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WenShwo Fang
Feng Chia University

Stephen M. Miller
University of Nevada, Las Vegas
University of Connecticut

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341 Mansfield Road, Unit 1063
Storrs, CT 06269–1063
Phone: (860) 486–3022
Fax: (860) 486–4463
http://www.econ.uconn.edu/

Output Growth and Its Volatility: The Gold Standard through the Great Moderation

WenShwo Fang
Department of Economics
Feng Chia University
100 WenHwa Road
Taichung, TAIWAN
wsfang@fcu.edu.tw

and

Stephen M. Miller*
Department of Economics
University of Nevada, Las Vegas
4505 Maryland Parkway
Las Vegas, Nevada, USA 89154-6005
stephen.miller@unlv.edu

Abstract:

This study examines the relationship between U.S. output growth and its volatility over the period 1875:Q1 to 2008:Q2. We examine the data for outliers and apply corrections when found. Next, we search for possible effects of structural breaks in the growth rate and its volatility. In so doing, we employ autoregressive generalized conditional heteroskedasticity and autoregressive exponential general conditional heteroskedasticity specifications of the process describing output growth rate and its volatility with and without structural breaks in the mean and volatility processes. We discover one break in the mean process – 1936:Q2 – and three breaks in the volatility process – 1916:Q4, 1950:Q3, and 1983:Q4 (or 1984:Q3). After accommodating the breaks in the mean and volatility processes, the integrated generalized autoregressive conditional heteroskedasticity effect proves spurious. Finally, our data analyses and empirical results suggest that higher output-growth volatility stimulates output growth and that higher output growth reduces its volatility. Moreover, the evidence shows that the time-varying variance falls sharply once we incorporate the three structural breaks in the unconditional variance of output.

Keywords: economic growth and volatility, structural change, IGARCH

JEL classification: C32; E32; O40

* *Corresponding author*

1. Introduction

Researchers occasionally consider the possible structural changes in the duration of recessions and expansions. For example, Diebold and Rudebusch (1992), Cover and Pecorino (2005), and Young and Du (2008) all investigate the possibility of break points in the business cycle, examining the duration of recessions and expansions. Diebold and Rudebusch (1992) and Cover and Pecorino (2005) use the NBER reference cycle data in their analysis. Young and Du (2008) also examine detrended real GDP growth in addition to the NBER reference cycles.

Researchers more frequently explore the possible structural change in the volatility of real GDP growth. For example, Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Ahmed *et al.* (2004), among others, document a structural change in the volatility of U.S. GDP growth, finding a rather dramatic reduction in GDP volatility that some have labeled the Great Moderation. Stock and Watson (2003), Bhar and Hamori (2003), Mills and Wang (2003), and Summers (2005) show a structural break in the volatility decline of the output growth rate for Japan and other G7 countries, although the break occurs at different times.

Researchers now most frequently employ an autoregressive model for the mean equation of real GDP growth and some form of a generalized autoregressive conditional heteroskedasticity (GARCH) modeling strategy to examine the volatility of real GDP growth. Most such studies, however, assume a stable GARCH or exponential GARCH (EGARCH) process, capturing the movement in volatility. The neglect of potential structural breaks in the unconditional or conditional variances of output growth leads to high persistence in the conditional volatility or integrated GARCH (IGARCH). That is, typically all persistence measures fall close to one.

The evidence of a structural change in output growth volatility combined with finding high persistence in conditional volatility motivates us to revisit the issue of conditional volatility in real

GDP growth rates for the US, using a much longer time-series data set – 1875:Q1 to 2008:Q2.[1] We report that three structural breaks exist in the variance resulting in high volatility persistence – 1916:Q4, 1950:Q3, and 1983:Q4 (or 1982:Q3). This issue is well known at the theoretical level;[2] but, the only empirical examination for the U.S. appears in Fang and Miller (2008). This paper contributes to the literature by providing some new evidence from the US that focuses on a longer time horizon extending back to the last quarter of the 19th century. First, excess kurtosis in the growth rate drops substantially or disappears in GARCH or EGARCH models, once we modify outliers in the data set. Non-normally distributed residuals may emerge by not modeling the extraordinary change in the growth series. Second, the IGARCH effect or high volatility persistence remains, when we introduce one structural break in the mean equation. Third, the time-varying variance falls sharply, only when we incorporate the three breaks in the variance equation. The IGARCH effect proves spurious due to nonstationary variance of output growth. Fourth, the GARCH(1, 1) model finds significant effects of our more correct specification of output volatility on output growth or of output growth on its volatility.

Using U.S. quarterly real GDP data, Fang and Miller (2008) report that the long-term

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[1] The sample ends at the beginning of the financial crisis and the Great Recession, which may emerge as another break point in output growth and its volatility.

[2] Diebold (1986) first argues that structural changes may confound persistence estimation in GARCH models. He notes that Engle and Bollerslev's (1986) integrated GARCH (IGARCH) may result from instability of the constant term of the conditional variance (i.e., nonstationarity of the unconditional variance). Neglecting such changes can generate spuriously measured persistence with the sum of the estimated autoregressive parameters of the conditional variance heavily biased towards one. Lamoureux and Lastrapes (1990) provide confirming evidence that ignoring discrete shifts in the unconditional variance, the misspecification of the GARCH model can bias upward GARCH estimates of persistence in variance. Including dummy variables to account for such shifts diminishes the degree of GARCH persistence. More recently, Mikosch and Stărică (2004) prove that the IGARCH model makes sense when non-stationary data reflect changes in the unconditional variance. Hillebrand (2005) shows that in the presence of neglected parameter change-points, even a single deterministic change-point can cause GARCH to measure volatility persistence inappropriately. Alternatively, Hamilton and Susmel (1994) and Kim *et al.* (1998) suggest that the long-run variance dynamics may include regime shifts, but within a given regime, it may follow a GARCH process. Kim and Nelson (1999), Bhar and Hamori (2003), Mills and Wang (2003), and Summers (2005) apply this approach of Markov switching heteroskedasticity with two states to examine the volatility of real GDP growth and identify structural changes.

growth rate of output does not shift and its variance declines. This combination may imply immediately a weak relationship between growth and volatility.[3] In contrast, for our much longer time-series data set from 1875:Q1 to 2008:Q2 rather than the post-WWII sample of Fang and Miler (2008), we find that structural changes emerge in the variance as well as the mean of the real GDP growth rate identified by the multiple structural change test of Bai and Perron (1998, 2003). If the long-term mean growth rate fell substantially, which we find, the implication of the Great Moderation for the relationship between output growth and its volatility is not straightforward and requires model-based calculations.

The rest of the paper unfolds as follows. Section 2 discusses the data, detects and corrects outliers, models the unstable GARCH process of output growth volatility, and identifies the break dates in the mean and the conditional variance. Section 3 presents empirical results with changes in the mean and the variance and identifies two areas of misspecification of the GARCH modeling of output growth volatility. Section 4 considers evidence on the relationship between the output growth rate and its volatility. Finally, Section 5 concludes.

2. Data Analysis and Modeling

Output growth rates (y_t) equal the percentage change in the logarithm of seasonally adjusted quarterly real GDP (Y_t) with base year 2000 over the period 1875:Q1 to 2008:Q2. That is, we create the quarterly real GDP series, involving one splice. The original data come from Balke and Gordon (1986) from 1875:Q1 to 1983:Q4 (base year = 1972) and the US Bureau of Economic Analysis from 1947:Q1 to 2008:Q2 (base year = 2000). We splice the 1875:Q1 to 1983:Q4 real GDP series to the 1947:Q1 to 2008:Q2 real GDP series in 1947:Q1, measured in 2000 prices.

[3] Stock and Watson (2003) interpret the moderation in output volatility with no change in the mean growth rate as shorter recessions and longer expansions in the US, linking to the literature on durations of recessions and expansions..

Descriptive Statistics

Table 1 reports descriptive statistics for the growth rate of the spliced quarterly real GDP. The US experiences a mean growth rate of 0.82 percent for the full 134-year sample with the highest rate of 7.96 in 1879:Q4 and the lowest rate of -8.76 in 1893:Q3. Output volatility, represented by the standard deviation, equals 2.24. Under the assumptions of normality, standard measures of skewness and kurtosis possess asymptotic distributions of N(0, 6/T) and N(0, 24/T), respectively, where T(=533) equals the sample size. The skewness statistic displays an asymmetric distribution characterized by negative skewness, meaning that in the sample period, a greater probability exists of large decreases in real GDP growth than large increases. The kurtosis statistic exhibits leptokurticity with fat tails, meaning that extreme changes occur more frequently with a higher kurtosis. The Jarque-Bera test rejects normality. Ljung-Box Q and Q^2 statistics test for autocorrelation up to nine lags. The Ljung-Box statistics ($LB\ Q$) indicate autocorrelation in the growth rates, while the Ljung-Box statistics for squared rates ($LB\ Q^2$) suggest time-varying variance in the series. Autocorrelation and heteroskedasticity suggest ARMA processes for the mean and the variance equations to capture the dynamic structure and to generate white-noise residuals.

Autoregressive Model of Output Growth Rate

Table 1 also reports the results of the AR model constructed for the growth rate series. Based on the Schwarz Bayesian Criterion (SBC), four lags, an AR(4) process, prove adequate to capture growth dynamics and produce uncorrelated residuals. That is, the mean growth rate equation equals the following:

$$y_t = a_0 + \sum_{i=1}^{4} a_i y_{t-i} + \varepsilon_t \,, \tag{1}$$

where the growth rate $y_t \equiv 100 \times (\ln Y_t - \ln Y_{t-1})$, $\ln Y_t$ equals the natural logarithm of real GDP, and ε_t equals the serially uncorrelated error term.

The AR(4) model proves problematic in several areas. First, we reject normality of the error term with significant skewness and kurtosis. Second, the significant Ljung-Box Q-statistics for squared rates indicate time-varying variance in the series, although the insignificant Ljung-Box Q-statistics suggest no autocorrelation. We expect to resolve these two issues of misspecification by modeling outliers and changes in the mean and the variance equations. That is, the likelihood of biasing the estimated volatility persistence parameters toward one and the skewness and leptokurtosis in the distribution of output growth should vanish after adjustment of the GARCH model with various changes.

Outlier Detection and Correction

Economic and financial time series frequently include outliers.[4] An outlier observation appears inconsistent with other observations in the growth rates. To the best of our knowledge, however, researchers typically overlook their existence and effect when modeling output growth and its volatility.[5] The combined task of detecting outliers and correcting them faces similar problems to the lag-length selection process in time-series modeling. Too many outliers in a data series deteriorate the quality of that data; too few (i.e., correcting too many outliers) may prevent the capture of important structural changes in the data series.

Table A1 in the Appendix identifies the outliers in the growth rate of real GNP, using the

[4] Balke and Fomby (1994) analyze fifteen post-World War II U.S. macroeconomic time series using the outlier identification procedure based on Tsay (1988) and find that outliers may prove important for U.S. macroeconomic data, and such aberrant observations may lead to large ARCH test statistics. van Dijk, Franses, and Lucas (1999) demonstrate that neglecting additive outliers frequently leads to a rejection of the null hypothesis of homoskedasticity, when it is in fact true. Tolvi (2001) and Charles and Darné (2006), however, show another possibility. That is, outliers can hide the ARCH tests of the series. After correcting the data for outliers, returns series sometimes display strong evidence of ARCH. Franses and Ghijsels (1999) and Charles and Darné (2005, 2006) apply the method of Chen and Liu (1993) to correct for additive outlier and show that correcting for additive outliers reduces excess kurtosis in GARCH models and improves forecasts of stock market volatility.

[5] Fang and Miller (2008) provide an exception. They develop the method that we generally follow in this paper.

following selection criterion: $\left| y_t - Mean \right| > k \cdot SD$, where k measures the stringency imposed on outlier detection. When $k=4$, we identify only one outlier and when $k=2$, we indentify 45 outliers. Finally, with $k=3$, we find 11 outliers. We focus on the results for $k=3$. Of the 11 outliers, three represent high growth rates, while eight represent low (negative) growth rates.

We apply the Franses and Ghijsels (1999) method to correct additive outliers in GARCH models. In the correction process, we, first, estimate the AR(4)-GARCH(1,1) model for the growth rate series and replace the observed growth rates with outlier-corrected values.

Table 2 reports descriptive statistics for the outlier-corrected growth rate. Comparing Table 2 to Table 1, we corrected eight negative outliers but only three positive outliers and the skewness statistic moves form a significant negative value in Table 1 to an insignificant positive value in Table 2. Nonetheless, even though the test statistics both decrease in value, we still observe significant kurtosis and non-normality in the outlier-corrected growth rate series.

Table 2 also reports the results of estimating the AR(4) model for the growth rate of real GNP assuming a homoskedastic error process. We note that the error process does not exhibit skewness or kurtosis and we cannot reject the null hypothesis of a normal error structure. We do find evidence of heteroskedastic errors, which leads to our analysis of a GRACH process for the error process in the AR(4) mean equation.

Identifying Structural Change

Using the outlier-corrected data, we look for structural changes in the volatility for GDP growth in sequential steps. First, we estimate equation (1) allowing for the possibility of structural breaks in its intercept and slope coefficients. Specifically, we use the statistical techniques of Bai and Perron (1998, 2003) to estimate multiple break dates without prior knowledge of when those breaks occur. After finding any breaks in the mean of y_t, we use that model specification to obtain series of estimated residuals, $\hat{\varepsilon}_t$. Second, we search for breaks in the variance by testing for parameter constancy in the conditional mean of the absolute value of the residuals $\hat{\varepsilon}_t$ as shown in Cecchetti *et al*. (2005) and Herrera and Pesavento (2005).

Bai and Perron (1998, 2003) propose several tests for multiple breaks. We adopt one procedure and sequentially test the hypothesis of m breaks versus m+1 breaks using $\sup F(m+1|m)$ statistics, which detects the presence of m+1 breaks conditional on finding *m* breaks and the supremum comes from all possible partitions of the data for the number of breaks tested. In the application of the test, we search for up to five breaks. If we reject the null of no break at the 5-percent significance level, we, then, estimate the break date using least squares, to divide the sample into two subsamples according to the estimated break date, and to perform a test of parameter constancy for both subsamples. We repeat this process by sequentially increasing *m* until we fail to reject the hypothesis of no additional structural change. In the process, rejecting *m* breaks favors a model with *m+1* breaks, if the overall minimal value of the sum of squared residuals over all the segments, including an additional break, falls sufficiently below the sum of squared residuals from the model with *m* breaks. The break dates selected include the ones associated with this overall minimum. We search for multiple breaks in the series of output growth using the GAUSS code made available by Bai and Perron (2003).

Table 3 displays the results of testing for breaks in the mean and the variance, their critical values at the 5-percent significance level (in brackets). Pure and partial structural breaks refer to the situations where the test permits all coefficients to change (pure) and only the intercept coefficient to change (partial). When testing for pure structural breaks, the value of the $\sup F(5|0)$ test proves significant for *m=5*, suggesting the existence of at least one break in the growth rate series. The sequential $\sup F(m+1|m)$ exhibits significance only for *m=1*. That is, given the existence of one break, $sup\,F(2|1)=16.6217$ suggests that only one break exists. The break date occurs at 1936:Q2 with 95% confidence interval [1912:Q3 to 1962:Q2]. The procedure also identifies three structural breaks in the variance of growth rates at 1916:Q4, 1950:Q3, and 1982:Q4 with 95-percent confidence intervals [1905:Q4 to 1922:Q4], [1948:Q4 to 1956:Q1], and [1982:Q2 to 1990:Q3]. Thus, three structural changes in the *GARCH* process govern volatility.

Considering partial structural breaks leads to the following conclusions. First, we do not find a break in the intercept of the mean equation. That is, the structural break in the mean equation reflects entirely shifts in the slope coefficients of the AR(4) process, that is, coefficients of the second, third, and fourth lags (see Tables 5 and 6). We still identify three structural breaks in the variance at 1916:Q4, 1950:Q3 and 1982:Q3 with 95-percent confidence ranges of [1906:Q1 to 1920:Q4], [1949:Q2 to 1956:Q1], and [1979:Q3 to 1990:Q1].

Table 4 reports the structural stability tests for the unconditional variance as well as the mean of the growth rate by splitting the sample into sub-periods according to the break dates. Panels A and B report the pure and partial structural breaks, respectively. For the unconditional mean, a *t*-statistic tests for the equality of means under unequal variances for two different samples, while a variance-ratio statistic tests for the equality of the unconditional variances.

In Panel A, the mean growth rates in each sub-sample do not differ significantly, since the *t*-statistic cannot reject the null hypothesis of equal means. The structural break identified in the mean for the pure structural break test occurs only in the slope coefficients and not the intercept (see Tables 5 and 6). The standard deviations significantly differ between all four sub-periods. The standard deviation rises from 1.5877 between 1876:Q1 and 1916:Q4 to 2.4199 between 1917:Q1 to 1950:Q3 and then falls to 1.1303 between 1950:Q4 to 1983:Q4 before falling further during the Great Moderation to 0.5232 between 1982:Q4 to 2008:Q2. In panel B, no structural break exists for the mean equation. The standard deviations, once again, significantly differ between all four sub-periods. The standard deviation rises from 1.5877 between 1876:Q1 and 1916:Q4 to 2.4199 between 1917:Q1 to 1950:Q3 and then falls to 1.1333 between 1950:Q4 to 1982:Q3 before falling further during the Great Moderation to 0.5648 between 1982:Q4 to 2008:Q2.

Figure 1 plots the observed real GDP growth rate. The eye can catch the decrease in the volatility around 1950 and then another decrease around 1982. The increase in volatility documented around 1916 does not appear so obvious.

GARCH Modeling of Output Volatility

To consider the effect of the Great Moderation on the volatility persistence of output growth in GARCH specifications, we include dummy variables in the conditional variance equation, which equal unity from the break date forward, zero otherwise, in the GARCH and EGARCH processes, respectively, as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3, \tag{2}$$

$$\log \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \log \sigma_{t-1}^2 + \gamma_1 D_1 + \gamma_2 D_2 + \gamma_3 D_3, \tag{3}$$

where D_1 = 1 for $t > 1916:Q3$, 0, otherwise; D_2 = 1 for $t > 1950:Q2$, 0, otherwise; and D_3 = 1 for $t > 1983:Q3$, 0, otherwise. The dummy variables accommodate the extraordinary changes. Since the volatility first increases and then declines twice, we expect a significant positive estimate for γ_1 and significant negative estimates for γ_2 and γ_3 to capture the break in the variance process. In equation (3), asymmetry in the response exists if $\alpha_2 \neq 0$. Moreover, negative (positive) shocks generate higher volatility than positive (negative) shocks of the same magnitude when $\alpha_2 < 0$ ($\alpha_2 > 0$).

Although the data do not suggest a significant change in the mean of the growth rate of real

GNP, we do find a significant change in the structure of the mean equation, that is, the coefficients of the AR(4) process shift in 1936:Q2. To accommodate the structural change in the mean equation, we specify that equation as follows:

$$y_t = a_0 + \sum_{i=1}^{4} a_i y_{t-i} + d_0 D + \sum_{i=1}^{4} d_i D y_{t-i} + \varepsilon_t \,, \tag{4}$$

where we define $b_i = a_i + d_i$, $i = 0,1,2,3,4$, and D=1 for $t > 1936 : Q1$, 0, otherwise.

Tables 5 and 6 report the results of estimating the mean equation in an AR(4) process and its volatility as a GARCH(1,1). Column 1 reports the results for the raw, uncorrected data whereas column 2 reports the results for the outlier corrected data, where we replace 11 quarterly growth rates with adjusted values. Columns 3 and 4 lists the results for the outlier corrected data and incorporating the mean structural shift dummy variable (column 3) and then both the mean and variance shift dummy variables (column 4).

Estimating the mean model with a GARCH(1,1) specification for the error term (column 1) leads to an IGARCH outcome with significant skewness and kurtosis as well as non-normality. When we use the outlier-corrected data (column 2), we still experience the IGARCH outcome but the significant skewness and kurtosis disappear and normality appears. The IGARCH remains when we also accommodate the structural shift in the mean equation (column 3).

Including both the structural shifts in the mean and the volatility equations (column 4) eliminates the IGARCH. The coefficients of the structural dummy variables in the volatility equation (i.e., γ s) prove significant. We see a significant increase in the volatility between the 1876:Q1 to 1916:Q4 and the 1917:Q1 to 1950:Q3 periods. Then we find significant decreases in volatility between the 1917:Q1 to 1950:Q3 and 1950:Q4 to 1983:Q4 periods and between the 1950:Q4 to 1983:Q4 and the 1984:1 to 2008:Q2 periods. Further, the Ljung-Box Q-statistics of the standardized residuals and the squared standardized residuals show no evidence of autocorrelation and heteroskedasticity, providing support for the specification of the GARCH or the EGARCH. The significant LR statistic at the 5-percent level indicates no IGARCH effect.

The results of the symmetric or asymmetric GARCH models suggest that the time-varying variance in the growth rate may reflect major structural changes in the implementation of monetary policy, although other rationalizations may make sense as well. The first period between 1876:Q1 to 1916:Q4 reflects the gold standard and that ended with the start of WWI. The second sub-period between 1917:Q1 to 1950:Q3 include the two World Wars and the inter-War period where countries sought unsuccessfully to return to the gold standard. The third sub-period between 1950:Q4 to 1983:Q4 begins near the Treasury Federal Reserve Accord whereby the Federal Reserve System received more independence in the conduct of monetary policy. Finally, the last period, called the Great Moderation, begins shortly after the drastic reduction in deflation engineered by the Volker Federal Reserve though early 2008.

In sum, previous studies assume implicitly that a stable GARCH process governs conditional growth volatility. The neglect of the structural breaks in the variance implies misspecification of the conditional variance. This leads to the conclusion of a significant IGARCH effect. Moreover, taking no account of possible outliers and breaks in the growth rates entails excess kurtosis, and, thus, a significant Jarque-Bera test. Fang and Miller (2008) pioneered the adjustment for outliers and the inclusion of structural breaks in the volatility of the output growth rate, leading to the disappearance of the IGARCH effect. In fact, they found for post WWII data that the proper specification reduced to a simple ARCH model. We extend the method of Fang and Miller (2008) to a longer data series and find four periods of different volatility identified by break points in 1916:Q4, 1950:Q3, and 1983:Q3. Our findings still imply an AR-GARCH or AR-EGARCH specification.

4. Relationship between Output Volatility and Economic Growth

The prior section considers the appropriate time-series specification of the volatility of the growth rate of real GDP. A number of authors examine the issue of how this volatility affects the growth rate of GDP. That is, does the decreased real GDP growth rate volatility cause a higher or lower real GDP growth rate? For example, applying a GARCH in mean (GARCH-M) model (Engle *et al*., 1987) and using post-war real quarterly GDP data, Henry and Olekalns (2002) discover a significant asymmetric GARCH effect and a negative link between volatility and real GDP growth for the U.S. without consideration of structural shift in the volatility process. In contrast, Fang and Miller (2008) find a weak GARCH effect and no link between volatility and growth for the U.S. with a structural break in the volatility process. This section pursues this question with our more appropriate time-series specification of the real GDP growth rate volatility. This issue is important because structural break in variance biases upward GARCH estimates of persistence in variance and, thus, vitiates the use of GARCH to estimate its mean effect.

In this section, the mean growth rate shown in equation (4) translates into the following:

$$y_t = a_0 + \sum_{i=1}^{4} a_i y_{t-i} + d_0 D + \sum_{i=1}^{4} d_i D y_{t-i} + \lambda \sigma_t + \varepsilon_t \qquad (5)$$

where σ_t equals the standard deviation of the conditional variance, σ_t^2, λ measures the volatility effect in the mean, and D=1 for $t > 1936 : Q1$, 0, otherwise.

Alternative theoretical models imply different results -- negative, positive, or independent relationships between output growth volatility and output growth. For example, the misperceptions theory, proposed originally by Friedman (1968), Phelps (1968), and Lucas (1972), argues that output fluctuates around its natural rate, reflecting price misperceptions due to monetary shocks. The long-run growth rate of potential output, however, reflects technology and other real factors. The standard dichotomy in macroeconomics implies no relationship between output volatility and its growth rate (i.e., $\lambda=0$). Martin and Rogers (1997, 2000) argue that learning-by-doing generates growth whereby production complements productivity-improving activities and stabilization policy can positively affect human capital accumulation and growth. One natural conclusion, therefore, implies a negative relationship between output growth volatility and growth (i.e., $\lambda<0$). In contrast, Black (1987) argues that high output volatility and high growth coexist. According to Blackburn (1999), a relative increase in the volatility of shocks increases the pace of knowledge accumulation and, hence, growth, implying a positive relation between output growth volatility and growth (i.e., $\lambda>0$).

More recently, Fountas *et al.* (2006) consider the possibility of a two-way relationship between output growth and its volatility. The authors first estimate a bivariate GARCH specification of output growth and inflation. And then they recover the means and conditional variances for output growth and inflation to run a second-stage four-variable vector-autoregressive model to conduct Granger-causality tests. Using G7 examples, they find that output growth volatility positively affects output growth in all the seven countries, except Japan, and output growth negatively affects output growth volatility in Japan, Germany, and the U.S. and a zero effect in the rest of the countries. That is, a bi-directional causality between output growth and its volatility exists in Germany and the U.S., and one-way causality in Japan and the other four countries.

In a GARCH-M model, if output growth partly determines its volatility but is excluded in the variance equation, then the conditional variance equation is misspecified and GARCH-M estimates are not consistent (see Pagan and Ullah, 1988). Fountas and Karanasos (2006) and Fang and Miller (2008) develop a structural specification that incorporates the contemporaneous conditional volatility into the mean equation for output growth and lagged output growth into the conditional variance equation in their GARCH-M models. Contrary to Fountas *et al.* (2006), Fountas and Karanasos (2006) find, using annual industrial production data from 1860 to 1999, that the output growth rate volatility exhibits no effect on the growth rate, but the output growth rate affects its volatility negatively in the US. Similarly, Fang and Miller (2008), using quarterly post-WWII US data on real GDP growth, report that output growth rate volatility does not affect output growth, but that output growth does negatively affect its volatility.

To avoid the GARCH-M model suffering from an endogeneity bias, we augment the variance equations (4) and (5) to include lagged output growth, respectively, as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \theta\, y_{t-1} + \gamma_1\, D_1 + \gamma_2 D_2 + \gamma_3 D_3, \tag{6}$$

$$\begin{aligned} \log\, \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1\, \log \sigma_{t-1}^2 + \theta\, y_{t-1} \\ + \gamma_1\, D_1 + \gamma_2 D_2 + \gamma_3 D_3, \end{aligned} \tag{7}$$

where θ measures the level effect of the output growth in variance. To the best of our knowledge, no economic theory models explicitly the effect of output growth on its volatility. Theoretically, the sign of θ is unknown. Intuitively, Fountas *et al.* (2006) argue that either a negative or a positive relation may occur. That is, an increase in output growth leads to more inflation, if both the Friedman (1977) hypothesis and the Taylor (1979) effect hold, then higher inflation raises inflation volatility and higher inflation volatility trades off with output volatility. Thus, output growth and its volatility are negatively related (i.e., θ<0). Ungar and Zilberfarb (1993), however, show that higher inflation reduces inflation volatility, and thus a positive relation (i.e., θ>0) may also occur.

Tables 5 and 6 report the GARCH and EGARCH results, where we include the structural shift in the mean equations well as the three-time structural break in the variance process. Columns 5 and 6 report the results for the level effect in the variance equation only and the GARCH-M effect in the mean equation only, respectively. Column 7 lists the results for both the level effect in the variance equation and the GARCH-M effect in the mean equation simultaneously. Whether separate or together, Table 5 shows that the coefficient of the level effect in the variance equation and the GARCH-M effect in the mean equation prove significantly negative and positive, respectively, in the GARCH model. In sum, a higher variability leads to a higher growth rate and a higher growth rate leads to a lower variance. These findings match our period-by-period calculations of the mean values. That is, the second sub-period exhibited a higher variability and growth when compared to the first sub-period. Then the third and fourth sub-periods experienced lower variability and growth than their preceding sub-periods. These results, however, differ from those of Fountas and Karanasos (2006) and Fang and Miller (2008). Fountas and Karanasos (2006) use a long-sample of over 100 years, but they use annual data on industrial production. Fang and Miller (2008) do use quarterly data, but only for the post-WWII period.

Table 6 shows that the level and GARCH-M effects in the variance and mean equations, respectively, completely disappear in the EGARCH specification. In addition, the effect of innovations on the mean equation exert different effects on the logarithm of the standard deviation, where negative shocks exhibit a larger effect than positive shocks. That is, we find significant evidence of asymmetric effects. Moreover, the function value suggests that the AR-EGRACH specification dominates the AR-GARCH specification. As a consequence, our results suggest that prior findings of feedback between the volatility of the output growth rate and the output growth rate and vice versa may occur because researchers did not accommodate asymmetric responses in an EGARCH model.

4. Conclusion

This paper examines the effect of the Great Moderation on the relationship between quarterly real

GDP growth rate and its volatility in the U.S. over the period 1875:Q1 to 2008:Q2. First, we inspect the data for outliers and apply appropriate corrections on the outliers discovered. Second, we perform tests for structural breaks in the growth rate and its volatility. In so doing, we employ AR-GARCH and AR-EGARCH specifications of the process describing output growth rate and its volatility with and without structural breaks in the mean and volatility processes. Third, we identify one break in the mean process – 1936:Q2 – and three breaks in the volatility process – 1916:Q4, 1950:Q3, and 1983:Q4 (or 1984:Q3). After accommodating the breaks in the mean and volatility processes, the IGARCH effect proves spurious. Finally, our data analyses and empirical results suggest that output growth volatility positively affects output growth and that higher output growth negatively affects its volatility. Moreover, the evidence shows that the time-varying variance falls sharply once we incorporate the three structural breaks in the unconditional variance of output.

The independence between the output growth and its volatility needs careful interpretation. Endogenous growth theory, for example, does not imply any importance for the second moment. Blackburn and Galindev (2003) and Blackburn and Pelloni (2004) model the link between the mean and variance of the output growth rate explicitly. Different mechanisms of endogenous technological change and nominal or real shocks can lead to positive or negative relationship between growth and volatility. In his model, Blackburn (1999) shows for a linear endogenous learning function, the effect of the output growth-rate volatility on the output growth rate equals zero. A concave (convex) learning function generates a negative (positive) effect. That is, an independent relationship may exist with or without the Great Moderation. The disagreements between published findings highlights the sensitivity of the results to the country considered, the time period examined, the frequency of the data, and the methodology employed. This apparent inconclusiveness warrants further investigation of the relationship between growth and its volatility. Nevertheless, we conclude with a cautionary note that failure to model structural breaks in the volatility of output growth and/or failure to model volatility asymmetries may lead researchers to conclude falsely that output volatility affects output growth.

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Table 1: Descriptive Statistics for Quarterly Real GNP, 1875:Q1-2008:Q2

Panel A. Quarterly Real GNP Growth

Sample size	533	***LB Q*** **(1)**	81.6645* [0.0000]	***LB*** Q^2 **(1)**	63.0489* [0.0000]
Mean	0.8228	***LB Q*** **(2)**	87.5309* [0.0000]	***LB*** Q^2 **(2)**	89.3816* [0.0000]
Standard deviation	2.2383	***LB Q*** **(3)**	98.0657* [0.0000]	***LB*** Q^2 **(3)**	115.7437* [0.0000]
Maximum	7.9649	***LB Q*** **(4)**	98.6400* [0.0000]	***LB*** Q^2 **(4)**	131.9787* [0.0000]
Minimum	-8.7565	***LB Q*** **(5)**	103.6247* [0.0000]	***LB*** Q^2 **(5)**	152.5296* [0.0000]
Skewness	-0.5790* [0.0000]	***LB Q*** **(6)**	103.6517* [0.0000]	***LB*** Q^2 **(6)**	168.8023* [0.0000]
Kurtosis	2.8927* [0.0000]	***LB Q*** **(7)**	103.7492* [0.0000]	***LB*** Q^2 **(7)**	182.4710* [0.0000]
Normality test	215.6173* [0.0000]	***LB Q*** **(8)**	103.8328* [0.0000]	***LB*** Q^2 **(8)**	253.3080* [0.0000]
ADF(n)	-10.8341(3)*	***LB Q*** **(9)**	108.0243* [0.0000]	***LB*** Q^2 **(9)**	268.1344* [0.0000]

Panel B. Quarterly Real GNP Growth AR(4) Estimates

$$y_t = a_0 + \sum_{i=1}^{4} a_i y_{t-i} + \varepsilon_t$$

a_0	a_1	a_2	a_3	a_4		
0.5205*	0.4427*	-0.1269*	0.2050*	-0.1612*		
(0.1001)	(0.0429)	(0.0461)	(0.0460)	(0.0428)		
LB Q **(6)**	***LB Q*** **(12)**	***LB*** Q^2 **(6)**	***LB*** Q^2 **(12)**	**Skewness**	**Kurtosis**	**Normality**
0.9081	10.8531	120.1226*	200.3953*	-0.3407*	3.6410*	302.4423*
[0.9888]	[0.5415]	[0.0000]	[0.0000]	[0.0014]	[0.0000]	[0.0000]
LM **(1)**	***LM*** **(2)**	***LM*** **(3)**	***LM*** **(4)**	***LM*** **(5)**	***LM*** **(6)**	
24.2678*	46.6051*	59.2766*	59.5945*	61.1667*	62.3875*	
[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	

Note: Standard errors appear in parentheses; p-values appear in brackets; 0.0000 indicates less than 0.00005. The measures of skewness and kurtosis are normally distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T equals the number of observations. ADF(n) equals the augmented Dickey-Fuller unit-root test with lags n selected by the SBC. *LB* $Q(k)$ and *LB* $Q^2(k)$ equal Ljung-Box Q-statistics distributed asymptotically as χ^2 with k degrees of freedom, testing for level and squared terms for autocorrelations up to k lags.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 2: **Descriptive Statistics for Quarterly Real GNP, 1875:Q1-2008:Q2 (Critical Value by k=3)**

Panel A. Quarterly Real GNP Growth

Sample size	533	***LB Q* (1)**	88.3310* [0.0000]	***LB*** Q^2 **(1)**	73.3568* [0.0000]
Mean	0.9163	***LB Q* (2)**	103.3467* [0.0000]	***LB*** Q^2 **(2)**	108.7103* [0.0000]
Standard deviation	1.6246	***LB Q* (3)**	117.0545* [0.0000]	***LB*** Q^2 **(3)**	166.0098* [0.0000]
Maximum	5.7512	***LB Q* (4)**	117.3284* [0.0000]	***LB*** Q^2 **(4)**	214.9633* [0.0000]
Minimum	-4.2890	***LB Q* (5)**	122.1291* [0.0000]	***LB*** Q^2 **(5)**	232.7550* [0.0000]
Skewness	0.0383 [0.7188]	***LB Q* (6)**	122.2763* [0.0000]	***LB*** Q^2 **(6)**	259.6555* [0.0000]
Kurtosis	0.5496* [0.0100]	***LB Q* (7)**	122.3000* [0.0000]	***LB*** Q^2 **(7)**	270.8399* [0.0000]
Normality test	6.8386* [0.0327]	***LB Q* (8)**	122.3246* [0.0000]	***LB*** Q^2 **(8)**	305.5707* [0.0000]
ADF(n)	-14.9648(0)*	***LB Q* (9)**	126.5871* [0.0000]	***LB*** Q^2 **(9)**	331.8204* [0.0000]

Panel B. Quarterly Real GNP Growth AR(4) Estimates

$$y_t = a_0 + \sum_{i=1}^{4} a_i y_{t-i} + \varepsilon_t$$

a_0	a_1	a_2	a_3	a_4		
0.5185*	0.4143*	-0.0330	0.1405*	-0.0948*		
(0.0827)	(0.0430)	(0.0462)	(0.0460)	(0.0429)		
***LB Q* (6)**	***LB Q* (12)**	***LB*** Q^2 **(6)**	***LB*** Q^2 **(12)**	**Skewness**	**Kurtosis**	**Normality**
9.9923	15.9125	291.1085*	505.8934*	0.0757	0.2343	1.7164
[0.1249]	[0.1952]	[0.0000]	[0.0000]	[0.4781]	[0.2744]	[0.4239]
***LM* (1)**	***LM* (2)**	***LM* (3)**	***LM* (4)**	***LM* (5)**	***LM* (6)**	
38.8847*	77.2316*	94.7072*	110.8972*	115.6021*	117.2286*	
[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	

Note: See Table 1.
* denotes 5-percent significance level.
** denotes 10-percent significance level.

Table 3: Bai and Perron (1998) Structural Break Test and Break Date

	Pure Structural Break		Partial Structural Break	
	Mean	**Variance**	**Mean**	**Variance**
$Sup\ F(1\|0)$	14.0145	73.4623*	2.0179	76.7664*
	[18.2300]	[8.5800]	[8.5800]	[8.5800]
$Sup\ F(2\|0)$	15.5203	51.3302*	1.7890	54.9842*
	[15.6200]	[7.2200]	[7.2200]	[7.2200]
$Sup\ F(3\|0)$	14.0973*	81.1428*	1.7463	62.9688*
	[13.9300]	[5.9600]	[5.9600]	[5.9600]
$Sup\ F(4\|0)$	13.2340*	60.9688*	1.6418	49.6966*
	[12.3800]	[4.9900]	[4.9900]	[4.9900]
$Sup\ F(5\|0)$	13.5140*	48.2529*	0.9565	39.8136*
	[10.5200]	[3.9100]	[3.9100]	[3.9100]
UD_{Max}	15.5203	81.1428*	2.0179	76.7664*
	[18.4200]	[8.8800]	[8.8800]	[8.8800]
WD_{Max}	23.4183*	116.8129*	2.8229	90.6498*
	[19.9600]	[9.9100]	[9.9100]	[9.9100]
$Sup\ F(2\|1)$	16.6217	43.7844*	1.5136	32.3612*
	[19.9100]	[10.1300]	[10.1300]	[10.1300]
$Sup\ F(3\|2)$	12.1317	43.7844*	1.6373	35.6135*
	[20.9900]	[11.1400]	[11.1400]	[11.1400]
$Sup\ F(4\|3)$	5.5802	2.8801	1.4164	0.0713
	[21.7100]	[11.8300]	[11.8300]	[11.8300]
$Sup\ F(5\|4)$	—	0.2401	—	0.0713
		[12.2500]		[12.2500]
Break date	1936:2	1916:4	NA	1916:4
		1950:3		1950:3
		1983:4		1982:3
95% Confidence Interval	1912:3-1962:4	1905:4-1922:4	NA	1906:1-1920:4
		1948:4-1956:1		1949:2-1956:1
		1982:2-1990:3		1979:3-1990:1

Note: Critical values for the 5-percent significance level appear in parentheses. In the detection process, we require 15% of the full sample as the minimal length of any partition. Thus, — indicates that no more place exists to insert an additional break given the minimal length requirement.

* denotes 5-percent significance level.

Table 4. Cross-Sample Structural Stability Test

Panel A. Pure Structural Break Specification

	Sub-sample	Sub-sample 1 (1876:1-1936:2)	Sub-sample 2 (1936:2-2008:2)
Mean	Sub-sample 1 (1876:1-1936:2)	*0.9124*	
	Sub-sample 2 (1936:2-2008:2)	-0.0491 [0.9607]	*0.9196*

	Sub-sample	Sub-sample 1 (1876:1-1916:4)	Sub-sample 2 (1917:1-1950:3)	Sub-sample 3 (1950:4-1983:4)	Sub-sample 4 (1984:1-2008:2)
Standard Deviation	Sub-sample 1 (1876:1-1916:4)	*1.5877*			
	Sub-sample 2 (1917:1-1950:3)	0.4304* [0.0000]	*2.4199*		
	Sub-sample 3 (1950:4-1983:4)	1.9728* [0.0000]	4.5831* [0.0000]	*1.1303*	
	Sub-sample 4 (1984:1-2008:2)	9.2063* [0.0000]	21.3875* [0.0000]	4.6665* [0.0000]	*0.5232*

Panel B. Partial Structural Break Specification

	Sub-sample	Sub-sample 1 (1876:1-1916:4)	Sub-sample 2 (1917:1-1950:3)	Sub-sample 3 (1950:4-1982:3)	Sub-sample 4 (1982:4-2008:2)
Standard Deviation	Sub-sample 1 (1876:1-1916:4)	*1.5877*			
	Sub-sample 2 (1917:1-1950:3)	0.4304* [0.0000]	*2.4199*		
	Sub-sample 3 (1950:4-1982:3)	1.9624* [0.0000]	4.5591* [0.0000]	*1.1333*	
	Sub-sample 4 (1982:4-2008:2)	7.8999* [0.0000]	18.3526* [0.0000]	4.0254* [0.0000]	*0.5648*

Note: P-values appear in brackets; 0.0000 indicates less than 0.00005. A t-statistic under unequal variances tests for structural change in the unconditional mean between the different regimes. *F* test equals the unconditional variance ratio test between the samples *i* and *j*, and is asymptotically distributed as $F(df_i, df_j)$, where *df* denotes the degrees of freedom.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 5: GARCH Model Estimation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a_0	0.4892* (0.0735)	0.5031* (0.0702)	0.5663* (0.1317)	0.6164* (0.1299)	0.5842* (0.1306)	0.2497 (0.2206)	0.1891 (0.2246)
a_1	0.3941* (0.0504)	0.3482* (0.0461)	0.4204* (0.0645)	0.4142* (0.0633)	0.4135* (0.0636)	0.4418* (0.0634)	0.4298* (0.0637)
a_2	0.0549 (0.0569)	0.0921* (0.0471)	-0.0331 (0.0654)	-0.0411 (0.0646)	-0.0374 (0.0656)	-0.0574 (0.0637)	-0.0453 (0.0642)
a_3	-0.0012 (0.0491)	0.0034 (0.0470)	0.1118** (0.0646)	0.1243** (0.0698)	0.1314** (0.0697)	0.1172** (0.0690)	0.1194** (0.0690)
a_4	-0.0780 (0.0479)	-0.0664 (0.0438)	-0.1959* (0.0661)	-0.1940* (0.0690)	-0.1904* (0.0688)	-0.1995* (0.0671)	-0.1647* (0.0679)
b_0			-0.1207 (0.1589)	-0.1504 (0.1507)	-0.1423 (0.1519)	-0.0053 (0.1936)	0.0826 (0.1888)
b_1			-0.1117 (0.0909)	-0.0865 (0.0880)	-0.0947 (0.0904)	-0.0803 (0.0887)	-0.1328 (0.0877)
b_2			0.2058* (0.0914)	0.1930* (0.0913)	0.20448* (0.0911)	0.2187* (0.0901)	0.2146* (0.0884)
b_3			-0.1739* (0.0909)	-0.2073* (0.0916)	-0.2173* (0.0923)	-0.2076* (0.0905)	-0.1770* (0.0913)
b_4			0.2291* (0.0884)	0.2258* (0.0900)	0.2351* (0.0905)	0.2484* (0.0896)	0.2081* (0.0890)
λ						0.2630* (0.1206)	0.2344* (0.1150)
α_0	0.0039 (0.0069)	0.0082 (0.0067)	0.0085 (0.0063)	0.5933* (0.2261)	0.5937* (0.1680)	0.3235* (0.1175)	0.6130* (0.1582)
α_1	0.1219* (0.0279)	0.1192* (0.0225)	0.1264* (0.0225)	0.0967* (0.0420)	0.0825** (0.0493)	0.0900* (0.0313)	0.1158* (0.0422)
β_1	0.8889* (0.0206)	0.8798* (0.0220)	0.8732* (0.0213)	0.6148* (0.1290)	0.6787* (0.1157)	0.7439* (0.0706)	0.6497* (0.0890)
γ_1				0.6312* (0.2934)	0.4318* (0.1925)	0.2755* (0.1420)	0.3807* (0.1812)
γ_2				-0.8755* (0.3436)	-0.6697* (0.2408)	-0.3995* (0.1585)	-0.6143* (0.2107)
γ_3				-0.2816* (0.1326)	-0.2313* (0.0867)	-0.1710* (0.0755)	-0.2503* (0.0844)
δ					-0.0890* (0.0308)		-0.0960* (0.0243)
LR	0.5551 [0.4566]	0.0092 [0.9233]	0.0009 [0.9750]	7.3333* [0.0070]	8.7611* [0.0032]	8.9238* [0.0029]	11.5363* [0.0007]
Function value	-986.0413	-867.6055	-861.5766	-846.5214	-846.3065	-848.2876	-844.8263
***LB Q* (6)**	4.2322 [0.6452]	8.0472 [0.2346]	4.8343 [0.5652]	5.5430 [0.4762]	5.9473 [0.4291]	6.4416 [0.3755]	5.3606 [0.4984]
***LB Q* (12)**	9.1435 [0.6906]	11.2388 [0.5085]	7.4793 [0.8243]	10.4005 [0.5808]	10.8923 [0.5381]	11.2514 [0.5075]	10.4588 [0.5757]
***LB* Q^2 (6)**	5.2126 [0.5168]	1.7684 [0.9397]	2.1661 [0.9038]	6.0501 [0.4176]	5.6035 [0.4690]	4.9441 [0.5509]	5.9228 [0.4318]
***LB* Q^2 (12)**	26.5926* [0.0088]	5.6750 [0.9315]	5.1228 [0.9537]	13.6767 [0.3218]	11.0833 [0.5217]	8.0936 [0.7777]	9.8225 [0.6315]
Skewness	-0.3502* [0.0010]	-0.1422 [0.1829]	-0.1263 [0.2366]	-0.0621 [0.5608]	-0.0529 [0.6199]	-0.0391 [0.7137]	-0.0304 [0.7753]
Kurtosis	2.0828* [0.0000]	0.0651 [0.7611]	0.1234 [0.5649]	-0.3234 [0.1314]	-0.3023 [0.1585]	-0.3657 [0.1011]	-0.3348 [0.1183]
Normality	106.4392* [0.0000]	1.8770 [0.3911]	1.7441 [0.4180]	2.6456 [0.2663]	2.2622 [0.3226]	2.9756 [0.2258]	2.5530 [0.2789]

Note: Column (1) without outlier corrected, column (2)-(7) with outlier corrected. Standard errors appear in parentheses; p-values appear in brackets; *LB* $Q(k)$ and *LB* $Q^2(k)$ equal Ljung-Box Q-statistics, testing for standardized residuals and squared standardized residuals for autocorrelations up to k lags, where the degrees of freedom are reduced by the number of estimated coefficients in the mean equation. LR equals the likelihood ratio statistic, following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 6:EGARCH Model Estimation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a_0	0.4474* (0.0749)	0.4774* (0.0697)	0.6217* (0.1155)	0.5771* (0.1284)	0.5885* (0.1237)	0.2919 (0.2335)	0.3057 (0.2236)
a_1	0.4323* (0.0543)	0.3499* (0.0455)	0.4476* (0.0626)	0.4245* (0.0622)	0.4493* (0.0609)	0.4377* (0.0635)	0.4582* (0.0612)
a_2	0.0402 (0.0544)	0.0949* (0.0468)	-0.0489 (0.0642)	-0.0315 (0.0657)	-0.0507 (0.0645)	-0.0373 (0.0652)	-0.0515 (0.0643)
a_3	-0.0042 (0.0471)	-0.0021 (0.0473)	0.0912 (0.0665)	0.1125 (0.0714)	0.1222** (0.0683)	0.1189** (0.0650)	0.1236** (0.0694)
a_4	-0.0699 (0.0456)	-0.0535 (0.0437)	-0.1927* (0.0632)	-0.1937* (0.0667)	-0.2198* (0.0616)	-0.1964* (0.0650)	-0.2282* (0.0607)
b_0			-0.2784* (0.1373)	-0.1398 (0.1491)	-0.1393 (0.1431)	0.0292 (0.1891)	0.0226 (0.1766)
b_1			-0.1288 (0.0887)	-0.1164 (0.0860)	-0.1569** (0.0861)	-0.1323 (0.0868)	-0.1626** (0.0857)
b_2			0.2432* (0.0892)	0.2060* (0.0910)	0.2280* (0.0897)	0.2114* (0.0905)	0.2275* (0.0895)
b_3			-0.1538** (0.0888)	-0.1958* (0.0927)	-0.2060* (0.0902)	-0.2050* (0.0916)	-0.2119* (0.0907)
b_4			0.2560* (0.0863)	0.2439* (0.0877)	0.2665* (0.0836)	0.2503* (0.0865)	0.2773* (0.0830)
λ						0.1848 (0.1248)	0.1882 (0.1251)
α_0	-0.1967* (0.0312)	-0.1679* (0.0313)	-0.1475* (0.0301)	0.0163 (0.0635)	-0.0256 (0.0685)	0.0119 (0.0600)	-0.0196 (0.0660)
α_1	0.2714* (0.0474)	0.2145* (0.0405)	0.1873* (0.0384)	0.1471* (0.0668)	0.1306** (0.0715)	0.1489* (0.0623)	0.1362* (0.0658)
α_2	-0.0663** (0.0380)	-0.0742* (0.0298)	-0.1046* (0.0305)	-0.1120* (0.0349)	-0.2074* (0.0886)	-0.1151* (0.0356)	-0.2096* (0.0825)
β_1	0.9910* (0.0075)	0.9847* (0.0096)	0.9858* (0.0091)	0.8171* (0.0657)	0.8321* (0.0683)	0.8224* (0.0585)	0.8191* (0.0617)
γ_1				0.1236* (0.0577)	0.1125* (0.0558)	0.1066* (0.0518)	0.1008* (0.0527)
γ_2				-0.2350* (0.0864)	-0.2114* (0.0933)	-0.2263* (0.0770)	-0.2254* (0.0840)
γ_3				-0.2918* (0.1017)	-0.2559* (0.0890)	-0.2647* (0.0899)	-0.2448* (0.0828)
δ					0.0446 (0.0341)		0.0446 (0.0310)
LR	1.4035 [0.2367]	2.4839 [0.1156]	2.3977 [0.1221]	7.7313* [0.0056]	6.0353* [0.0144]	9.1857* [0.0026]	8.5627* [0.0036]
Function value	-986.3654	-865.5957	-857.5925	-843.9671	-842.5171	-842.7940	-841.2278
***LB Q* (6)**	4.6256 [0.5926]	8.2200 [0.2224]	5.5077 [0.4805]	5.5687 [0.4731]	4.8422 [0.5642]	5.4986 [0.4816]	4.8398 [0.5645]
***LB Q* (12)**	10.1283 [0.6047]	11.4836 [0.4879]	8.2297 [0.7669]	10.5937 [0.5640]	9.7879 [0.6345]	10.6308 [0.5607]	9.8977 [0.6249]
***LB* Q^2 (6)**	5.4832 [0.4834]	2.3737 [0.8823]	1.9877 [0.9208]	4.6772 [0.5858]	3.8734 [0.6938]	4.3816 [0.6251]	3.6731 [0.7208]
***LB* Q^2 (12)**	34.1805* [0.0006]	6.5298 [0.8870]	4.4710 [0.9733]	11.0131 [0.5277]	9.5219 [0.6578]	9.3488 [0.6728]	8.5080 [0.7442]
Skewness	-0.4115* [0.0001]	-0.1461 [0.1713]	-0.1297 [0.2245]	-0.0697 [0.5300]	-0.0839 [0.4500]	-0.0684 [0.5380]	-0.0842 [0.4522]
Kurtosis	2.3218* [0.0000]	-0.0268 [0.9002]	-0.0438 [0.8379]	-0.3516 [0.1150]	-0.3613 [0.1053]	-0.3578 [0.1087]	-0.3621 [0.1074]
Normality	133.7566* [0.0000]	1.8978 [0.3871]	1.5257 [0.4663]	2.9162 [0.2326]	3.2349 [0.1983]	2.9912 [0.2241]	3.1971 [0.2021]

Note: See Table 5.
* denotes 5-percent significance level.
** denotes 10-percent significance level.

Table A1: Outlier Information

Panel A. Descriptive Statistics

	Obs.	Mean	SD	Q1	Q2	Q3	IQD	f_1	f_3	F_1	F_3
Quarterly GDP Growth	533	0.8228	2.2383	0.0784	0.8632	1.8444	1.7660	-2.5706	4.4934	-5.2196	7.1424

Panel B. Frequency of events:

$\lvert y_t - \text{Mean} \rvert > k \cdot \text{SD}$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
Quarterly GDP Growth	**1932Q1**	**1933Q1**	**1893Q3**	–
	1930Q1	**1933Q3**		
	1934Q1	**1908Q1**		
	1935Q4	**1918Q2**		
	1941Q2	**1879Q4**		
	1942Q4	**1919Q1**		
	1934Q2	**1945Q4**		
	1888Q1	**1933Q4**		
	1931Q3	**1945Q3**		
	1876Q1	**1937Q4**		
	1938Q3	**1893Q3**		
	1918Q4			
	1946Q1			
	1896Q1			
	1938Q1			
	1934Q3			
	1936Q2			
	1935Q1			
	1897Q1			
	1939Q4			
	1914Q4			
	1921Q1			
	1930Q4			
	1929Q4			
	1901Q1			
	1932Q2			
	1930Q3			
	1899Q1			
	1920Q2			
	1931Q4			
	1933Q2			
	1891Q3			
	1920Q4			
	1907Q4			
	1933Q1			
	1933Q3			
	1908Q1			
	1918Q2			
	1879Q4			
	1919Q1			
	1945Q4			
	1933Q4			
	1945Q3			
	1937Q4			
	1893Q3			

Table A2: Outlier Information

Date	$y_t \le f_1$
1893Q3	-8.75653
1937Q4	-7.44208
1945Q3	-7.40205
1933Q4	-7.29332
1945Q4	-7.14014
1919Q1	-6.4836
1908Q1	-6.03746
1933Q1	-5.99004
1907Q4	-5.69925
1920Q4	-5.62506
1931Q4	-5.26931
1920Q2	-5.14992
1930Q3	-5.01223
1932Q2	-5.00756
1929Q4	-4.82265
1930Q4	-4.74572
1921Q1	-4.65817
1914Q4	-4.62649
1934Q3	-4.34376
1938Q1	-4.31652
1896Q1	-4.26888
1946Q1	-4.2563
1918Q4	-4.10921
1931Q3	-4.00449
1888Q1	-3.96425
1930Q1	-3.69189
1932Q1	-3.67096
1932Q3	-3.55135
1946Q2	-3.3992
1893Q4	-3.3346
1924Q2	-3.19257
1903Q4	-3.10203
1940Q1	-2.93112
1958Q1	-2.71872

Date	$y_t \ge f_3$
1918Q1	4.512044
1915Q4	4.575253
1894Q4	4.597397
1922Q2	4.727923
1924Q4	4.981125
1941Q3	5.132264
1934Q1	5.365708
1935Q4	5.402271
1941Q2	5.439613
1942Q4	5.464017
1934Q2	5.573941
1876Q1	5.66863
1938Q3	5.678989
1936Q2	6.081649
1935Q1	6.192347
1897Q1	6.21144
1939Q4	6.263423
1901Q1	6.627313
1899Q1	6.770195
1933Q2	6.961935
1891Q3	7.12518
1933Q3	7.649726
1918Q2	7.818015
1879Q4	7.964976

Date	y_t	$\lvert y_t - \text{Mean} \rvert$
1932Q1	-3.67096	4.493761953
1930Q1	-3.69189	4.514693847
1934Q1	5.365708	4.542908248
1935Q4	5.402271	4.579470987
1941Q2	5.439613	4.616813279
1942Q4	5.464017	4.641217004
1934Q2	5.573941	4.751141175
1888Q1	-3.96425	4.787049654
1931Q3	-4.00449	4.827290495
1876Q1	5.66863	4.84583036
1938Q3	5.678989	4.856188877
1918Q4	-4.10921	4.932006244
1946Q1	-4.2563	5.079102959
1896Q1	-4.26888	5.091681146
1938Q1	-4.31652	5.139317996
1934Q3	-4.34376	5.16655683
1936Q2	6.081649	5.258848876
1935Q1	6.192347	5.369547142
1897Q1	6.21144	5.388640261
1939Q4	6.263423	5.440622852
1914Q4	-4.62649	5.449290714
1921Q1	-4.65817	5.480966947
1930Q4	-4.74572	5.568515057
1929Q4	-4.82265	5.645451233
1901Q1	6.627313	5.8045127
1932Q2	-5.00756	5.830355026
1930Q3	-5.01223	5.835033189
1899Q1	6.770195	5.947395048
1920Q2	-5.14992	5.97272059
1931Q4	-5.26931	6.092113299
1933Q2	6.961935	6.139135088
1891Q3	7.12518	6.302380216
1920Q4	-5.62506	6.447861121
1907Q4	-5.69925	6.52205031
1933Q1	**-5.99004**	**6.812839527**
1933Q3	**7.649726**	**6.826925919**
1908Q1	**-6.03746**	**6.860256347**
1918Q2	**7.818015**	**6.995214788**
1879Q4	**7.964976**	**7.142176493**
1919Q1	**-6.4836**	**7.306395305**
1945Q4	**-7.14014**	**7.962940516**
1933Q4	**-7.29332**	**8.116115799**
1945Q3	**-7.40205**	**8.22484672**
1937Q4	**-7.44208**	**8.264884475**
1893Q3	**-8.75653**	**9.579333878**

Table A3: Outlier Corrected by Franses and Ghijsels (1999) Additive Outlier Detection

Quarterly GNP Growth AR(4)-GARCH(1,1)

Location	Date	Value	$k = 3$	Size	Lambda
231	1933Q4	-7.29332	※	107.488	12.6302
70	1893Q3	-8.75653	※	71.55	9.10659
229	1933Q2	6.961935		55.6713	7.5209
278	1945Q3	-7.40205	※	50.7756	7.14605
247	1937Q4	-7.44208	※	49.6185	7.29989
62	1891Q3	7.12518		44.1903	6.79465
234	1934Q3	-4.34376		33.2772	5.19547
232	1934Q1	5.365708		33.6301	5.40379
100	1901Q1	6.627313		32.1756	5.32333
236	1935Q1	6.192347		32.1198	5.43303
15	1879Q4	7.964976	※	31.5535	5.50271
92	1899Q1	6.770195		30.9937	5.54228
84	1897Q1	6.21144		30.7512	5.67833
80	1896Q1	-4.26888		29.7588	5.68557
127	1907Q4	-5.69925		29.1644	5.76458
256	1940Q1	-2.93112		28.9075	5.82894
171	1918Q4	-4.10921		28.5819	6.01163
215	1929Q4	-4.82265		27.7132	6.0588
228	1933Q1	-5.99004	※	25.6877	5.78083
195	1924Q4	4.981125		24.9043	5.71216
169	1918Q2	7.818015	※	22.3191	5.31931
172	1919Q1	-6.4836	※	22.4206	5.478
177	1920Q2	-5.14992		22.4714	5.65842
222	1931Q3	-4.00449		20.9017	5.43069
48	1888Q1	-3.96425		20.5925	5.45283
179	1920Q4	-5.62506		18.9762	5.18591
218	1930Q3	-5.01223		18.3766	5.18073
193	1924Q2	-3.19257		17.7098	5.05962
279	1945Q4	-7.14014	※	17.2481	5.07236
241	1936Q2	6.081649		17.2423	5.20527
128	1908Q1	-6.03746	※	16.5971	5.17551
250	1938Q3	5.678989		16.3673	5.25823
155	1914Q4	-4.62649		16.3767	5.4062
255	1939Q4	6.263423		16.6695	5.60622
223	1931Q4	-5.26931		16.395	5.74878
230	1933Q3	7.649726	※	16.2552	5.87036(cv)
111	1903Q4	-3.10203		13.8871	5.11026
296	1950Q1	4.03606		12.0111	4.52453
181	1921Q2	2.549648		12.0458	4.6522
165	1917Q2	3.497603		11.3617	4.48978
254	1939Q3	3.6776		10.9287	4.37755
261	1941Q2	5.439613		10.4382	4.23318
60	1891Q1	-2.43559		10.2445	4.23973

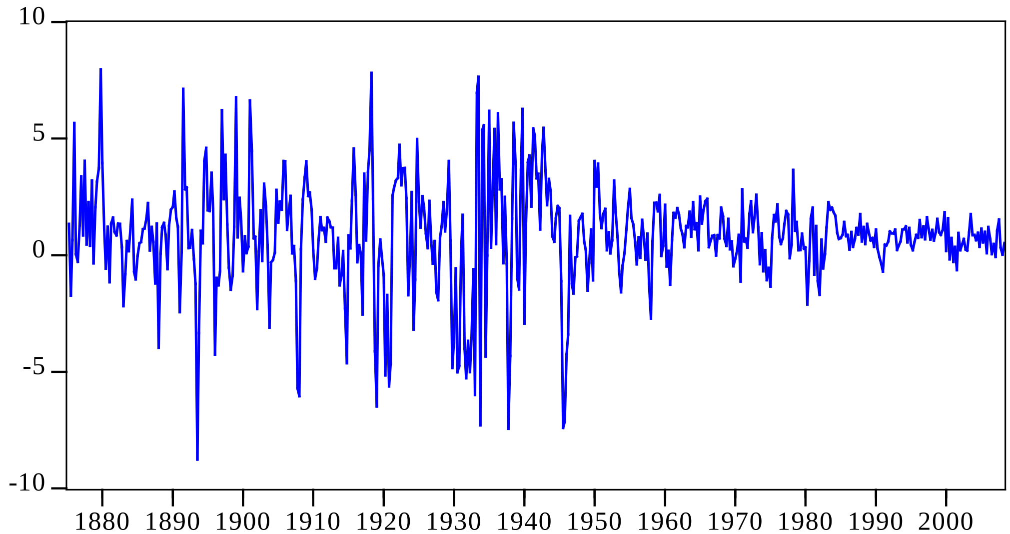

Figure 1. Real GNP Growth Rate