \frac{\hat{\sigma}^2}{\left(1 - \frac{k_1+k_2}{T}\right)^2}$$

where $\hat{\sigma}^2 = \frac{1}{T}\sum_{i=1}^{T} u_t^2 = \frac{SSR}{T}$. Table 10, in the appendix, shows the selection result using the two criteria, where $k_1 = 2$, $k_2 = 2$ case is chosen.

The full sample estimation result of the nonlinear model, using the CBO's GDP gap and the current PER, is shown in Table 3. The Wald test for nonlinearity strongly supports the nonlinear relationship in the policy reaction function. But, the nonlinearity mostly arises from the inflation coefficient; the test cannot reject the linearity in the stock price gap coefficient. Moreover, all three coefficients of stock price are highly statistically insignificant, which implies that the nonlinear description of the stock price coefficient is not appropriate in the full sample.

Table 3. **Estimation results of the nonlinear model (full sample)**

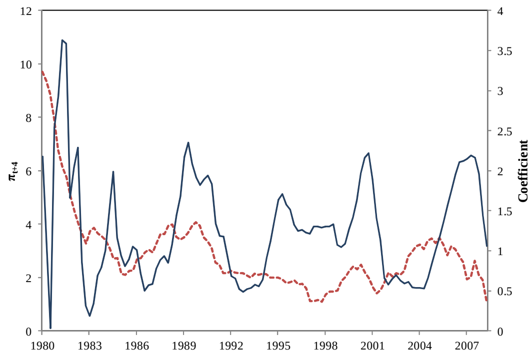
α_0	α_1	α_2	ρ	γ_0	γ_1	γ_2
1.184**	0.418**	0.014	0.806**	0.099**	0.017	-0.008
(0.307)	(0.057)	(0.026)	(0.027)	(0.024)	(0.018)	(0.010)

Note: a) H-P filtered cycle of the federal funds rate is used as δ_t
b) the current PER and CBO's GDP gap are used.
c) $^{(*)}$, *, ** indicates statistical significant at 10%, 5%, and 1% levels respectively

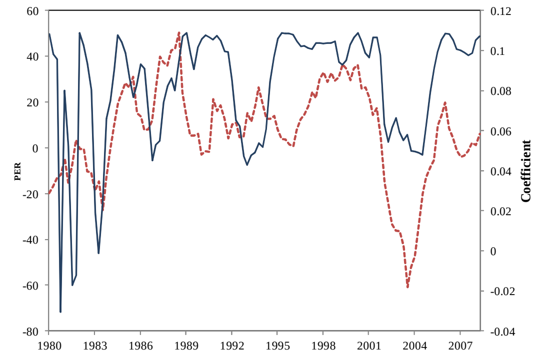
Wald test: H_0: $\alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = 0$ p-value = 0.000
H_0: $\alpha_1 = \alpha_2 = 0$ p-value = 0.000
H_0: $\gamma_1 = \gamma_2 = 0$ p-value = 0.496
J-statistic = 14.991 p-value = 0.776

The beauty of our nonlinear model is that we can obtain the time-varying path of the coefficients in (6) by mapping the values of the estimated quadratic function corresponding to the interest rate (δ_t) of each historical date. Figure 2 shows the time-varying movement of the inflation and stock price gap coefficients. The inflation coefficient shows a fluctuation around 1.2 and the movement is highly dependent on the future inflation process; that is, the Fed responds more actively to inflation (coefficient $\geq$ 1) when π_{t+4} is rising, while it is accommodative if π_{t+4} is low. For instance, the coefficient increased to more than two, facing the inflation hike at around 1980 and 1990, and these responses are followed by the downshift of inflation of 1982 and 1992. This pattern shows a feasible illustration of the Fed's behavior that the Fed concerns more about expected inflation pressure rather than deflationary one.

Figure 2(b) shows that the stock price gap coefficient is positive most of the time after mid-1980s, with a temporary fluctuation. However, the response pattern to the movement of PER is unclear. We find a comovement after 2000, but it shows a countercyclical pattern in 1990s. This asymmetry might indicate that there is a structural break in the response to the stock price movement. But we do not discuss



(a) inflation coefficient (dotted line = π_{t+4})



(b) stock price coefficient (dotted line = s_t)

FIGURE 2. **Time varying path of the coefficients**

TABLE 4. **Estimation results of the nonlinear model with the linear stock variable coefficient (full sample)**

α_0	α_1	α_2	ρ	γ
1.150**	0.408**	0.024	0.817**	0.073**
(0.309)	(0.057)	(0.025)	(0.024)	(0.010)

Note: a) H-P filtered cycle of the federal funds rate is used as δ_t
b) the current PER and CBO's GDP gap are used.
c) $^{(*)}$, *, ** indicates statistical significant at 10%, 5%, and 1% levels, respectively

Wald Test: H_0: $\alpha_1 = \alpha_2 = 0$ p-value = 0.000
J-statistic = 15.176 p-value = 0.854

more here because the graph is drawn based on the highly insignificant coefficients. We will discuss this issue in detail in the next section.

Considering that the nonlinearity in the stock price gap is weak by the Wald test, we also estimate the model with the stock price coefficient restricted to a linear effect, as below:

$$E[e_t Z_t] = E\left[\{R_t - \rho R_{t-1} - (1-\rho)(\theta + (\alpha(\delta_t))\pi_{t+k} + \beta y_{t+k} + \gamma s_t\} Z_t\right] = 0. \quad (9)$$

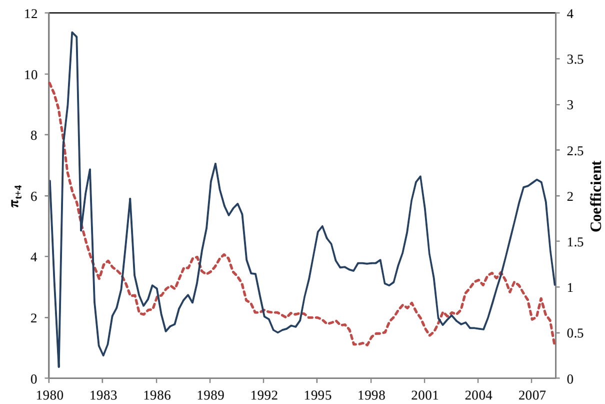


FIGURE 3. **Time varying path of the coefficients with linear stock variable coefficient (dotted line=π_{t+4})**

Table 4 and Figure 3 show the estimation result and the time varying path of the inflation coefficient, respectively. The inflation coefficient is similar to Table 3, and comoves with π_{t+4}. The response to the stock price coefficient is positive and statistically significant. The estimation result using the lagged PER and HP filtered GDP gap are similar and does not present in this paper.

5. Structural break in the nonlinear model

Although the nonlinear model in Section 4 captures the time varying process of coefficients in the liner model (5), the coefficient is a quadratic function of the stationary process δ_t, which does not cover permanent shifts in the coefficient such as structural breaks. Therefore, examining the existence of a structural break is still reasonable in the nonlinear model. This section performs the structural break test, and modifies the estimated model, based on the result of the test. This test evaluates if 1991:II, based on the change of the inflation process, is the right break point, if it exists. Note that the breakpoint in the inflation process does not always directly indicate the breakpoint in the model, because a model generally depends on the lead

Table 5. **Test results for structural breaks**

	without restriction				with linear PER coefficient			
	CBO		H-P filter		CBO		H-P filter	
	s_t	s_{t-1}	s_t	s_{t-1}	s_t	s_{t-1}	s_t	s_{t-1}
supF	1538	538	4132	1880	1089	226	3153	1242
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
break point	89:III	89:IV	90:II	90:II	89:III	90:II	90:II	90:III

or lagged effect in the model. Accordingly, we use tests in which the breakpoint is unknown, and estimate the break point. There are various powerful tests to detect a structural break when the breakpoint is unknown, such as **?**'s *supF*, **?**'s *expF*, and **?**'s *qll*. This paper uses the *supF* test only because the test provides the estimated breakpoint as a byproduct of the test. The *supF* test is defined as,

$$supF = \sup_{t_1 \leq \tau \leq T-t_2} F(\tau) \tag{10}$$

where $F(\tau)$ is the Wald test for a structural break at a given break point τ, T is the sample size, and t_1 and t_2 are the trimming parameters, which are conventionally set as either $0.02 \cdot T$ or $0.05 \cdot T$. If the source of the break is well identified, we do not have to search for a supremum over the entire sample period. Instead, by focusing on the periods where the source occurs, we can improve the power of the test. We make a minor modification of *supF* in this section, so that the supremum is taken six quarters back, and forth, from 1991:II, assuming that the lead or lagged effect of the inflation break on the reaction function would be within a year.

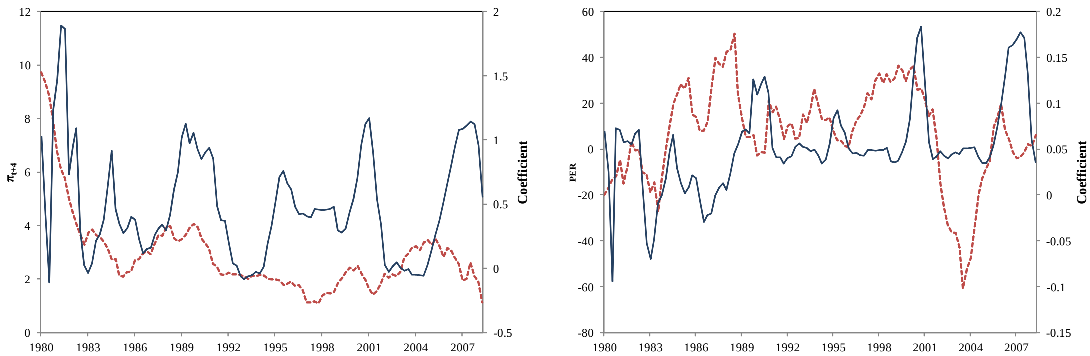
Restricting the *sup* range is also beneficial, in that the HAC estimator generally provides a poor covariance estimate for GMM when the breakpoint is close to the end of the sample period, resulting in poor test performance. This problem is serious when the instruments set is large, as in our model. This modification of *supF* is basically to choose the trimming parameter as $t_1 = 0.34 \cdot T$ and $t_2 = 0.45 \cdot T$. The critical values for these trimming parameters are not provided in **?**. Thus, we obtain the p-value by simulating the asymptotic distribution of the test function, which is $\sup_{0.34 \leq s \leq 0.45} B(s)'B(s)/(s(1-s))$ where $B(s)$ is the standard multivariate Brownian bridge. The nonlinear model with a structural break is now defined as,

$$R_t = \rho R_{t-1} + (1-\rho)\left(\theta + (\alpha_0 + \alpha_1 \delta_t + \alpha_2 \delta^2)\pi_{t+k} + \beta y_{t+k} \ + \right.$$
$$(\gamma_0 + \gamma_1 \delta_t + \gamma_2 \delta_t^2) s_t) - D_t \left[\rho^1 R_{t-1} - (1-\rho^1)\left(\theta + (\alpha_0 + \alpha_1^1 \delta_t + \alpha_2^1 \delta^2)\pi_{t+k} \ \right]\right.$$
$$+ \ \beta^1 y_{t+k} + (\gamma_0^1 + \gamma_1^1 \delta_t + \gamma_2^1 \delta_t^2) s_t] \tag{11}$$

where $D_t = 1_{[t>\tau]}$. The test results are shown in Table 5, where we find strong evidence of a structural break in all models. Even if the nonlinear model captures the time variation of the coefficients, we find that considering a structural break is still of high importance. The break points range from 1989:III to 1990:III, which is moderately different from **?**'s 1991:II. We estimate with different break point (1989:III-1990:III) and find no significant change in time-varying pattern of the coefficients although using CBO and s_{t-1} shows some instabilities. Accordingly, we split the sample into 1979:IV-1989:III and 1989:IV - 2008:II for CBO's GDP gap and 1979:IV-1990:I and 1990:II - 2008:II for HP-filtered GDP gap, and estimate the model separately. The coefficients of split samples are asymptotically equivalent to those in Equation (11).

The estimation result in the split samples, using CBO's GDP GAP and the current PER, is shown in Table 6, and Figure 4 presents the corresponding time varying movement of the coefficients. The Wald test strongly supports the nonlinearity in the inflation and stock price coefficients in both sub-sample periods.

Figure 4(a) shows the monetary policy response to the expected inflation. As in the full sample case in Figure 2(a), the inflation coefficient highly depends on the level of expected inflationary pressure. That is, high future inflation forces the Fed to be



(a) inflation coefficient(dotted line = π_{t+4}) (b) stock price coefficient(dotted line=PER)

FIGURE 4. **Time varying path of the coefficients(split samples)**

TABLE 6. **Estimation result of the nonlinear model (subsample)**

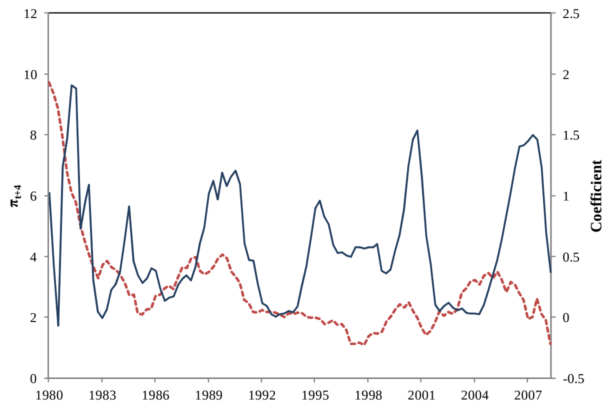
	α_0	α_1	α_2	ρ	γ_0	γ_1	γ_2
1979:IV - 1989:III	0.472** (0.045)	0.215** (0.013)	0.011 (0.003)	0.316** (0.023)	0.030** (0.010)	0.027** (0.002)	-0.004$^{(*)}$ (0.002)
	Wald Test: H_0: $\alpha_1 = \alpha_2 = 0$					p-value = 0.000	
	H_0: $\gamma_1 = \gamma_2 = 0$					p-value = 0.000	
	J-statistic = 9.264					p-value = 0.979	
	α_0	α_1	α_2	ρ	γ_0	γ_1	γ_2
1989:IV - 2008:II	0.376$^{(*)}$ (0.226)	0.291** (0.073)	0.021 (0.076)	0.752** (0.043)	0.041** (0.014)	0.022** (0.007)	0.017* (0.008)
	Wald Test: H_0: $\alpha_1 = \alpha_2 = 0$					p-value = 0.000	
	H_0: $\gamma_1 = \gamma_2 = 0$					p-value = 0.011	
	J-statistic = 13.234					p-value = 0.867	

notes: a) $^{(*)}$, *, ** indicates statistical significance at 10%, 5%, 1% levels, respectively.
b) the current PER and CBO's GDP gap are used.

more aggressive, while the Fed rarely responds when the inflation is expected to be stable. However, unlike the linear model case and **?**'s finding, we find no substantial change in the inflation coefficient after the break. The coefficient dynamics still reflects aggressive response to the inflationary pressure and more to accommodative in the stable inflation periods even after 1991. We conjecture that the reason why the post-break inflation coefficient is insignificant in linear model is because the inflation has been more stable since 1991, rather than the Fed's behavior has changed. We also estimate the model with moderate shifts in the breakpoint, which reveals no significant changes in the pattern.

Figure 4(b) shows a significant change in the stock price coefficient after the break: it fluctuates below zero in 1980s , it returns to positive after 1989. This graph seems to be an evidence of the Fed's response to stock market movements. To examine the response in more detail, we need to distinguish two different monetary policy responses to the stock price. one is the standard policy that responds to stock price when it conveys information about future path of inflation and output, and there is widespread agreement about the appropriateness of this type of response. The other is so called the bubble policy that preemptively reacts to reduce the possible stock price bubble, which is more contentious. If the positive reaction coefficient is due to the preemptive response to stock price bubbles, it is expected that the coefficient magnitude would be bigger in periods in which stock price growth begins increasing or stays high. However, the magnitude of the coefficient is relatively small when the PER growth continuously increases, such as in the period of the dotcom bubble in the late 90s, and it often hikes at the end of the bubble periods such as 2000:IV and 2007:II. This dynamic runs counter to the notion of the Fed's preemptive act against the asset price bubble. Consequently, the Fed's preemptive action against bubbles is not successful to explain the positive coefficient after 1991.

It is still possible that, as a general concern, the positive coefficient is due to the misspecification of (6) such as incorrect forecast horizons or information set(instruments). On the other hand, it can be true that the Fed reacts to bubbles, but its identification



(a) inflation coefficient

FIGURE 5. **Time varying path of the coefficients (subsample)**

of bubbles are not in timely manner because, as **?** points out, economists do not have a good measure of bubbles, and this identification problem results in the lagged response as Figure 4(b). The estimated policy reaction function itself does not provide a clear answer. Our contribution is restricted to the finding that the positive response to the stock price in conventional linear Taylor rule does not necessarily implies the Fed's active reaction against bubbles.

Table 11 and Figure 5 show the estimation results and corresponding time varying inflation coefficients. The inflation coefficient pattern is quite close to that in Figure 5. The stock price gap coefficient, which is assumed to be linear, is positive and statistically significant after the break.

6. Conclusion

The Fed's behavior raises many macroeconomic and econometric issues. After observing the non-robustness of the linear set-up around the breakpoint, we construct an alternative nonlinear model. Allowing for the time variations of reaction coefficients provides a more robust view of the Fed's behavior over the past thirty years. It is hard to find a change in the Fed's fundamental responds to inflationary pressure after 1990. The Fed appears to respond to the stock price change when the linear estimation is used. But the time varying pattern of the responses does not necessarily suggest that the response is due to preemptive acts against stock market bubbles. Examining the precise reason is beyond the scope of this paper and further research is required.

Our structural break test is restricted in that if focuses on the possibility of a break at around 1991 only. But other breaks at around 1987 and 1982 are still plausible, which would affect the estimation in the pre-break period. Thus, further investigation in periods between 1979 and 1991 are reasonable. We expect that the time variation could possibly capture those breaks, although it is not a perfect approach, in that temporary movement and structural break are observationally equivalent in a short time-period, so that we can avoid the cost of inaccurate HAC estimates in the short sample periods.

Appendix: Additional Tables and Figures

Table 7. Data Description

Variables	quarterly average basis[1)]
R_t	Federal Funds rates
π_t	Annualized growth rate of GDP deflator
y_t	Percentage deviation of real GDP from the CBO's PGDP or its Hodrick-Prescott (HP) trend y_t
γ_t	Two year growth rate of the S&P 500 price earning ratio (PER)
Instrumental variables (Z_t)	Four lags of π_t and y_t Four lags of quarterly growth rate in producer price index and M2 Four lags of the yield spread between long-term and short-term government bond[2)] Four lags of two year growth rate of the S&P 500 price earning ratio (PER)[3)] Current and two lags of GDP growth rate (when current PER is used)

Notes: a) All data except S&P 500 price earning ratio (PER) are obtained from web-based database (FRED) in Federal Reserve Bank of St. Louis.
b) We use yield spread between 10 year and 3 month U.S. Treasury bonds.
c) The data are obtained from Dr. Robert J. Shiller's homepage.

TABLE 8. Estimation result of the linear interest rate rule without stock price gap

Periods	CBO's GDP GAP		H-P filtered GDP GAP	
	α	ρ	α	ρ
1979:IV - 2011:I	2.374** (0.340)	0.888** (0.026)	2.955** (0.399)	0.874** (0.024)
1979:IV -2008:II	2.975(**) (0.599)	0.903** (0.021)	2.923** (0.445)	0.854** (0.038)
1979:IV - 1991:I	1.842* (0.457)	0.780** (0.026)	2.220** (0.494)	0.789** (0.038)
1991:II - 2008:II	1225.4 (n.a.)	1.017** (0.017)	4.343 (5.227)	0.967** (0.024)

Notes: a) Standard errors appear in parentheses.
b) (*), *, ** in α columns reject $H_0 : \alpha < 1$ at 10%, 5%, 1% levels, respectively.
c) (*), *, ** in ρ columns represent that the estimator is significance at 10%, 5%, 1% levels, respectively.

TABLE 9. **Estimated α with different break points (without stock price gap)**

	1990:II-	1990:III-	1990:IV-	1991:I-	1991:II-	1991:III-	1991:IV-	1992:I-
α	6.570** (2.079)	11.277** (3.289)	8.243 (7.964)	17.171 (16.082)	794.38 (n.a.)	883.65. (n.a.)	n.a. (n.a.)	n.a. (n.a.)

Notes: a) CBO's GDP gap are used
b) (*), *, ** reject $H_0 : \alpha < 1$ at 10%, 5%, 1% levels, respectively.

TABLE 10. **Selection of k_1 and k_2**

k_1	k_2	Mallow's C_L	Generalized cross-validation	Joint Wald Test (Null: linearity)	J-statistic(p-value)
2	2	0.8052	0.8195	0.0028	0.7363
2	3	0.8768	0.8959	0.0873	0.6589
2	4	0.9083	0.9323	0.0656	0.5832
3	2	1.0103	1.0324	0.0150	0.7641
3	3	0.9750	1.0007	0.0873	0.7562
3	4	0.9228	0.9518	0.1265	0.5582
4	2	0.6550	0.6724	0.0267	0.6012
4	3	0.6462	0.6665	0.0644	0.4675
4	4	1.2044	1.2486	0.6761	0.5896

Note: * implies final candidates. Also note that the quadratic function of α in $k_1 = 2$, $k_2 = 2$ case is not much different from forth-power function in $k_1 = 4$, $k_2 = 4$ cases in the quality sense, if not triple one

TABLE 11. **Estimation results of the nonlinear model with linear stock variable coefficient (subsample)**

	α_0	α_1	α_2	ρ	γ
1979:IV - 1989:III	0.471**	0.291**	0.014**	0.372**	0.002
	(0.056)	(0.014)	(0.023)	(0.031)	(0.011)
	Wald Test: H_0: $\alpha_1 = \alpha_2 = 0$			p-value = 0.000	
	J-statistic = 9.911			p-value = 0.987	
	α_0	α_1	α_2	ρ	γ
1989:IV - 2008:II	0.474*	0.350**	0.049	0.756**	0.052**
	(0.207)	(0.059)	(0.058)	(0.027)	(0.008)
	Wald Test: H_0: $\alpha_1 = \alpha_2 = 0$			p-value = 0.000	
	J-statistic =13.951			p-value = 0.903	

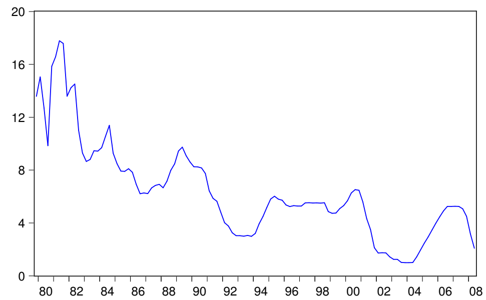
notes: a) $^{(*)}$, *, ** indicates statistical significance at 10%, 5%, 1% levels, respectively

b) the current PER and CBO's GDP gap are used.

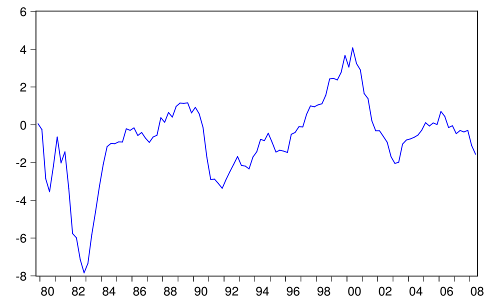
Table 12. **Estimated γ in the nonlinear model with linear γ coefficient**

	CBO PGDP & lagged PER		HP filter & current PER		HP filter & lagged PER	
	- 1990:III	1990:IV -	- 1990:III	1990:IV -	- 1990:III	1990:IV -
γ	-0.009	0.063**	-0.010	0.050**	-0.023**	0.056**
	(0.008)	(0.017)	(0.010)	(0.003)	(0.008)	(0.006)

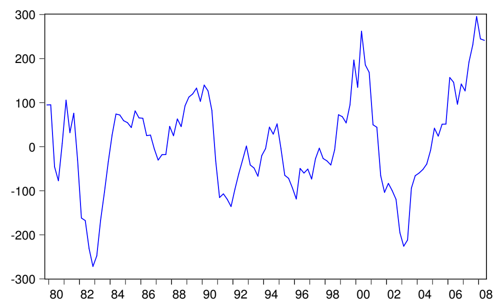
Notes: a) (*), *, ** indicate that the estimator is significance at 10%, 5%, 1% levels, respectively.



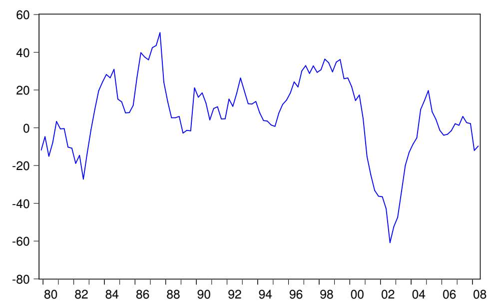
(a) Federal fund rate



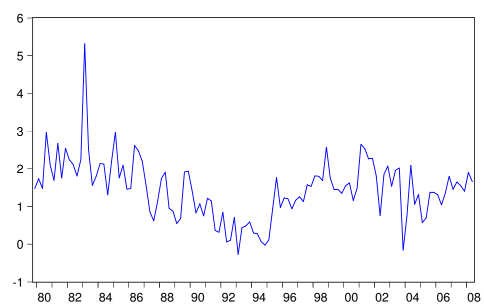
(b) GDP gap (usign CBO's PGDP)



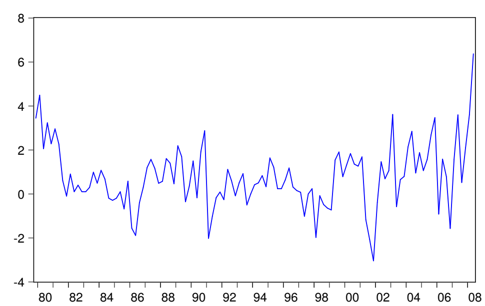
(c) GDP gap (usign HP filter)



(d) PER (two year growth rate)

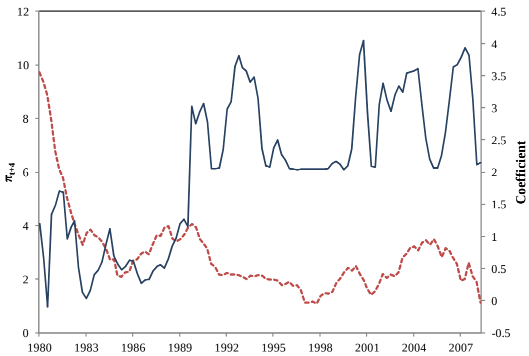


(e) M2 growth

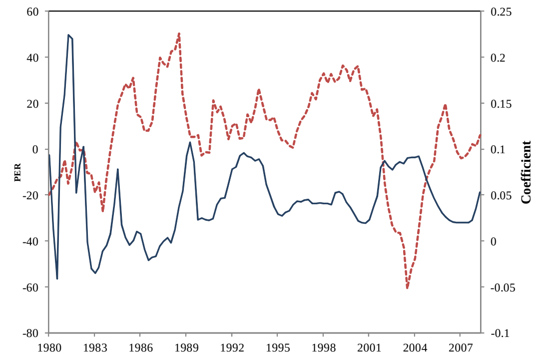


(f) PPI growth

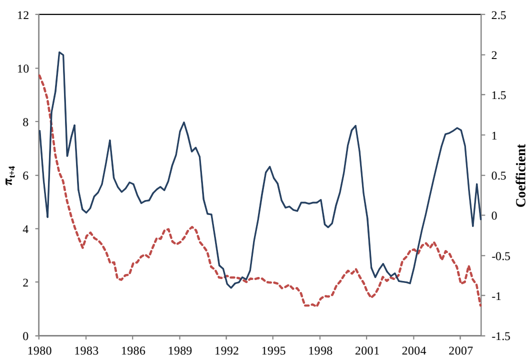
Figure 6. **Graphs of Each Variable**



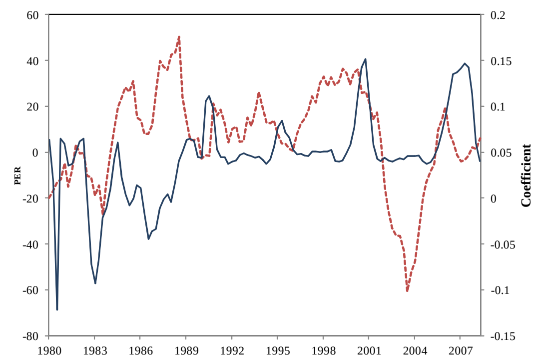
(a) CBO's PGDP & lagged PER



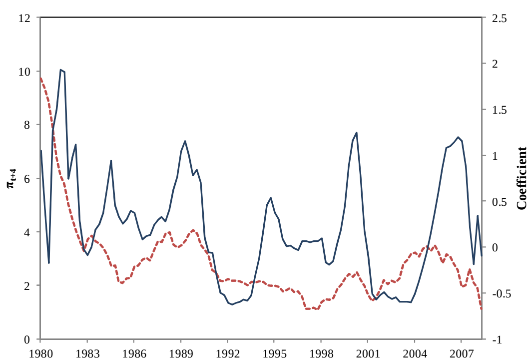
(b) CBO's PGDP & lagged PER



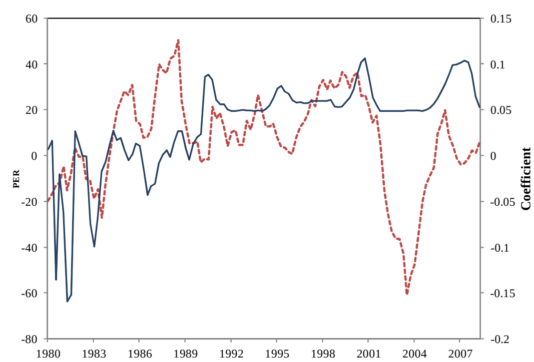
(c) HP filter & current PER



(d) HP filter & current PER



(e) HP filter & lagged PER

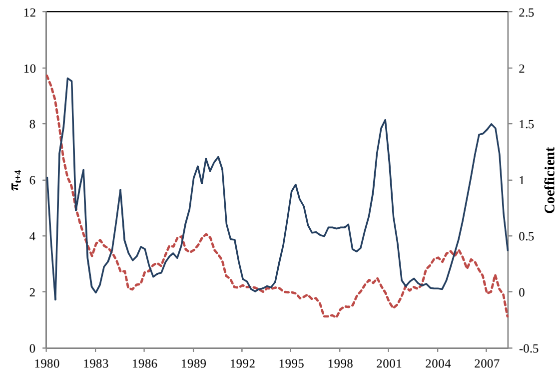


(f) HP filter & lagged PER

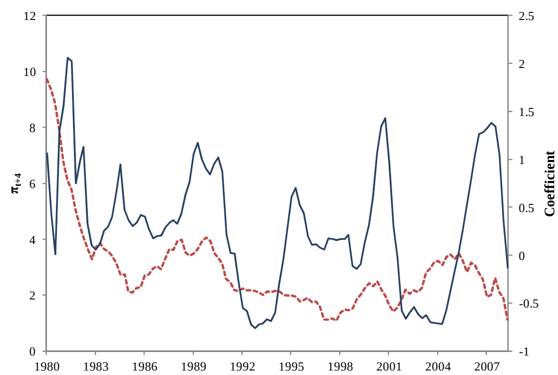
[inflation coefficient] **[stock price coefficient]**

Note: dotted lines in the left graph = π_{t+4}, and in the right graph=s_t.

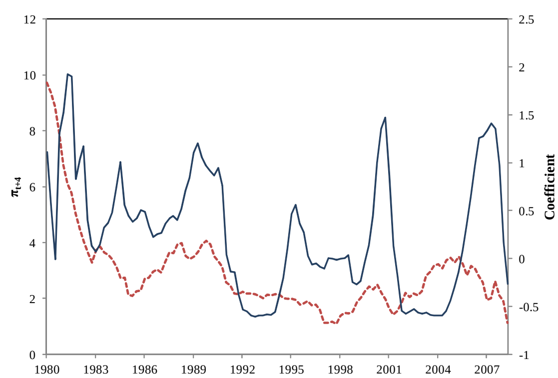
Figure 7. **Time varying path of the coefficients**



(a) CBO's PGDP & lagged PER



(b) HP filter & current PER



(c) HP filter & lagged PER

FIGURE 8. **Time varying path of the inflation coefficients (linear stock price coefficient)**