University of Connecticut

Department of Economics Working Paper Series

Expected Currency Excess Returns and International Business Cycles

Sanglim Lee
University of Connecticut

Working Paper 2012-15

August 2012

365 Fairfield Way, Unit 1063
Storrs, CT 06269-1063
Phone: (860) 486-3022
Fax: (860) 486-4463
http://www.econ.uconn.edu/

Expected Currency Excess Returns and International Business Cycles*

Sanglim Lee†

September 6, 2012

Abstract

It is well known that the uncovered interest parity condition does not hold empirically, implying that investments in high-interest rate currencies in foreign currency markets result in a positive expected excess return. Verdelhan (2010) successfully explains this phenomenon by referring to exogenous consumption processes and external habit formation. In this paper, I extend his model by using an international real business cycle model (Backus, Kehoe, and Kydland 1994) with internal habit formation. When the production-based stochastic discount factor is used, this benchmark model, driven by total factor productivity, accounts for this empirical evidence as well.

JEL Classification: E32, E44, F31, F44

Key Words: Currency Excess Return, Real Business Cycle, Forward Premium Puzzle

---

*I am grateful to Christian Zimmermann for his valuable guidance and comments. I would also like to thank Francis Ahking and Uluc Aysun for their helpful discussions and feedback. All existing errors are my own.

†Department of Economics, University of Connecticut-Storrs, sanglim.lee@uconn.edu

1 Introduction

The term *currency carry trade* refers to an investment strategy in foreign exchange markets in which an investor borrows from a low-interest currency, such as the Japanese yen, and uses the funds to invest in a high-interest currency, such as the Australian dollar. Several studies have revealed that this strategy has been consistently profitable over the last three decades (Lustig and Verdelhan 2007; Burnside et al. 2010). However, according to uncovered interest rate parity (UIP), this arbitrage investment scheme should not be successful. The UIP condition implies that two assets that are identical in every way except for their currency denominations should have the same returns, as the interest differential between two assets should be offset by the expected change in the exchange rate between the two countries.

In this paper, I study this discrepancy between the positive expected currency excess returns observed in the empirical data and the zero-excess returns implied by the UIP condition, and I explore what accounts for this anomaly in foreign exchange markets. Verdelhan (2010) successfully explains the expected excess returns by referring to exogenous consumption processes and external habit formation. His model is a consumption-based asset pricing model and thus a partial equilibrium framework in the sense that the consumption process is exogenous and the production sector is not considered.

However, it would be worthwhile to investigate whether stylized facts regarding both the foreign exchange markets and international business cycles can be successfully replicated in the general equilibrium framework. Motivated by this idea, I extend his model by using a two-country, two-good international real business cycle (IRBC) model (Backus, Kehoe, and Kydland 1994, henceforth BKK) with internal habit formation, and I explore whether the benchmark model (the standard BKK model plus internal habit formation), which is driven by total factor productivity (TFP) shocks, can reproduce the empirical evidence for both expected excess returns in foreign exchange markets and international business cycle properties.

For comparison purposes (the standard BKK model vs. the benchmark model), I set the calibration parameters to be similar to those in BKK (1994). The model simulation results show that the standard BKK model cannot produce the expected excess return. However, the success of the benchmark model is mixed. When the consumption-based stochastic discount factor (SDF) is used, the benchmark model also fails to generate the expected currency excess returns. However, when production-based SDF is used, it successfully generates the expected excess returns.

The role of the internal habit formation is essential in the benchmark model, thus explaining the result of the expected excess returns being the risk premium. Intuitively, the higher habit formation causes the investors to be more risk averse, since a high level of habit formation forces investors to smooth consumption a lot, as they are averse to volatility of consumption induced by TFP shocks. The sensitivity analysis in Section 4.6 supports this intuition by showing that the propensity to be more risk averse caused by higher habit formation yields higher expected excess returns. This result implies that the expected excess returns can be justified by the risk-return trade-off principle.

Several recent papers have stated that the average excess currency returns of the carry trade are not temporary anomalies that can be arbitraged away. Lustig and Verdelhan (2007) compare the performance of a levered carry-trade strategy with the performance of an un-levered buy-and-hold strategy in the U.S. stock market. The carry-trade strategy involves taking short positions in a portfolio of low-interest rate currencies and long positions in a portfolio of high-interest rate currencies. They show that the carry trade consistently outperforms the stock market over the sample period between 1971 and 2005.

Burnside et al. (2010) also present a similar result, showing that the Sharpe ratio of the carry trade is substantially larger than that of the U.S. stock market from 1976 to 2008 (0.982 vs. 0.443, respectively). While the average excess return of the U.S. stock market is larger than the payoff of the carry trade (0.065 vs. 0.050, respectively), the excess returns of the U.S. stock market are much more volatile than the excess return of the carry trade (0.148 vs. 0.051, respectively).

It is well known in the literature that these average excess currency returns of the carry trade are possible due to the violation of the UIP condition. A negative slope coefficient in the UIP regression implies that when an individual currency has a higher interest differential than its average, it tends to appreciate rather than depreciate. This feature makes it possible for the carry trade return to be profitable. Thus, one of the purposes of this paper is to see whether the benchmark model delivers the negative slope coefficient in the UIP regression.

1.1 Literature Review

There is a sizable amount of literature that examines the expected currency excess returns. Hansen and Hodrick (1980) is the first study to document the deviation from the UIP hypothesis. Engel (1996) provides a comprehensive survey of the literature and shows that the coefficient estimates in the majority of empirical studies on the UIP hypothesis are significantly different from one and are even negative.

Fama (1984) argues that the forward bias may be caused by a time-varying risk premium that is more volatile than the expected rate of currency depreciation. However, Lewis (1995) states that the risk aversion level of agents would have to be unrealistically high to explain the issue of the forward bias with a risk premium. In this sense, the empirical anomaly can be seen as another version of the equity premium puzzle, which is first discussed by Mehra and Prescott (1985). This means that the traditional consumption-based asset pricing models face limitations and fail to explain the violation of the UIP condition and the forward bias puzzle. Several recent studies, however, allow for departures from traditional settings.

Moore and Roche (2002) examine whether stylized facts on forward and spot rates can be reproduced by a standard Cash-in-Advance (CIA) model that incorporates the concept of external habit persistence proposed by Campbell and Cochrane (1999). Although they are able to explain most of the volatility and persistence of forward and spot rates, they fail to reproduce the negative Fama slope coefficient.

Lustig and Verdelhan (2007) argue that the aggregate consumption risk can explain the carry trade premium on a large sample of countries over the period of 1952 to 2002. By forming portfolios of currencies based on interest rates, they find that the high interest rate currency returns are strongly procyclical, while low-interest rate currency returns are acyclical. Because high interest currencies co-move more with U.S. consumption growth risk than low interest currencies do, they argue that higher interest rate currencies expose U.S. investors to much more U.S. consumption growth risk, thus requiring that these currencies yield higher returns.

Verdelhan (2010) also incorporates external habit formation to explain the carry trade return. In his model, the risk premium is time varying such that the risk premium on foreign currency is high in bad times and low in good times. By using the external habit preferences proposed by Campbell and Cochrane (1999), Verdelhan (2010) uses time-varying countercyclical risk aversion and risk premium to explain the UIP puzzle.

However, in the literature, most of the studies use a partial equilibrium framework in that consumption process is assumed to be exogenous.[1] In this paper, however, I con-

[1] In the literature, Farhi and Gabaix (2011) refer to rare disaster risks and Alvarez, Atkeson, and

sider adding production sectors into the model to determine whether the IRBC model driven by TFP shocks is able to generate the expected currency excess returns. In Section 2, I present the benchmark model. In Section 3, I explain calibration parameters. In Section 4, I discuss the simulation results and conduct sensitivity analysis. In Section 5, I summarize the main results and conclude the paper.

2 The Model

2.1 Suggestions for the Model

Before introducing the benchmark model, in this section, I would like to briefly explain why I study the currency excess return by using an IRBC model with internal habit formation. First, as mentioned previously, Verdelhan's model is a consumption-based asset-pricing model. It is a partial equilibrium framework in the sense that the consumption process is exogenous and the production sector is not considered. However, knowledge of whether stylized facts regarding the business cycle and the financial markets can be successfully replicated in the general equilibrium framework would be more useful. For this reason, I adopt an IRBC model in which the consumption process is endogenously determined and the production sector is included. Accordingly, while consumption growth shocks explain the risk premium in Verdelhan's consumption-based asset-pricing model, TFP shocks explain the currency excess return in the general equilibrium model. Moreover, it is intuitively more plausible that TFP shocks induce counter-cyclical risk aversion and risk premium rather than consumption growth shocks, because TFP is regarded as the fundamental driver of macroeconomic fluctuations.

Kehoe (2009) use endogenously segmented markets to explain the forward premium puzzle in a general equilibrium framework.

Second, I follow Verdelhan (2010) in incorporating habit formation into the utility function. The concept of habit formation implies that agents are averse to variations in habit-adjusted levels of consumption rather than to variations in consumption itself. Hence, small changes in consumption growth can generate very large variations in habit-adjusted consumption growth, and it may contribute to a resolution of the equity premium puzzle as formulated by Mehra and Prescott (1985). While the constant relative risk aversion (CRRA) preference used in Mehra and Prescott (1985) requires a very high degree of risk aversion, resulting in a counterfactual high risk-free rate, habit formation can help explain the currency excess return with moderate risk aversion. The difference between the present study and that of Verdelhan (2010) is that, while he uses external habit levels that depend on aggregate consumption, I use internal habit levels that depend on individual consumption. While external habit levels can be interpreted as a social externality, internal habit levels do not exert any externality. Thus, the internal habit assumption enables the social planner solution to be identical with the decentralized solution according to the second welfare theorem. The social planner solution is useful in the computation of the model's equilibrium.

Third, the IRBC model can be used to study both asset prices and business cycle phenomena. In this sense, my study is in part related to Jermann (1998); he combines habit formation and capital adjustment costs in a standard one-sector model and reproduces stylized facts related to the equity premium in stock markets and business cycle statistics. Similarly, in this paper, I use habit formation and capital adjustment costs in a standard IRBC model and study currency excess returns in foreign exchange markets and business cycle properties.

I use the two-country, two-good IRBC model proposed by Backus, Kehoe, and Kydland (1994). The main difference between this and a standard BKK model is that the benchmark model includes habit formation in household preferences and capital adjustment costs in capital accumulation. Each country produces a different intermediate good with its own specific technology. The intermediate good firms in a domestic country produce one good labeled a, while the intermediate good firms in a foreign country produce another good labeled b. The final goods in each country consist of bundles of both intermediate goods a and b from the two countries, and the final goods are used for consumption and investment. In what follows, I mainly describe the domestic country, since it is assumed that the world economy consists of two symmetric countries. Foreign country variables are denoted by an asterisk.

2.2 Preferences

Each household derives utility from both consumption and leisure. In addition, each household has internal consumption habits that operate on lagged household consumption. This is unlike external habits or the catching-up-with-the-Joneses specification developed by Abel (1990), which assumes lags of aggregate consumption in utility. In comparison with external consumption habit, internal consumption habit does not exert any externality. Under the internal consumption habit assumption, the period utility for a household is given by:

$$U(C_t, C_{t-1}, 1 - L_t) = \frac{1}{1-\gamma}[(C_t - hC_{t-1})^{\mu}(1 - L_t)^{1-\mu}]^{1-\gamma}, \tag{1}$$

where C_t is consumption, L_t is labor, h is habit intensity, μ is consumption share, and γ is risk aversion. I assume complete asset markets in this economy, so there are complete contingent claims denominated in the units of one of the countries' tradable goods. In each period t, the economy experiences one event $s_t \in S$ in which S is a possibly infinite set and s^t is the history of events up to time t. Let $B(s^t, s_{t+1})$ be the quantity of bonds purchased by households after history s^t that pay one unit of good a in period $t+1$ if and only if the state of the economy is s_{t+1}. Let $Q(s^t, s_{t+1})$ be the price in units of good a of these bonds. When markets are complete, a consumer's total resources are given by his or her factor income and his or her income from bonds. Then the budget constraint for the representative household is:

$$\begin{aligned} & C(s^t) + I(s^t) + q^a(s^t) \sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \\ & \quad = q^a(s^t)[w(s^t)L(s^t) + r(s^t)K(s^t)] + q^a(s^t)B(s^{t-1}, s_t). \end{aligned} \tag{2}$$

where $q^a(s^t)$ is the price of good a.

The capital stocks evolve according to the following dynamic equation:

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2}(K_{t+1} - K_t)^2, \tag{3}$$

where I_t is the investment, and δ is depreciation. The term $\frac{\phi}{2}(K_{t+1} - K_t)^2$ represents the capital adjustment costs used by Mendoza (1991, 1995). In general, these are introduced to moderate the variability of capital accumulation. However, Jermann (1998) states that they can also help improve asset-pricing implications when habit formation is used in the standard real business cycle model. Because habit formation induces excess smoothness in consumption, it is difficult to induce substantial risk premia (Rouwenhorst 1995). This is because agents can easily alter their production plans to smooth their consumption under a habit formation framework. However, the introduction of some inflexibility into the production sector, such as capital adjustment costs, resolves this problem.[2]

[2]To incorporate factor market inflexibility, which reduces the elasticity of capital supply, Jermann (1998) uses convex adjustment costs, while Boldrin et al. (2002) assume two-sector technology with limited inter-sectoral factor mobility.

The household in each country maximizes utility in Equation (3) subject to the budget constraint in Equation (4) and the law of motion for capital in Equation (5).

2.3 Technology

There are two sectors in each country: a traded intermediate goods sector and a non-traded final goods sector. Each country specializes in the production of an intermediate good,[3] and households supply labor and rent capital to firms producing intermediate goods. The intermediate good that firms produce is determined according to a Cobb-Douglas production function that takes capital and labor as inputs. The maximization problem for intermediate good firms is given by:

$$\max_{L_t, K_t} Y_t - w_t L_t - r_t K_t, \tag{4}$$

subject to the production function:

$$Y_t = z_t K_t^{\theta} L_t^{1-\theta}, \tag{5}$$

where K_t is capital, w_t and r_t are wages and rental rates, and z_t is TFP.

The law of motion for TFP is given by:

$$z_{t+1} = A z_t + \epsilon_{t+1}, \tag{6}$$

where

$$z_{t+1} = \begin{pmatrix} z_{1t+1} \\ z_{2t+1} \end{pmatrix}, \tag{7}$$

[3]As mentioned before, the domestic country specializes in the production of an intermediate good a while the foreign country specializes in the production of an intermediate good b.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{8}$$

$$\epsilon_{t+1} = \begin{pmatrix} \epsilon_{1t+1} \\ \epsilon_{2t+1} \end{pmatrix}, \tag{9}$$

$$\epsilon_{t+1} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_2 \end{bmatrix} \right). \tag{10}$$

As seen in Equations (8) to (12), the technology shocks for the two countries follow a first-order vector autoregressive (VAR) process. A is an autoregressive coefficient matrix, and ϵ_{t+1} is a bivariate vector of normal random variables with a zero-mean and a variance-covariance matrix Ω. The diagonal elements of A determine the degree of persistence in productivity, and the off-diagonal elements of A incorporate a spillover effect between the innovations of the two countries. If the off-diagonal elements of Ω are nonzero, this allows for correlation of contemporaneous shocks between two countries.

After intermediate goods are produced, households trade the intermediate goods in a frictionless international market. Some of the intermediate goods are used domestically for the production of final goods, while the rest are exported.[4] The final good firms produce their goods by combining domestic and foreign intermediate goods using a form of technology with constant returns to scale or an Armington aggregator. The maximization problem for the final good firms is given by:

$$\max_{a_t, b_t} F_t - q_t^a a_t - q_t^b b_t, \tag{11}$$

subject to the production function:

[4]In Section (2.3), a^* represents the domestic intermediate good exported to the foreign country, while b^* represents the foreign intermediate good exported to the domestic country.

$$F_t = [w_1 a_t^{(\sigma-1)/\sigma} + w_2 b_t^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}, \tag{12}$$

where ω_1 determines the home bias in the composition of final goods. The parameter σ denotes the elasticity of substitution between intermediate goods a and b. q_t^a and q_t^b are the respective prices of goods a and b in units of the final good. The final goods can be either consumed or invested.

2.4 Equilibrium

An equilibrium is an allocation and a set of prices for all s^t and for all $t \geq 0$, such that, given these prices, all agents and both types of firms optimize their behavior, while markets clear.

Market clearing for intermediate goods *a* and *b* requires that:

$$Y_t = a_t + a_t^*, \tag{13}$$

$$Y_t^* = b_t + b_t^*. \tag{14}$$

Market clearing for final goods requires that:

$$F_t = C_t + I_t, \tag{15}$$

$$F_t^* = C_t^* + I_t^*. \tag{16}$$

In the complete market framework, market clearing for bonds requires that:

$$B(s^t, s_{t+1}) + B(s^t, s_{t+1})^* = 0, \forall s_{t+1} \in S. \tag{17}$$

Because there are no distortions and the bond market is complete, the social planner solution is identical to the decentralized solution according to the second welfare theorem. Thus, the equilibrium allocation in this economy can be computed as the solution to the social planner's problem.

The planner's problem can be written in recursive form as:

$$V(K_t, K_t^*, z_t, z_t^*) = \tag{18}$$
$$\max_{D_t} \left\{ U(C_t, C_{t-1}, 1 - L_t) + U(C_t^*, C_{t-1}^*, 1 - L_t^*) + \beta E_t[V(K_{t+1}, K_{t+1}^*, z_{t+1}, z_{t+1}^*)] \right\},$$

subject to:

$$z_t K_t^{\theta} L_t^{1-\theta} = a_t + a_t^*, \tag{19}$$

$$z_t^* K_t^{*\theta} L_t^{*1-\theta} = b_t + b_t^*, \tag{20}$$

$$F_t = C_t + K_{t+1} - (1-\delta)K_t + \frac{\phi}{2}(K_{t+1} - K_t)^2, \tag{21}$$

$$F_t^* = C_t^* + K_{t+1}^* - (1-\delta)K_t^* + \frac{\phi}{2}(K_{t+1}^* - K_t^*)^2. \tag{22}$$

where D_t is the vector of the social planner's control variables, which are given by $\{C_t, C_t^*, L_t, L_t^*, K_{t+1}, K_{t+1}^*, a_t, a_t^*, b_t, b_t^*\}$.

I now turn to the definition of the real exchange rate and risk-free rates to express the expected currency excess return.[5] In general, the real exchange rate q_t is expressed as:

[5]While most papers test the UIP condition on nominal variables, I use real variables, as in Verdelhan (2010), for comparison purposes. According to Verdelhan, recent studies reveal that real variables are more relevant in explaining the empirical evidence for the UIP condition: Various measures of CPI inflation are unrelated to currency returns (Hollifield and Yaron 2003); real aggregate consumption growth risk is priced on currency markets (Lusting and Verdelhan 2007).

$$RER_t = q_t = \frac{s_t \times P^*}{P}, \tag{23}$$

where s_t is the nominal exchange rate defined as units of domestic (e.g., U.S.) currency per unit of foreign (e.g., Euro) currency, P_t^* is the foreign price of a foreign consumption basket, and P_t is the domestic price of a domestic consumption basket. Following Chari, Kehoe, and McGrattan (2002), the real exchange rate can be defined as:

$$RER_t = q_t = \frac{U_c(C_t^*, C_{t-1}^*, 1 - L_t^*)}{U_c(C_t, C_{t-1}, 1 - L_t)}. \tag{24}$$

As seen, the real exchange rate in the model is the ratio of the marginal utilities of consumption of households in the two countries. This relationship is developed by the no-arbitrage condition for bond holdings between domestic and foreign consumers. Although the law of one price holds, the real exchange rate does not identically equal one because the consumption baskets differ across countries in this economy. An increase in q_t means that foreign goods become more expensive relative to U.S. goods. In this case, the U.S. dollar experiences a real depreciation, because households need more U.S. consumption baskets to purchase one foreign basket. Similarly, a decline in q_t is described as a real appreciation of the U.S. dollar.

Next, to obtain risk-free rates, the domestic and foreign stochastic discount factors (SDF) are recovered from equilibrium marginal rates of substitution derived from households' first-order conditions as:

$$M_{t+1} = \beta \frac{U_c(C_{t+1}, C_t, 1 - L_{t+1})}{U_c(C_t, C_{t-1}, 1 - L_t)}, \tag{25}$$

$$M_{t+1}^* = \beta \frac{U_c(C_{t+1}^*, C_t^*, 1 - L_{t+1}^*)}{U_c(C_t^*, C_{t-1}^*, 1 - L_t^*)}, \tag{26}$$

and accordingly the domestic and foreign risk-free rates which is yielded from risk-free bonds held from period t to $t+1$ are defined as:

$$r_t = \frac{1}{E_t(M_{t+1})}, \tag{27}$$

$$r_t^* = \frac{1}{E_t(M_{t+1}^*)}. \tag{28}$$

Finally, by using Equations (26), (29), and (30), expected currency excess returns rx_{t+1} can be defined as:

$$rx_{t+1} = \Delta q_{t+1} + r_t^* - r_t. \tag{29}$$

Equation (31) implies that the domestic investor buying the foreign bond gains the interest rate spread $r_t^* - r_t$. However, the investor experiences an expected loss if the expected value of q_{t+1} decreases or the foreign currency is expected to depreciate in real terms.

3 Calibration and Solution

Most of the parameters are set to be similar to those in the standard BKK (1994) model for comparison purposes (the standard BKK model vs. the benchmark model). As in BKK (1994), the preference discount factor β is 0.99, and the coefficient of relative risk aversion is 2. The share of consumption μ in the habit utility function is calculated at 0.3314, which is slightly different from 0.34 in BKK (1994) due to the habit formation in household preferences. The habit intensity h is set to 0.8, because studies on asset pricing (Constantinides 1990, Jermann 1998, Boldrin et al. 2001) find

Table 1: Calibration

Preference	Discount factor	$\beta = 0.99$
	Consumption share	$\mu = 0.3313$
	Risk aversion	$\gamma = 2$
	Habit intensity	$h = 0.8$
Technology	Capital share	$\theta = 0.36$
	Capital adjustment costs	$\phi = 0.285$
	Depreciation rate	$\delta = 0.025$
	Import share	$b/Y = 0.15$
	Elasticity of substitution	$\sigma = 1.5$
	between intermediate goods	
	Armington aggregator weights	$\omega_1 = 0.897$
		$\omega_2 = 0.282$
Estimated parameters	Productivity	$A = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$
	Std Dev innovations	$\sigma_1 = 0.00852$
		$\sigma_2 = 0.00852$
	Corr innovations	$\sigma_{1,2} = 0.2580$

that consumption habits characterized by values in the range of 0.7 to 0.9 can help explain the equity premium puzzle. Hence, I chose the middle value of the range. As in BKK (1994), the capital share in production is 0.36, the depreciation rate is 0.025, and the elasticity of substitution between domestic and foreign goods in the Armington aggregator is 1.5. The Armington aggregator weights ω_1 and ω_2 are calculated at 0.897 and 0.282, respectively, using an import share $b/Y = 0.15$ and $Y = 1$ at the steady state. I present its calculation details in the Appendix. The parameter ϕ for capital adjustment costs is set to 0.285 so that the investment volatility in the benchmark model is similar to that of the standard BKK model for comparison purposes. Finally, I adopt the estimated parameters for technology shocks used in BKK (1994). They estimate the parameters of the technology process in Equation (10) using Solow residuals for the U.S and for an aggregate of six European countries.[6] As presented in Table 2, shocks to productivity are highly persistent (a_{11}=a_{22}=0.906), and there is some evidence of spillover effect (a_{12}=a_{21}=0.088). In addition, the innovations to productivity are positively correlated across countries ($\sigma_{1,2} = 0.2580$). With the parameter values reported in Table (2), I solve the model numerically using second-order perturbation methods.

4 Results

4.1 Case of Consumption-Based SDF

In this section, I compare the standard BKK model with the benchmark model with the habit formation regarding impulse responses to determine which model is able to successfully account for the empirical evidence for expected excess returns.

[6] The six European countries are Austria, Finland, Germany, Italy, Switzerland, and the United Kingdom.

4.1.1 Impulse Response Analysis

Figures (1) and (2) show the impulse responses of the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} to a 1% increase in a domestic country's TFP shocks for each model.

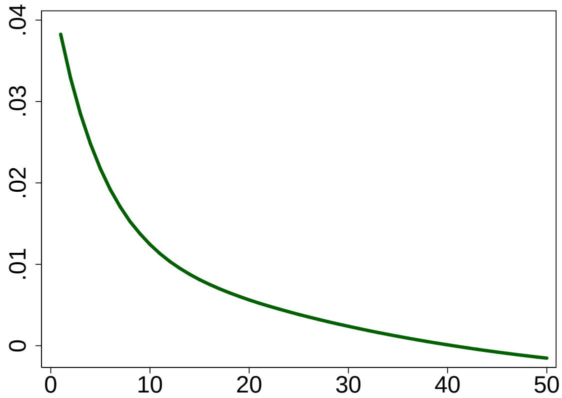
First, consider whether the impulse responses are consistent with the UIP condition presented in Equation (1). In the case of the standard BKK model shown in Figure (1), the interest rate spread $r_t - r_t^*$ is positive at first and then negative, finally converging at the zero line. Similarly, the expected change in the real exchange rate Δq_{t+1} is also positive at first and then negative, finally converging at the zero line.

These results show that the impulse responses in the BKK model are consistent with the UIP condition, because when domestic interest rates are relatively higher than foreign interest rates, domestic currency is expected to depreciate, and thus the interest rate spreads are offset by the expected change in the real exchange rate.[7]

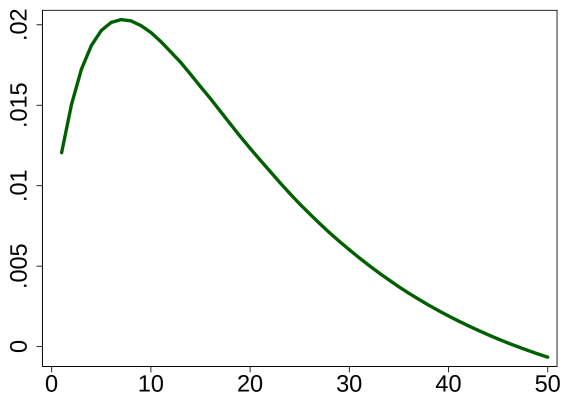
The case of the habit formation model shown in Figure (2) also exhibits that its impulse responses are consistent with the UIP condition. However, the main difference between this and Figure (1) is that the impulse responses of the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} react contrary to a 1% increase in a domestic country's TFP shocks. In other words, when consumption-based SDF is used in the habit formation model, domestic interest rates become countercyclical. This induces the interest rate spread $r_t - r_t^*$ to be negative. Since the expected change in the real exchange rate Δq_{t+1} is also negative, it supports the UIP condition by showing that, when domestic interest rates are relatively lower than foreign interest rates, domestic

[7] When Δq_{t+1} is negative, the real exchange q_t is expected to decrease at time $t+1$ (foreign currency depreciates). On the other hand, when Δq_{t+1} is positive, the real exchange q_t is expected to increase at time $t+1$ (foreign currency appreciates).

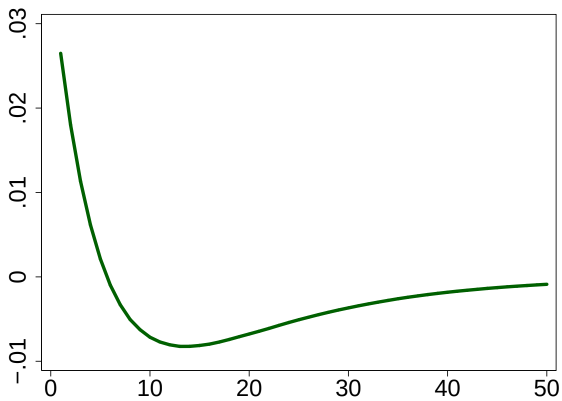
Figure 1: Impulse Responses for 1% Productivity Shock in Country 1 (BKK with a Consumption-Based SDF, $h = 0$ and $\phi = 0$)



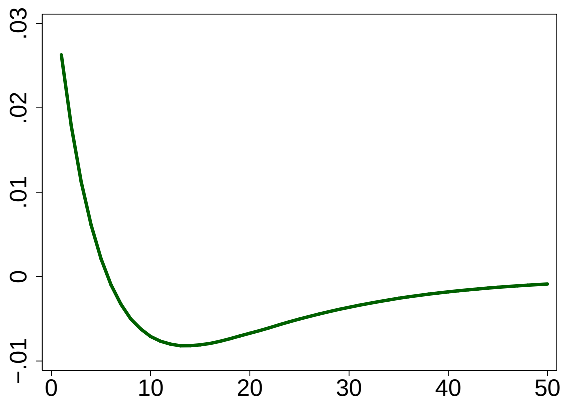
(a) r_t



(b) r_t^*

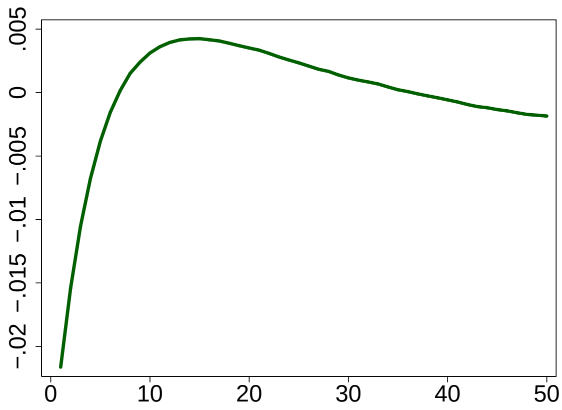


(c) $r_t - r_t^*$

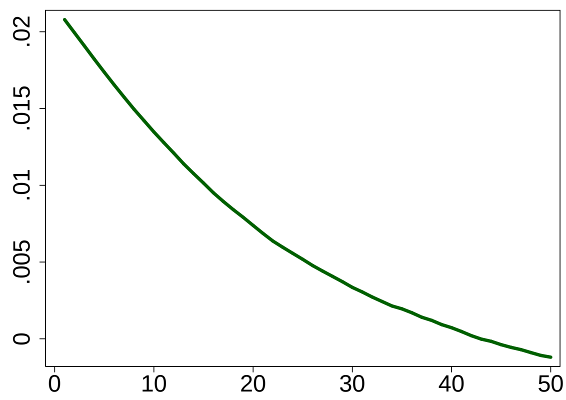


(d) $\triangle q_{t+1}$

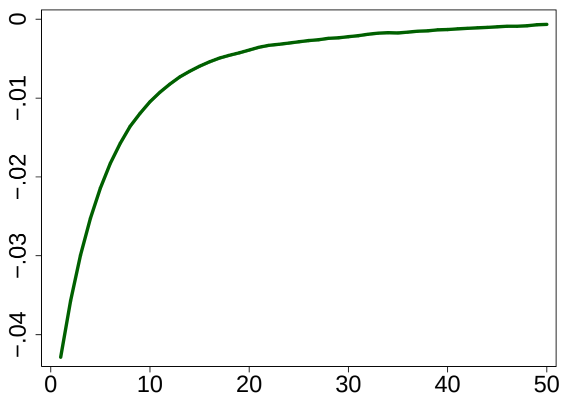
Figure 2: Impulse Responses for 1% Productivity Shock in Country 1 (HABIT with a Consumption-Based SDF, $h = 0.8$ and $\phi = 0.285$)



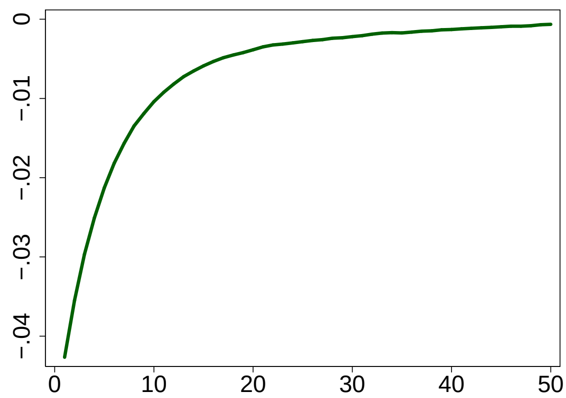
(a) r_t



(b) r_t^*



(c) $r_t - r_t^*$



(d) $\triangle q_{t+1}$

currency is expected to appreciate.

Both results imply that, when consumption-based SDF is used in a production economy, it is difficult to generate the negative Fama coefficient regardless of the existence of the habit formation. The main reason is that the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} move in the same direction for domestic country's TFP shocks. In the next section, however, I use production-based SDF to see whether the result is improved.

4.2 Case of Production-Based SDF

Under the general equilibrium framework, which is a competitive economy with complete financial markets, it is well known that the marginal rate of intertemporal substitution is tied to the marginal rate of intertemporal transformation. In this section, I take advantage of this fact as the rationale for using production-based SDF in the model and attempt to generate the negative Fama coefficient. This motivation mainly comes from Balvers and Huang (2007), and thus I follow their procedures.

In the social planner's problem defined in Equation (18), the first-order condition for the capital is:

$$U_c(C_t, C_{t-1}, 1 - L_t) = \beta E_t[V_K(K_{t+1}, K_{t+1}^*, z_{t+1}, z_{t+1}^*)] \tag{30}$$

The envelope condition is:

$$V_K(K_t, K_t^*, z_t, z_t^*) = U_c(C_t, C_{t-1}, 1 - L_t)F_K(z_t, K_t, L_t), \tag{31}$$

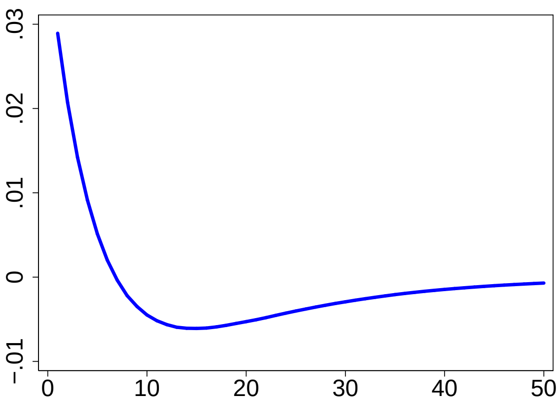
where $F_K(z_t, K_t, L_t)$ is the marginal product of capital and is equal to $\theta \frac{Y_t}{K_t} - \delta$.

Updating Equation (31) by one period and combining with Equation (30) yields a production-based SDF as:

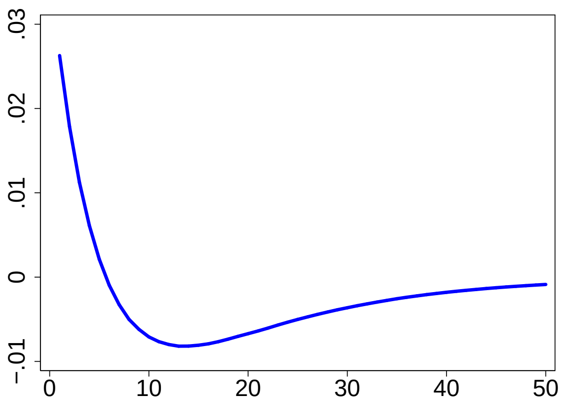
$$M_{t+1} = \beta \frac{U_c(C_{t+1}, C_t, 1 - L_{t+1})}{U_c(C_t, C_{t-1}, 1 - L_t)} = \left(\frac{V_K(K_{t+1}, K^*_{t+1}, z_{t+1}, z^*_{t+1})}{E_t V_K(K_{t+1}, K^*_{t+1}, z_{t+1}, z^*_{t+1})} \right) \left(\frac{1}{F_K(z_{t+1}, K_{t+1}, L_{t+1})} \right). \tag{32}$$

4.2.1 Impulse Response Analysis

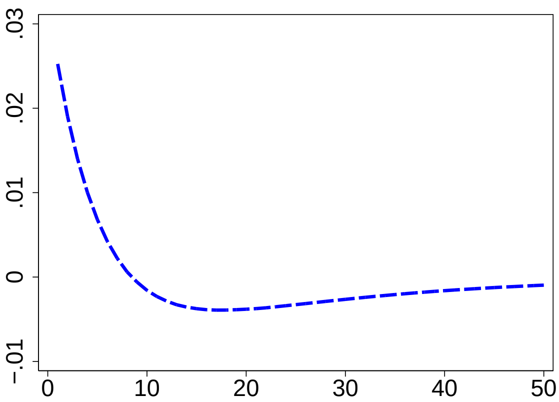
Figure 3: Impulse Responses for 1% Productivity Shock in Country 1 (BKK vs. HABIT with a Production-Based SDF)



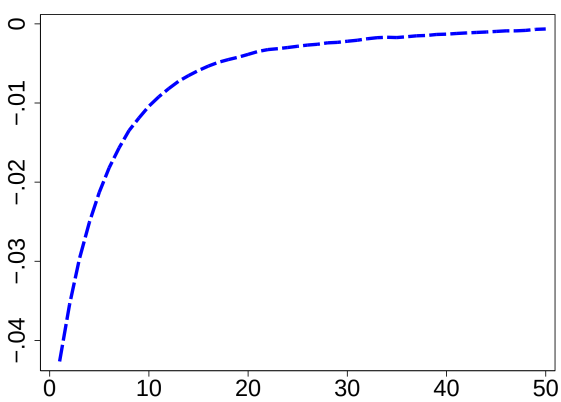
(a) $r_t - r^*_t$ (BKK)



(b) $\triangle q_{t+1}$ (BKK)

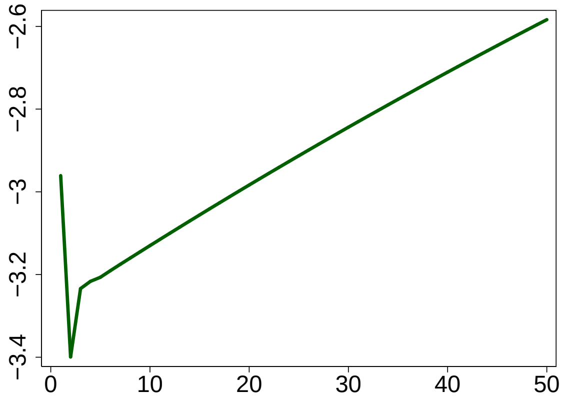


(c) $r_t - r^*_t$ (HABIT)

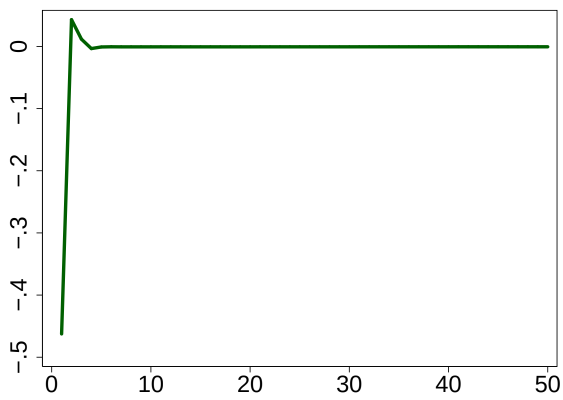


(d) $\triangle q_{t+1}$ (HABIT)

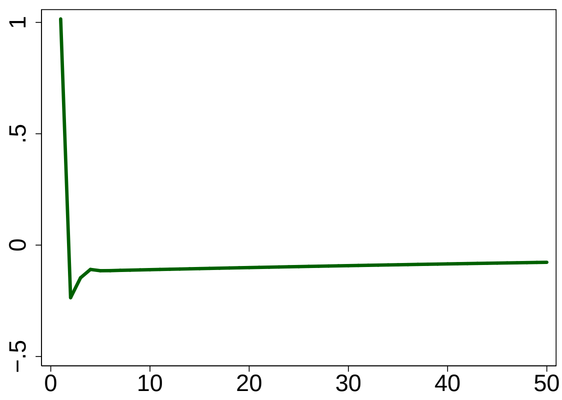
Figure 4: Impulse Responses from VAR



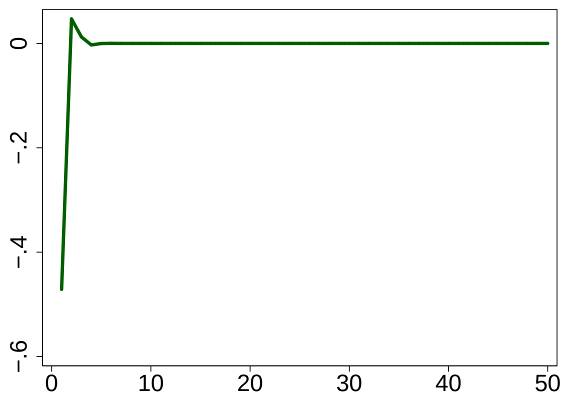
(a) $r_t - r_t^*$ (U.S. vs. Austria)



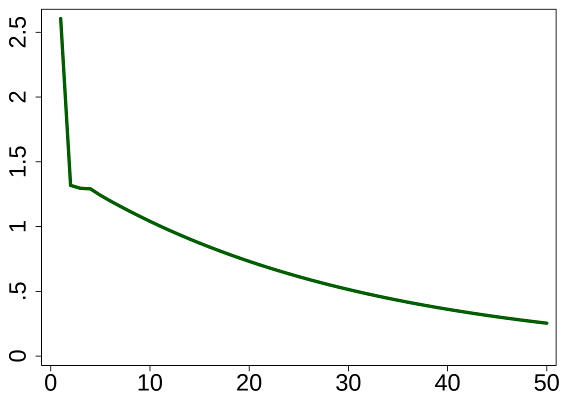
(b) $\triangle q_{t+1}$ (U.S. vs. Austria)



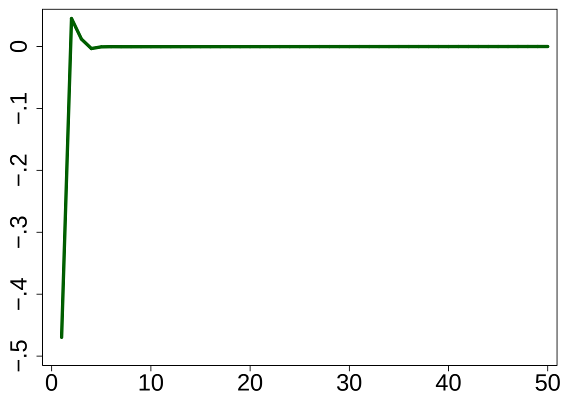
(c) $r_t - r_t^*$ (U.S. vs. Finland)



(d) $\triangle q_{t+1}$ (U.S. vs. Finland)

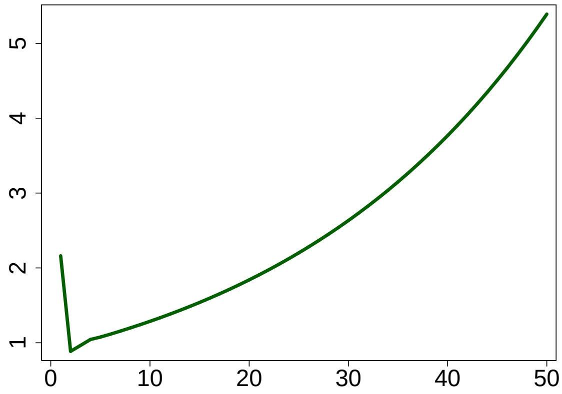


(e) $r_t - r_t^*$ (U.S. vs. Germany)

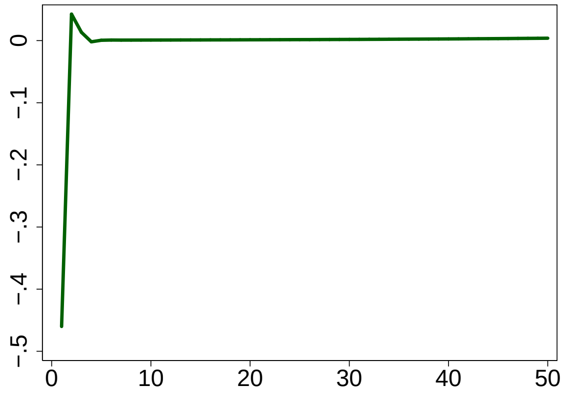


(f) $\triangle q_{t+1}$ (U.S. vs. Germany)

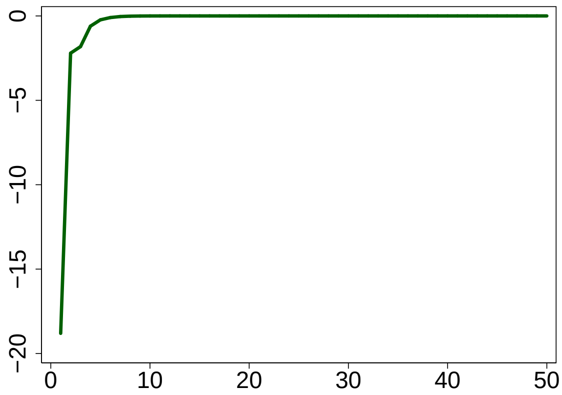
Figure 5: Impulse Responses from VAR



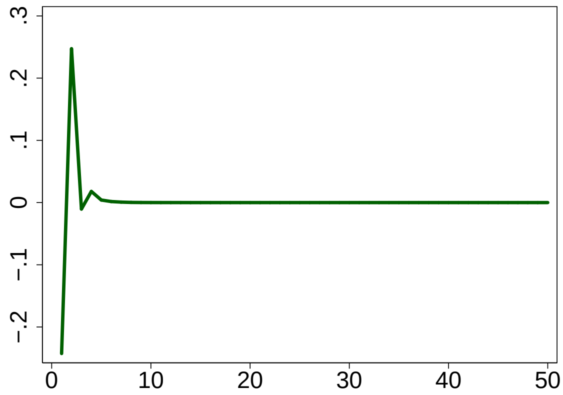
(a) $r_t - r_t^*$ (U.S. vs. Italy)



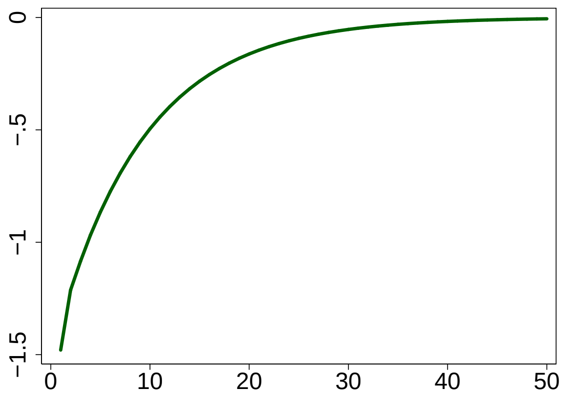
(b) $\triangle q_{t+1}$ (U.S. vs. Italy)



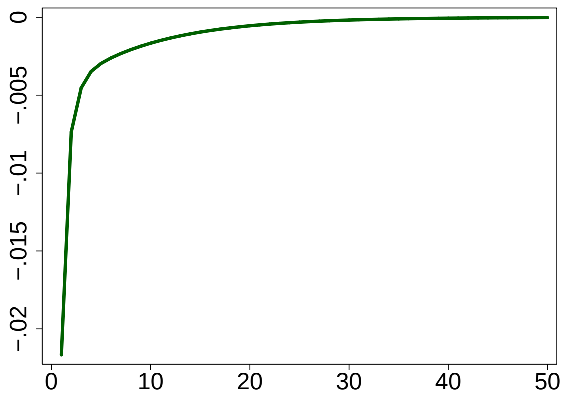
(c) $r_t - r_t^*$ (U.S. vs. Switzerland)



(d) $\triangle q_{t+1}$ (U.S. vs. Switzerland)



(e) $r_t - r_t^*$ (U.S. vs. U.K.)



(f) $\triangle q_{t+1}$ (U.S. vs. U.K.)

After using the production-based SDF in Equation (32), the impulse responses shown in Figure (3) are not consistent with the UIP condition. The interest rate spread $r_t - r_t^*$ is positive at first and then negative, finally converging at the zero line, while the expected change in the real exchange rate $E_t q_{t+1} - q_t$ is at first negative and then converges at the zero line. These results are compatible with the violation of the UIP condition, because when domestic interest rates are relatively higher than foreign interest rates, domestic currency appreciates rather than depreciates.

Because the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} move in the opposite direction for domestic country's TFP shocks, the negative Fama coefficient is expected in this case. I confirm this relationship in the next section.

As an additional analysis, I present the impulse responses to a 1% TFP shock in the U.S. from a vector autoregression (VAR) in Figure (4) and (5). The VAR includes the interest spread $r_t - r_t^*$, the expected change in the real exchange rate Δq_{t+1}, and the Solow residuals in the U.S. data from 1999 to 2011.[8] Six countries are selected for this analysis: Austria, Finland, Germany, Italy, Switzerland, and the United Kingdom. BKK (1992) use these countries and the U.S. to estimate the parameters for the bivariate shock process.

Overall, these figures show that a positive TFP shock leads the U.S. interest rate to be relatively higher than that of its counterpart, and causes the U.S. currency to appreciate against the foreign currency. Three countries including Finland, Germany, and Italy show that the behaviors of interest rates and exchange rates are consistent with the

[8] All interest rate and exchange rate data are obtained from the IFS and the Haver Analytics. I calculate the Solow residuals in the U.S. with the data of GDP (99A..ZF), Gross Fixed Capital Formation (93E.RZF), and Employment (67E..ZF) from the IFS.

violation of the UIP condition.[9] These empirical results from the VAR suggest that TFP shocks would be an important source in driving the U.S. interest rate and exchange rate, and provide support for the benchmark model with the habit formation which is driven by TFP shocks.

4.3 Fama Regression Coefficient

Table 2: Simulation Results of Fama Regression

	BKK C-based	BKK P-based	Habit C-based	Habit P-based	Verdelhan (2010)
h	0	0	0.8	0.8	
ϕ	0	0	0.285	0.285	
μ	0.34	0.34	0.3313	0.3313	
$\rho(r_t - r_t^*, \Delta q_{t+1})$	1.000	0.996	1.000	-0.971	
$\sigma(r_t - r_t^*)$	0.0006	0.0006	0.0008	0.0005	
$\sigma(\Delta q_{t+1})$	0.0006	0.0006	0.0008	0.0008	
β	0.995	0.934	0.997	-1.546	-0.99

As shown in Equation (2), the slope coefficient in the Fama regression β is

$$\beta = \frac{Cov(r_t - r_t^*, q_{t+1} - q_t)}{Var(r_t - r_t^*)}. \tag{33}$$

This slope coefficient for each model is presented in Table (3). As expected, only the habit formation model with production-based SDF has a negative slope coefficient ($\beta = -1.546$), and this result confirms the validity of the impulse response analysis

[9] The results are relatively robust to the ordering of variables except for the case where the variables are ordered as Δq_{t+1}, the Solow residuals, and $r_t - r_t^*$.

described in the previous section. Since only this model shows the native relationship between the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1}, the corresponding consequence is the negative Fama coefficient.

4.4 Business Cycle Properties

Table 3: Simulation Results of Business Cycle

	Data	BKK	The benchmark model
		$h = 0$	$h = 0.8$
		$\phi = 0$	$\phi = 0.2565$
		$\mu = 0.34$	$\mu = 0.3313$
$\sigma(c_t)/\sigma(y_t)$	0.49	0.49	0.39
$\sigma(i_t)/\sigma(y_t)$	3.15	3.41	3.64
$\rho(y_t, c_t)$	0.76	0.90	0.63
$\rho(y_t, i_t)$	0.90	0.93	0.96
$\rho(y_t, y_t^*)$	0.70	0.10	0.16
$\rho(c_t, c_t^*)$	0.46	0.81	0.90
$\rho(c_t/c_t^*, q_t)$	-0.39	0.92	0.62

Note: Business cycle statistics are from Backus, Kehoe and Kydland (1994).

Table (4) shows the business cycle properties for the standard BKK and the benchmark models, respectively. Overall, the benchmark model matches the stylized facts of the business cycle: The relative volatility of consumption is less than that of investment, consumption and investment are both positively correlated with GDP, and the cross-country GDP and consumption correlation are positive, respectively.

In comparison with the standard BKK model, the benchmark model generates some different results: The benchmark model has smaller consumption volatility and smaller correlation between consumption and GDP ($\sigma(c)/\sigma(y) = 0.39$ and $\rho(y, c) = 0.63$). However, the benchmark model has a higher cross-country consumption correlation ($\rho(c, c^*) = 0.90$) and a higher cross-country GDP correlation ($\rho(y, y^*) = 0.16$).

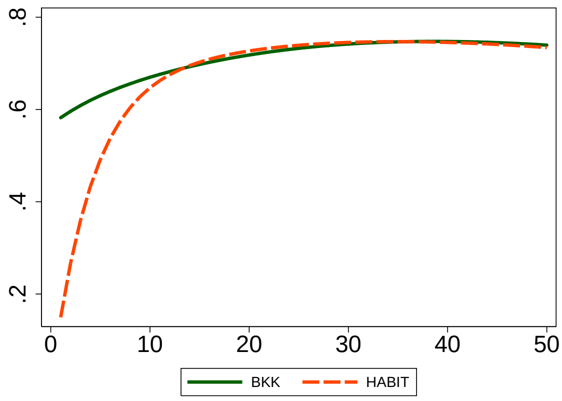
All of these results seem to arise from the presence of habit formation effects. In the sensitivity analysis section, I discuss how this habit formation affects business cycle properties and helps generate the negative Fama coefficient by varying key parameter values.

4.5 Interpretation

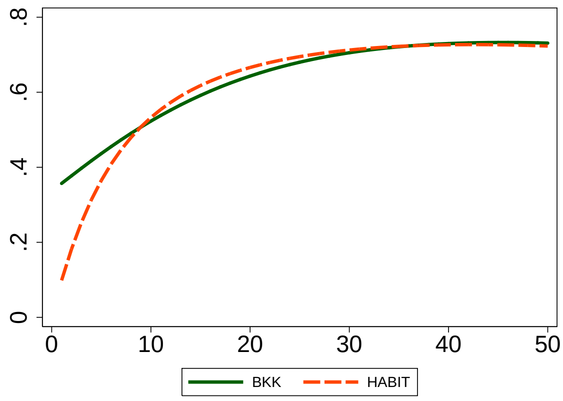
In this section, I discuss why the standard BKK model and the benchmark model with the habit formation cannot generate the negative Fama coefficient when consumption-based SDF is used. The impulse responses of consumption and labor for each country are presented in Figure (6) to facilitate our discussion here. As mentioned before, the main reasons are that the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} are positively correlated and that both of them are closely tied with the marginal utility of consumption.

First, consider the real exchange rate, which is defined in Equation (24). In the two-country economy with complete financial markets, it is well known in the literature that risk sharing induces a positive cross-country consumption correlation. Therefore, when there are positive TFP shocks in the domestic country, consumption in both countries increases in similar pattern. However, labor supply adjusts differently such that it increases in the domestic country but decreases in the foreign country, because the domestic country is more productive than its foreign counterpart. Consequently, the

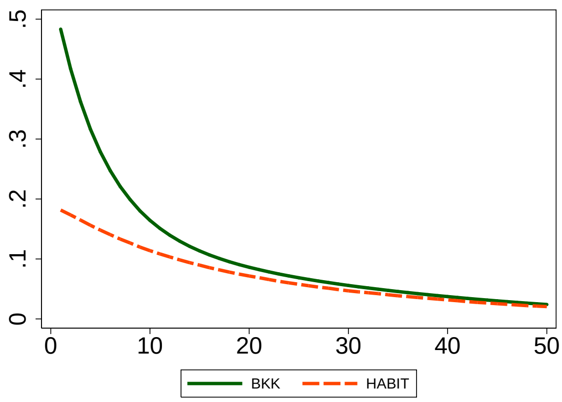
Figure 6: Impulse Responses for 1% Productivity Shock in Country 1 (BKK vs. HABIT)



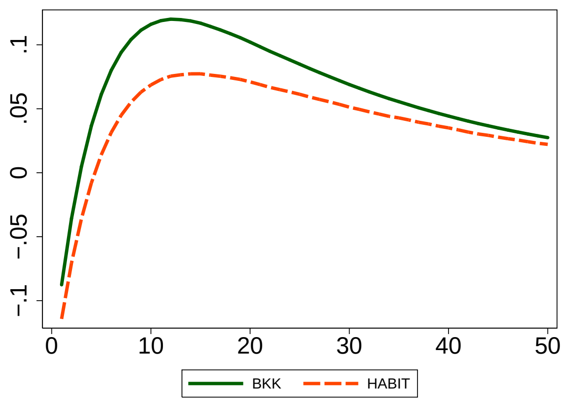
(a) Consumption in Country 1



(b) Consumption in Country 2



(c) Labor in Country 1



(d) Labor in Country 2

marginal utility of consumption in the domestic country is less than that in the foreign country, and the real exchange rate q_t increases.

In the case of the BKK model, a positive TFP shock in the domestic country continues to increase expected consumption and labor supply in the domestic country compared to those in the foreign country. Because expected future marginal utility in the domestic country is less than that of the foreign country with this expectation, the impulse response of the expected change in real exchange rate Δq_{t+1} is positive, as shown in Figure (1).

However, when the habit formation is added to the model, it makes consumption very smooth and less responsive to a positive TFP shock in domestic country. Furthermore, with this horizontal consumption path, households choose to spend more leisure rather than supply labor. This logic is similar to Lettau and Uhlig (2000), who point out that households with the habit formation do not have to build up the capital stock as much to maintain the higher consumption as in the no-habit case. Since consumption is less responsive and labor supply is reduced in the domestic country, labor supply in the foreign country is relatively more reduced in the two-country economy. As a result, a positive TFP shock in the domestic country induces expected future marginal utility in the domestic country to be larger than that in the foreign country, and thus the impulse response of the expected change in real exchange rate Δq_{t+1} is negative, as shown in Figure (2).

Second, consider the risk-free interest rates. When consumption-based SDF is used, the interest rate is defined as the inverse of the marginal rate of intertemporal substitution. In the no-habit case in the BKK model, positive TFP shocks in the domestic country continue to increase expected consumption and labor supply. Because the current marginal utility in this case is larger than the expected future one, the interest rate is procyclical, and the impulse response of the interest rates spread $r_t - r_t^*$ is positive as shown in Figure (1).

However, in the habit case for a positive TFP shock in the domestic country, consumption is unresponsive and labor supply is reduced in a similar way as that described above. As a result, the current marginal utility is less than the expected future one, and the interest rate is countercyclical. The impulse response of the interest rate spread $r_t - r_t^*$ is negative as shown in Figure (2). All of these results are robust when there is no spillover effect across countries.

The bottom line is that since the interest rate spread $r_t - r_t^*$ and the expected change in the real exchange rate Δq_{t+1} are positively correlated regardless of no-habit and habit cases when consumption-based SDF is used, the model cannot generate the negative Fama coefficient.

This obstacle is able to be overcome through the use of the production-based SDF, because in this case the interest rate is modeled as the marginal product of capital and is procyclical. Thus, expected currency excess returns can be explained in a similar way as in Verdelhan (2010): When the domestic interest rate is lower than the foreign interest rate, it implies that the domestic country experiences worse times than its foreign counter part, and the domestic households are more risk averse than their foreign counterparts. Thus, the domestic household requires a positive risk premium.

As for the Backus and Smith puzzle (1993), the benchmark model cannot fully account for the disconnection between the real exchange rate and relative consumptions. As seen in Table (4), the correlation between the two variables is counterfactually high, although the habit formation as non-time-separable utility reduces the link between the two variables compared to the no-habit case (0.62 vs. 0.92, respectively). This tight link is caused by the model structure in which the real exchange rate is defined as the ratio of the marginal utility of consumption in the two countries.

4.6 Sensitivity Analysis

Table 4: Sensitivity Analysis

	Large Elasticity			Small Elasticity		
	$\sigma = 2.5$			$\sigma = 0.5$		
	$h = 0.1$	$h = 0.5$	$h = 0.99$	$h = 0.1$	$h = 0.5$	$h = 0.99$
Business Cycle						
$\sigma(c)/\sigma(y)$	0.52	0.47	0.08	0.58	0.52	0.07
$\sigma(i)/\sigma(y)$	2.73	3.05	3.21	3.21	3.56	4.59
$\rho(y, c)$	0.90	0.83	0.19	0.97	0.88	0.28
$\rho(y, i)$	0.98	0.97	0.92	0.96	0.95	0.99
$\rho(y, y^*)$	-0.02	-0.02	-0.21	0.14	0.16	-0.50
$\rho(c, c^*)$	0.74	0.80	1.00	0.57	0.65	1.00
Fama Regression						
β	-0.930	-1.122	-1.552	-1.692	-2.156	-3.282

As discovered in the previous section, the habit formation plays a significant role in generating the negative Fama coefficient. However, the magnitude of the coefficient seems to be small. According to the literature, it is known that reducing the elasticity of substitution between intermediate goods σ increases the volatility of the terms of trade and thus the real exchange rate. In this section, I consider how changes in the habit intensity h and the elasticity of substitution affect the Fama coefficient.

As shown in Table (5), the Fama coefficient becomes more negative when σ is small and h is larger. It is a straightforward interpretation that the low value of σ increases the volatility of Δq_{t+1} and contributes to the more negative value of the Fama coefficient. The risk-return trade-off principle can explain how the high level of h affects the Fama coefficient. Intuitively, the higher habit formation induces investors to become more risk averse. Because a high level of habit formation forces the investors to smooth consumption extremely, investors dislike volatility in consumption induced by TFP shocks. Thus, the response of consumption to TFP shocks becomes more muted as the extent of habit formation increases. Table (5) verifies this intuition by showing that when $\sigma = 0.5$, the relative volatility of consumption decreases from 0.61 to 0.09 as the parameter for habit formation increases from 0.1 to 0.99.

The risk aversion caused by habit formation is related to expected currency excess returns. As discussed earlier, a higher expected return for carry trade is made possible with a more negative Fama coefficient. Therefore, as the investors become more risk averse, the Fama coefficient becomes more negative. Table 5 displays this relationship. This result is consistent with the risk-return trade off, implying that higher risk aversion is associated with higher expected returns for carry trade.

5 Conclusion

It is well known that the UIP condition does not hold empirically, meaning that expected excess returns result from investments in high-interest rate currencies in foreign currency markets. Verdelhan (2010) successfully explains this anomaly by using external habit formation and exogenous consumption process. In this paper, I explore whether the international real business cycle model (Backus, Kehoe, and Kydland 1994) with internal habit formation, which is driven by TFP shocks, can also explain this phenomenon in a general equilibrium.

As opposed to Vadelhan (2010), in which a consumption-based SDF is used, the benchmark model with internal habit formation cannot generate the negative Fama coefficient due to the countercyclical interest rates. However, when production-based SDF is used by taking advantage of the fact that the marginal rate of intertemporal substitution is tied to the marginal rate of intertemporal transformation, the model has procyclical interest rates and the negative Fama coefficient. In this case, expected currency excess returns can be explained in a similar manner as Verdelhan (2010), such that when the domestic interest rate is lower than the foreign interest rate, the domestic households are more risk averse than their foreign counterparts and require a positive risk premium.

Sensitivity analysis supports this risk premium argument by indicating that the greater tendency to be risk-averse caused by higher habit formation yields higher expected currency excess return.

6 Appendix

6.1 First-Order Conditions

The first-order conditions for the planner's problem are completely presented as follows:

$$
\begin{aligned}
C_t : \lambda_t = {} & [(C_t - hC_{t-1})^{\mu}(1-L_t)^{1-\mu}]^{1-\gamma}\frac{\mu}{C_t - hC_{t-1}} \\
& - \beta h E_t \left([(C_{t+1} - hC_t)^{\mu}(1-L_{t+1})^{1-\mu}]^{1-\gamma}\frac{\mu}{C_{t+1} - hC_t} \right), \qquad (34) \\
C_t^* : \lambda_t^* = {} & [(C_t^* - hC_{t-1}^*)^{\mu}(1-L_t^*)^{1-\mu}]^{1-\gamma}\frac{\mu}{C_t^* - hC_{t-1}^*} \\
& - \beta h E_t \left([(C_{t+1}^* - hC_t^*)^{\mu}(1-L_{t+1}^*)^{1-\mu}]^{1-\gamma}\frac{\mu}{C_{t+1}^* - hC_t^*} \right), \qquad (35)
\end{aligned}
$$

$$L_t : (1-\mu)[(C_t - hC_{t-1})^{\mu}(1-L_t)^{1-\mu}]^{1-\gamma}\frac{1}{1-L_t} = (1-\theta)\eta_t\frac{Y_t}{L_t}, \qquad (36)$$

$$L_t^* : (1-\mu)[(C_t^* - hC_{t-1}^*)^{\mu}(1-L_t^*)^{1-\mu}]^{1-\gamma}\frac{1}{1-L_t^*} = (1-\theta)\eta_t^*\frac{Y_t^*}{L_t^*}, \qquad (37)$$

$$K_{t+1} : \lambda_t(1+\phi(K_{t+1}-K_t)) = \beta E_t\left(\eta_{t+1}\theta\frac{Y_{t+1}}{K_{t+1}} + \lambda_{t+1}[(1-\delta)+\phi(K_{t+2}-K_{t+1})\right), \qquad (38)$$

$$K_{t+1}^* : \lambda_t^*(1+\phi(K_{t+1}^*-K_t^*)) = \beta E_t\left(\eta_{t+1}^*\theta\frac{Y_{t+1}^*}{K_{t+1}^*} + \lambda_{t+1}^*[(1-\delta)+\phi(K_{t+2}^*-K_{t+1}^*)\right), \qquad (39)$$

$$a_t : \eta_t = \lambda_t F_t^{1/\sigma}\omega_1 a_t^{-1/\sigma}, \qquad (40)$$

$$b_t : \eta_t^* = \lambda_t F_t^{1/\sigma}\omega_2 b_t^{-1/\sigma}, \qquad (41)$$

$$a_t^* : \eta_t = \lambda_t^* F_t^{*1/\sigma}\omega_2 a_t^{*-1/\sigma}, \qquad (42)$$

$$b_t^* : \eta_t^* = \lambda_t^* F_t^{*1/\sigma}\omega_1 b_t^{*-1/\sigma}, \qquad (43)$$

$$\eta_t : Z_t K_t^{\theta} L_t^{1-\theta} = a_t + a_t^*, \tag{44}$$

$$\eta_t^* : Z_t^* K_t^{*\theta} L_t^{*1-\theta} = b_t + b_t^*, \tag{45}$$

$$\lambda_t : F_t = C_t - K_{t+1} + (1-\delta)K_t - \frac{\phi}{2}(K_{t+1} - K_t)^2, \tag{46}$$

$$\lambda_t^* : F_t^* = C_t^* - K_{t+1}^* + (1-\delta)K_t^* - \frac{\phi}{2}(K_{t+1}^* - K_t^*)^2. \tag{47}$$

6.2 Steady-State Values

Because it is assumed that both countries are symmetric in this economy, the steady state value in a domestic country should be the same as that in a foreign country. This implies that $a = b^*$, $a^* = b$, and $Y = Y^*$. When the steady state value of Y is set to 1, the ratio a/b can be expressed as:

$$\frac{a}{b} = \frac{(1-b)/Y}{b/Y}. \tag{48}$$

Plugging an import share of $b/Y = 0.15$ into Equation (48) yields:

$$a = 0.85, \tag{49}$$

$$b = 0.15, \tag{50}$$

$$a^* = 0.15, \tag{51}$$

$$b^* = 0.85. \tag{52}$$

To determine the parameters of ω_1 and ω_2, I set intermediate goods output equal to final goods output in each country and divide Equation (40) by Equation (42) to get:

$$\left(\omega_1 a^{(\sigma-1)/\sigma} + \omega_2 b^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} = 1, \tag{53}$$

and

$$\frac{\omega_2}{\omega_1}\left(\frac{a}{a^*}\right)^{\frac{1}{\sigma}} = 1. \tag{54}$$

Plugging the above results and $\sigma = 1.5$ into the equation (53) and (54) yields:

$$\omega_1 = 0.897, \tag{55}$$

$$\omega_2 = 0.282. \tag{56}$$

In the steady state, Equations (38) and (39) become:

$$\frac{\lambda[1-\beta(1-\delta)]}{\beta\eta\theta} = \frac{Y}{K}, \tag{57}$$

$$\frac{\lambda^*[1-\beta(1-\delta)]}{\beta\eta^*\theta} = \frac{Y^*}{K^*}, \tag{58}$$

and Equations (40) and (43) become:

$$\eta = \lambda\omega_1 a^{\frac{-1}{\sigma}}, \tag{59}$$

$$\eta^* = \lambda^*\omega_1 b^{*\frac{-1}{\sigma}}. \tag{60}$$

Using Equations (57), (58), (59) and (60) yields:

$$K = 10.256, \tag{61}$$

$$K^* = 10.256. \tag{62}$$

Lastly, in the steady state, Equations (46) and (47) become:

$$C = Y - \delta K, \tag{63}$$

$$C^* = Y^* - \delta K^*, \tag{64}$$

which yields

$$C = 0.7435, \tag{65}$$

$$C^* = 0.7435. \tag{66}$$

Therefore, the steady state values of all other variables can also be calculated using the above results and the calibration parameters as follows:

6.3 Log-linearized Model

The following equations are the log-linearized first-order conditions around the steady-state values. For example, $\widehat{c}_t$ refers to $\frac{C_t - C}{C}$, where C denotes the steady-state value of C_t.

Optimality conditions for consumption

$$\widehat{\lambda}_t = \frac{1}{1-\beta h}\left(((1-\gamma)\mu - 1)\left(\frac{1}{1-h}\widehat{c}_t - \frac{h}{1-h}\widehat{c}_{t-1}\right) - (1-\gamma)(1-\mu)\frac{L}{1-L}\widehat{l}_t\right)$$
$$- \frac{\beta h}{1-\beta h}E_t\left(((1-\gamma)\mu - 1)\left(\frac{1}{1-h}\widehat{c}_{t+1} - \frac{h}{1-h}\widehat{c}_t\right) - (1-\gamma)(1-\mu)\frac{L}{1-L}\widehat{l}_{t+1}\right), \tag{67}$$

$$\widehat{\lambda}^*_t = \frac{1}{1-\beta h}\left(((1-\gamma)\mu - 1)\left(\frac{1}{1-h}\widehat{c}^*_t - \frac{h}{1-h}\widehat{c}^*_{t-1}\right) - (1-\gamma)(1-\mu)\frac{L^*}{1-L^*}\widehat{l}^*_t\right)$$
$$- \frac{\beta h}{1-\beta h}E_t\left(((1-\gamma)\mu - 1)\left(\frac{1}{1-h}\widehat{c}^*_{t+1} - \frac{h}{1-h}\widehat{c}^*_t\right) - (1-\gamma)(1-\mu)\frac{L^*}{1-L^*}\widehat{l}^*_{t+1}\right), \tag{68}$$

Table 5: Steady State Values

Home country	Foreign country
$C_{ss} = 0.743$	$C^*_{ss} = 0.743$
$L_{ss} = 0.269$	$L^*_{ss} = 0.269$
$K_{ss} = 10.256$	$K^*_{ss} = 10.256$
$Y_{ss} = 1$	$Y^*_{ss} = 1$
$F_{ss} = 1$	$F^*_{ss} = 1$
$a_{ss} = 0.85$	$a^*_{ss} = 0.15$
$b_{ss} = 0.15$	$b^*_{ss} = 0.85$
$\eta_{ss} = 5.549$	$\eta^*_{ss} = 5.549$
$\lambda_{ss} = 5.549$	$\lambda^*_{ss} = 5.549$
$r_{ss} = 0.0101$	$r^*_{ss} = 0.0101$
$I_{ss} = 0.256$	$I^*_{ss} = 0.256$
$q_{ss} = 1$	

Optimality conditions for labor

$$\widehat{\eta}_t + \widehat{y}_t - \widehat{l}_t = \mu(1-\gamma)\left(\frac{1}{1-h}\widehat{c}_t - \frac{h}{1-h}\widehat{c}_{t-1}\right) - [(1-\gamma)(1-\mu)-1]\frac{L}{1-L}\widehat{l}_t, \tag{69}$$

$$\widehat{\eta}^*_t + \widehat{y}^*_t - \widehat{l}^*_t = \mu(1-\gamma)\left(\frac{1}{1-h}\widehat{c}^*_t - \frac{h}{1-h}\widehat{c}^*_{t-1}\right) - [(1-\gamma)(1-\mu)-1]\frac{L^*}{1-L^*}\widehat{l}^*_t, \tag{70}$$

Optimality conditions for capital

$$\begin{aligned}&\lambda\widehat{\lambda}_t + \phi\lambda K E_t(\widehat{k}_{t+1} - \widehat{k}_t)\\ &= \beta E_t\left(\lambda\theta\frac{Y}{K}(\widehat{\eta}_{t+1} + \widehat{y}_{t+1} - \widehat{k}_{t+1}) + (1-\delta)\lambda\lambda_{t+1} + \phi\lambda K(\widehat{k}_{t+2} - \widehat{k}_{t+1})\right),\end{aligned} \tag{71}$$

$$\begin{aligned}&\lambda^*\widehat{\lambda}^*_t + \phi\lambda^* K^* E_t(\widehat{k}^*_{t+1} - \widehat{k}^*_t),\\ &= \beta E_t\left(\lambda^*\theta\frac{Y^*}{K^*}(\widehat{\eta}^*_{t+1} + \widehat{y}^*_{t+1} - \widehat{k}^*_{t+1}) + (1-\delta)\lambda^*\lambda^*_{t+1} + \phi\lambda^* K^*(\widehat{k}^*_{t+2} - \widehat{k}^*_{t+1})\right),\end{aligned} \tag{72}$$

Optimality conditions for intermediate goods

$$\widehat{\eta}_t = \widehat{\lambda}_t + \frac{1}{\sigma}\widehat{f}_t - \frac{1}{\sigma}\widehat{a}_t, \tag{73}$$

$$\widehat{\eta}^*_t = \widehat{\lambda}_t + \frac{1}{\sigma}\widehat{f}_t - \frac{1}{\sigma}\widehat{b}_t, \tag{74}$$

$$\widehat{\eta}_t = \widehat{\lambda}^*_t + \frac{1}{\sigma}\widehat{f}^*_t - \frac{1}{\sigma}\widehat{a}^*_t, \tag{75}$$

$$\widehat{\eta}^*_t = \widehat{\lambda}^*_t + \frac{1}{\sigma}\widehat{f}^*_t - \frac{1}{\sigma}\widehat{b}^*_t, \tag{76}$$

Resource constraints for intermediate goods

$$\widehat{z}_t + \theta\widehat{k}_t + (1-\theta)\widehat{l}_t = \frac{a}{Y}\widehat{a}_t + \frac{a^*}{Y}\widehat{a}^*_t, \tag{77}$$

$$\widehat{z}^*_t + \theta\widehat{k}^*_t + (1-\theta)\widehat{l}^*_t = \frac{b}{Y^*}\widehat{b}_t + \frac{b^*}{Y^*}\widehat{b}^*_t, \tag{78}$$

Resource constraints for final goods

$$\widehat{f}_t = \frac{C}{F}\widehat{c}_t + \frac{K}{F}\widehat{k}_{t+1} - (1-\delta)\frac{K}{F}\widehat{k}_t, \tag{79}$$

$$\widehat{f}_t^* = \frac{C^*}{F^*}\widehat{c}_t + \frac{K^*}{F^*}\widehat{k}_{t+1}^* - (1-\delta)\frac{K^*}{F^*}\widehat{k}_t^*, \tag{80}$$

Production functions for final goods

$$\widehat{f}_t = \left(1 + (w_2/w_1)(b/a)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\widehat{a}_t + \left(1 + (w_1/w_2)(a/b)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\widehat{b}_t, \tag{81}$$

$$\widehat{f}_t^* = \left(1 + (w_2/w_1)(a^*/b^*)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\widehat{b}_t^* + \left(1 + (w_1/w_2)(b^*/a^*)^{\frac{\sigma-1}{\sigma}}\right)^{-1}\widehat{a}_t^*, \tag{82}$$

Real exchange rate

$$\widehat{q}_t = \widehat{\lambda}_t^* - \widehat{\lambda}_t, \tag{83}$$

Real interest rates

$$\widehat{r}_t = \widehat{z}_t + (\theta-1)\widehat{k}_t + (1-\theta)\widehat{l}_t, \tag{84}$$

$$\widehat{r}_t^* = \widehat{z}_t^* + (\theta-1)\widehat{k}_t^* + (1-\theta)\widehat{l}_t^*, \tag{85}$$

Productivity shocks

$$\widehat{z}_t = \rho_1\widehat{z}_{t-1} + \rho_2\widehat{z}_{t-1}^* + \epsilon_t, \tag{86}$$

$$\widehat{z}_t^* = \rho_2\widehat{z}_{t-1}^* + \rho_1\widehat{z}_{t-1} + \epsilon_t^*. \tag{87}$$

References

[1] Abel, Andrew B, Asset prices under habit formation and catching up with the Joneses, *American Economic Review* (1990): 38-42

[2] Backus, David K, Kehoe, Patrick J and Kydland, Finn E, International real business cycles, *Journal of Political Economy* (1992): 745-775

[3] Backus, David K, Kehoe, Patrick J and Kydland, Finn E, Dynamics of the trade balance and the terms of trade: The J-Curve?, *American Economic Review* (1994): 84-103

[4] Balvers, Ronald J. and Huang, Dayong, Productivity-based asset pricing: Theory and evidence, *Journal of Financial Economics* (2007): 405-445

[5] Bansal, Ravi and Dahlquist, Magnus, The forward premium puzzle: different tales from developed and emerging economies, *Journal of International Economics* (2000): 115-144

[6] Boldrin, Michele, Christiano, Lawrence J., and Fisher, Jonas D. M., Habit persistence, asset returns and the business cycle, *American Economic Review* (2001): 149-166

[7] Burnside, Craig, Eichenbaum, Martin, Kleshchelski, Isaac, and Rebelo, Sergio, Do peso problems explain the returns to the carry trade?, *Working Paper* (2010)

[8] Campbell, John Y. and Cochrane, John H., By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* (1999): 539-580

[9] Chari, V.V., Kehoe, Patrick J and Mcgrattan, Ellen R, Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?, *Review of Economic Studies* (2002): 533-563

[10] Engel, Charles, The forward discount anomaly and the risk premium: A survey of recent evidence, *Journal of Empirical Finance* (1996): 123-192

[11] Eugene F. Fama, Forward and spot exchange rates, *Journal of Monetary Economics* (1984): 319-338

[12] Flood, Robert P and Rose, Andrew K, Uncovered interest parity in crisis, *International Monetary Fund* (2002): 252-266

[13] Francis, Bill B, Hasan, Iftekhar and Hunter, Delroy M, Emerging market liberalization and the impact on uncovered interest rate parity, *Journal of International Money and Finance* (2002): 931-956

[14] Frankel, J. and Poonawala, J., The forward market in emerging currencies: Less biased than in major currencies, *Journal of International Money and Finance* (2010): 585-598

[15] Froot, Kenneth A and Thaler, Richard H, Anomalies: foreign exchange, *Journal of Economic Perspectives* (1990): 179-912

[16] Hansen, Lars Peter and Hodrick, Robert J, Forward exchange rate as optimal predictors of future spot rates: an econometric analysis, *Journal of Political Economy* (1980): 829-853

[17] Heathcote, Jonathan and Perri, Fabrizio, Financial autarky and international business cycles, *Journal of Monetary Economics* (2002): 601-627

[18] Hollifield, Burton and Amir Yaron, The foreign exchange risk premium: real and nominal factors, *Working paper* (2003)

[19] Jermann, Urban J., Asset pricing in production economies, *Journal of Monetary Economics* (1998): 257-275

[20] Lettau, Martin and Uhlig, Harald, Can habit formation be reconciled with business cycle facts?, *Review of Economic Dynamics* (2000): 79-99

[21] Lewis, Karen K, Puzzles in international financial markets, *Handbook of International Economics* (1995): 1913-1971

[22] Lustig, Hanno and Verdelhan, Adrien, The cross section of foreign currency risk premia and consumption growth risk, *American Economic Review* (2007): 89-117

[23] Mehra, Rajnish and Prescott, Edward, The equity premium: a puzzle, *Journal of Monetary Economics* (1985): 145-161

[24] Mendoza, Enrique G, Real business cycles in a small open economy, *American Economic Review* (1991): 797-818

[25] Mendoza, Enrique G, The terms of trade, the real exchange rate, and economic fluctuation, *International Economic Review* (1995): 101-137

[26] Moore, Michael J. and Roche, Maurice J, Less of a puzzle: a new look at the forward forex market, *Journal of International Economics* (2002): 387-411

[27] Rabanal, Pau, Rubio-Ramirez F, Juan and Tuesta, Vicente, Cointegrated TFP process and international business cycles, *Journal of Monetary Economics* (2011): 156-171

[28] Rouwenhorst, K. Geert, Asset pricing implications of equilibrium business cycle models, *Frontiers of Business Cycle Research* (1995): Chapter 10

[29] Verdelhan, Adrien, A habit-based explanation of the exchange rate risk premium, *Journal of Finance* (2010): 123-146

[30] Zimmermann, Christian, International real business cycles among heterogeneous countries, *European Economic Review* (1997): 319-356