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Asset Bubbles in an Overlapping Generations Model with Endogenous Labor Supply[*]

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Abstract

This paper examines the effects of asset bubbles in an overlapping generations model with endogenous labor supply. We show analytically that asset bubbles can lead to an expansion in steady-state capital, investment, employment and output under certain conditions. The analytical results are followed by a specific numerical example.

Keywords: Asset Bubbles, Overlapping Generations, Endogenous Labor.

JEL classification: E22, E44.

1 Introduction

The existence and consequences of asset bubbles have long been a subject of interest to economists. In a seminal paper, Tirole (1985) showed that asset bubbles can exist in an overlapping generations economy with rational consumers and exogenous labor supply. A central implication of Tirole's model is that asset bubbles will always crowd out productive investment and reduce capital accumulation. Since labor supply is inelastic, this will then lead to a reduction in aggregate output. This negative relationship between asset bubbles and aggregate economic activities, however, is in contrast with empirical evidence. For instance, private nonresidential fixed investment in the U.S. has grown at an exceptionally high rate before the collapse of the internet bubble in the year 2000 and before the collapse of the recent housing price bubble. Martin and Ventura (2012) also provide evidence showing that asset bubbles in the U.S. and Japan are often associated with robust economic growth. In the present study, we show that this conflict between theory and evidence can be resolved when labor supply is endogenous.[1] Specifically, we show that asset bubbles can induce an expansion in steady-state capital, investment, employment and output if labor supply responds strongly and positively to changes in interest rate. This type of response is possible when the intertemporal elasticity of substitution (IES) in consumption is small and the Frisch elasticity of labor supply is large.[2] The intuition of this result will be explained later.

2 The Model

Consider an overlapping generations model in which each consumer lives two periods: young and old. In each period $t \geq 0$, a new generation of identical consumers is born. The size of generation t is given by $N_t = (1+n)^t$, with $n > 0$. All consumers have one unit of time endowment which can be allocated between work and leisure. Retirement is mandatory in the second period of life, so the labor supply of old consumers is zero.

Consider a consumer who is born at time $t \geq 0$. Let $c_{y,t}$ and $c_{o,t+1}$ denote his consumption when young and old, respectively, and let l_t denote his labor supply when young. The consumer's preferences

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[1] Our focus on endogenous labor supply is largely motivated by empirical evidence. Using data from the Current Employment Statistics (CES), we find that total employment and aggregate labor hours have moved closely with asset prices during the internet bubble episode and the recent housing bubble episode. In particular, both variables have grown at a higher than normal rate during the run-up phase of these bubbles. Further details of this are available from the authors upon request.

[2] Olivier (2000), Farhi and Tirole (2012) and Martin and Ventura (2012) have explored other channels through which asset bubbles can crowd in productive investment and foster economic growth in overlapping generations models. Miao and Wang (2012) have developed an infinite-horizon model in which asset bubbles can promote total factor productivity. None of these studies have explored the connections between endogenous labor supply and asset bubbles.

are represented by

$$\mathcal{U}\left(c_{y,t}, l_t, c_{o,t+1}\right) = \frac{c_{y,t}^{1-\sigma}}{1-\sigma} - A\frac{l_t^{1+\psi}}{1+\psi} + \beta\frac{c_{o,t+1}^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\sigma > 0$ is the inverse of the IES in consumption, $\psi \geq 0$ is the inverse of the Frisch elasticity of labor supply, $\beta \in (0,1)$ is the subjective discount factor and A is a positive constant. Let w_t be the market wage rate at time t. Then the consumer's labor income when young is $w_t l_t$. The consumer can save in two types of assets: physical capital and an intrinsically worthless asset.[3] The total supply of the intrinsically worthless asset is constant over time and is denoted by $M \geq 0$.[4] Denote savings in physical capital by s_t, and savings in the intrinsically worthless asset by m_t. The gross return from physical capital between time t and $t+1$ is given by R_{t+1}. The price of the intrinsically worthless asset at time t is p_t. No arbitrage means that these two assets must have the same return, so that $R_{t+1} = p_{t+1}/p_t$ for all $t \geq 0$. Taking $\{w_t, p_t, p_{t+1}, R_{t+1}\}$ as given, the consumer's problem is to choose an allocation $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}$ so as to maximize his lifetime utility in (1), subject to the budget constraints:

$$c_{y,t} + s_t + p_t m_t = w_t l_t, \qquad \text{and} \qquad c_{o,t+1} = R_{t+1} s_t + p_{t+1} m_t.$$

The first-order conditions for this problem are given by

$$w_t c_{y,t}^{-\sigma} = A l_t^{\psi}, \qquad \text{and} \qquad c_{y,t}^{-\sigma} = \beta R_{t+1} c_{o,t+1}^{-\sigma}. \tag{2}$$

Using these equations, we can obtain

$$c_{y,t} = \frac{c_{o,t+1}}{(\beta R_{t+1})^{\frac{1}{\sigma}}} = \frac{w_t l_t}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}},$$

$$l_t = A^{-\frac{1}{\sigma+\psi}} \left(1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}\right)^{\frac{\sigma}{\sigma+\psi}} w_t^{\frac{1-\sigma}{\sigma+\psi}},$$

$$s_t + p_t m_t = \Sigma\left(R_{t+1}\right) w_t l_t, \qquad \text{where} \qquad \Sigma\left(R_{t+1}\right) \equiv \frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}. \tag{3}$$

An increase in R_{t+1} has two opposing effects on saving which are captured by the function $\Sigma : \mathbb{R}_+ \to [0,1]$. First, an increase in R_{t+1} means that for the same amount of total savings, the consumer will

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[3] The latter is called "intrinsically worthless" because it has no consumption value and cannot be used for production. The only motivation for holding this type of asset is to resell it at a higher price in the next period.

[4] At time 0, all assets are owned by a group of "initial-old" consumers. The decision problem of these consumers is trivial and does not play any role in the following analysis.

receive more interest income when old. This creates an income effect which encourages consumption when young and discourages saving. Second, an increase in interest rate also lowers the relative price of future consumption. This creates an intertemporal substitution effect which discourages consumption when young and promotes saving. The relative strength of these two effects depends on the value of σ. In particular, the intertemporal substitution effect dominates when $\sigma < 1$. In this case, $\Sigma(\cdot)$ is a strictly increasing function. When $\sigma > 1$, the income effect dominates and $\Sigma(\cdot)$ is a strictly decreasing function. The two effects exactly cancel out when $\sigma = 1$. In this case, $\Sigma(\cdot)$ is a constant.

On the supply side of the economy, there is a large number of identical firms. In each period, each firm hires labor and physical capital from the competitive factor markets, and produces output according to

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \qquad \text{with } \alpha \in (0,1),$$

where Y_t denotes output produced at time t, K_t and L_t denote capital input and labor input, respectively. Since the production function exhibits constant returns to scale, we can focus on the choices made by a single price-taking firm. We assume that physical capital is fully depreciated after one period, so that R_t coincides with the rental price of physical capital at time $t \geq 0$. The representative firm's problem is given by

$$\max_{K_t, L_t} \left\{ K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t \right\},$$

and the first-order conditions are $R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$, and $w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$.

Given $M \geq 0$, a competitive equilibrium of this economy consists of sequences of allocations $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}_{t=0}^{\infty}$, aggregate inputs $\{K_t, L_t\}_{t=0}^{\infty}$, and prices $\{w_t, p_t, R_t\}_{t=0}^{\infty}$ such that (i) given $\{w_t, p_t, p_{t+1}, R_{t+1}\}$, the allocation $\{c_{y,t}, l_t, c_{o,t+1}, s_t, m_t\}$ is optimal for the consumers in generation $t \geq 0$, (ii) given $\{w_t, R_t\}$, the aggregate inputs $\{K_t, L_t\}$ solve the representative firm's problem at time $t \geq 0$, and (iii) all markets clear in every period, so that $L_t = N_t l_t$, $N_t m_t = M$ and

$$K_{t+1} = N_t s_t = \left[\frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma}-1}} \right] w_t N_t l_t - p_t N_t m_t, \qquad \text{for all } t \geq 0. \tag{4}$$

Let $k_t \equiv K_t/N_t$ be the quantity of physical capital per worker at time t, and let $a_t \equiv p_t m_t$ be the quantity of unproductive savings per young consumer. Then the equilibrium wage rate can be expressed

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as $w_t = (1 - \alpha) k_t^{\alpha} l_t^{-\alpha}$, and (4) can be rewritten as

$$(1 + n) k_{t+1} = (1 - \alpha) \left[\frac{\beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma} - 1}}{1 + \beta^{\frac{1}{\sigma}} R_{t+1}^{\frac{1}{\sigma} - 1}} \right] \left(\frac{k_t}{l_t} \right)^{\alpha} l_t - a_t. \tag{5}$$

The dynamics of a_t is determined by

$$a_{t+1} = p_{t+1} m_{t+1} = \frac{p_{t+1} m_{t+1}}{p_t m_t} a_t = \frac{R_{t+1}}{1 + n} a_t.$$

3 Stationary Equilibrium

3.1 Economy without Intrinsically Worthless Assets

Before analyzing the effects of asset bubbles, we first characterize the stationary equilibrium of an economy with zero supply of intrinsically worthless asset, i.e., $M = 0$ and $a_t = 0$ for all $t \geq 0$. A stationary equilibrium is a competitive equilibrium in which $k_t = k^*$, $l_t = l^*$ and $R_t = R^*$ for all $t \geq 0$. Substituting these conditions into (5) gives

$$\frac{\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1}}{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1}} \left(\frac{k^*}{l^*} \right)^{\alpha - 1} = \frac{1 + n}{1 - \alpha}$$

$$\Rightarrow \Lambda (R^*) \equiv \frac{\beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}}}{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1}} = \frac{(1 + n) \alpha}{1 - \alpha}. \tag{6}$$

Equation (6) follows from the fact that $R^* = \alpha (k^*/l^*)^{\alpha - 1}$. For any $\sigma > 0$, the function $\Lambda : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing with $\Lambda (0) = 0$ and $\lim_{R \to \infty} \Lambda (R) = \infty$. Hence, there exists a unique $R^* > 0$ that solves (6). The steady-state value of all other variables can be uniquely determined by

$$w^* = (1 - \alpha) \left(\frac{\alpha}{R^*} \right)^{\frac{\alpha}{1 - \alpha}}, \tag{7}$$

$$l^* = A^{-\frac{1}{\sigma + \psi}} \left[1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1} \right]^{\frac{\sigma}{\sigma + \psi}} (w^*)^{\frac{1 - \sigma}{\sigma + \psi}}, \tag{8}$$

$$k^* = l^* \left(\frac{\alpha}{R^*} \right)^{\frac{1}{1 - \alpha}}, \quad \text{and} \quad c_y^* = \frac{c_o^*}{(\beta R^*)^{\frac{1}{\sigma}}} = \frac{w^* l^*}{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1}}. \tag{9}$$

This establishes the following result.

Proposition 1 *A unique bubbleless steady state exists for any $\sigma > 0$. The steady-state values*

$\left\{ R^*, w^*, k^*, l^*, c_y^*, c_o^* \right\}$ *are determined by (6)-(9).*

3.2   Economy with Intrinsically Worthless Assets

Suppose now the economy has a strictly positive supply of intrinsically worthless assets, i.e., $M > 0$. In this subsection, we focus on stationary equilibria in which the price of these assets exceeds their fundamental value, i.e., $p_t = p^* > 0$. Formally, a "bubbly" steady state is a set of values $\left\{ \widetilde{a}^*, \widetilde{R}^*, \widetilde{w}^*, \widetilde{k}^*, \widetilde{l}^*, \widetilde{c}_y^*, \widetilde{c}_o^* \right\}$ that satisfies the following conditions: $\widetilde{a}^* > 0$, $\widetilde{R}^* = 1 + n$,

$$\widetilde{a}^* + (1+n)\,\widetilde{k}^* = (1-\alpha) \left[\frac{\beta^{\frac{1}{\sigma}} \left(\widetilde{R}^* \right)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} \left(\widetilde{R}^* \right)^{\frac{1}{\sigma}-1}} \right] \left(\frac{\widetilde{k}^*}{\widetilde{l}^*} \right)^{\alpha} \widetilde{l}^*, \tag{10}$$

and (7)-(9).[5] Substituting $\widetilde{a}^* > 0$ and $\widetilde{R}^* = 1 + n$ into (10) gives

$$\frac{1+n}{1-\alpha} < \left[\frac{\beta^{\frac{1}{\sigma}}(1+n)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}}(1+n)^{\frac{1}{\sigma}-1}} \right] \left(\frac{\widetilde{k}^*}{\widetilde{l}^*} \right)^{\alpha-1} \Rightarrow \frac{(1+n)\,\alpha}{1-\alpha} < \Lambda\,(1+n)\,. \tag{11}$$

Since $\Lambda\,(\cdot)$ is strictly increasing, (6) and (11) together imply that $R^* < 1+n$. This shows that $R^* < 1+n$ is a necessary condition for the existence of bubbly steady state. Suppose this condition is satisfied. Then substituting $\widetilde{R}^* = 1 + n$ into (7)-(9) yields a unique set of values for $\left\{ \widetilde{w}^*, \widetilde{k}^*, \widetilde{l}^*, \widetilde{c}_y^*, \widetilde{c}_o^* \right\}$. Using (10), we can obtain a unique value of a^*, which is strictly positive as $R^* < 1 + n$ and $\Lambda\,(\cdot)$ is strictly increasing. Hence, a unique bubbly steady state exists. This proves the following result.

Proposition 2 *A unique bubbly steady state exists if and only if $R^* < 1 + n$.*

Similar to Tirole (1985), our model predicts that equilibrium interest rate will increase in the presence of asset bubbles. When labor supply is exogenous, the steady-state value of per-worker capital is determined by $k^* = (\alpha/R^*)^{\frac{1}{1-\alpha}}$. Thus, a higher interest rate in the bubbly steady state means that there is fewer per-worker capital than in the bubbleless steady state, i.e., $\widetilde{k}^* < k^*$. When labor supply is endogenous, the value of k^* is jointly determined by l^* and R^* as shown in (9). If asset bubbles is associated with an increase in labor hours (i.e., $\widetilde{l}^* > l^*$), and if this increase is strong enough to overcome the increase in interest rate, then more capital will be accumulated in the bubbly steady state than in the bubbleless one, i.e., $\widetilde{k}^* > k^*$. The rest of this paper is intended to formalize this

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[5]Note that equations (7)-(9) must be satisfied in any steady state, regardless of the existence of asset bubbles.

idea. Two remarks are in order before we proceed. First, suppose $\widetilde{l}^* > l^*$ and $\widetilde{k}^* > k^*$ are true. Then per-worker output in the bubbly steady state is also higher than in the bubbleless steady state. Second, since the equilibrium interest rate is higher in the presence of asset bubbles, it follows from (9) that the capital-labor ratio in the bubbly steady state (i.e., $\widetilde{k}^*/\widetilde{l}^*$) must be lower than that in the bubbleless steady state (i.e., k^*/l^*).[6]

Suppose $R^* < 1 + n$. Then using (7)-(9), which are valid in both bubbleless and bubbly steady states, we can obtain

$$k^* = (1 - \alpha)^{\frac{1-\sigma}{\sigma+\psi}} A^{-\frac{1}{\sigma+\psi}} \left[1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{R^*}\right)^{\phi},$$

$$\widetilde{k}^* = (1 - \alpha)^{\frac{1-\sigma}{\sigma+\psi}} A^{-\frac{1}{\sigma+\psi}} \left[1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{1+n}\right)^{\phi},$$

where $\phi \equiv \frac{1}{1-\alpha} \left[1 + \frac{\alpha(1-\sigma)}{\sigma+\psi}\right] > 0$ for any $\sigma > 0$. Hence, $\widetilde{k}^* > k^*$ if and only if

$$\left[1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{1+n}\right)^{\phi} > \left[1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}\right]^{\frac{\sigma}{\sigma+\psi}} \left(\frac{\alpha}{R^*}\right)^{\phi}$$

$$\Leftrightarrow \left(\frac{R^*}{1+n}\right)^{\phi} > \left[\frac{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}}\right]^{\frac{\sigma}{\sigma+\psi}}. \tag{12}$$

Note that this condition *cannot* be satisfied if $\sigma \geq 1$. Since $R^* < 1+n$, we have $(R^*)^{\frac{1}{\sigma}-1} \geq (1+n)^{\frac{1}{\sigma}-1}$, whenever $\sigma \geq 1$. Condition (12) then implies

$$\left(\frac{R^*}{1+n}\right)^{\phi} > \left[\frac{1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}-1}}{1 + \beta^{\frac{1}{\sigma}} (1+n)^{\frac{1}{\sigma}-1}}\right]^{\frac{\sigma}{\sigma+\psi}} \geq 1,$$

which contradicts $R^* < 1+n$. Thus, a necessary condition for $\widetilde{k}^* > k^*$ is $\sigma < 1$. The intuition underlying this result is straightforward: In the presence of asset bubbles, equilibrium interest rate rises from R^* to $\widetilde{R}^* = 1+n$. Such an increase will create an income effect and an intertemporal substitution effect on the young's consumption. Since consumption and labor supply is inversely related, the income effect will discourage young consumers from working, whereas the intertemporal substitution effect will induce

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[6]Using the quarterly capital stock data compiled by Gomme and Rupert (2007) and the aggregate weekly hours index in the Current Employment Statistics (CES) data, we have constructed a time series of capital-labor ratio for the period 1964Q1 to 2004Q2. Since our model does not take into account long-term economic growth, we have removed the growth trend from the constructed series, and considered only the changes in the detrended capital-labor ratio. We find that this ratio has declined during most part of the run-up phase of the internet bubble episode (i.e., between January 1995 and January 2000). Further details of this are available from the authors upon request.

them to work more. Since $\widetilde{k}^* > k^*$ can happen only if $\widetilde{l}^* > l^*$, it is necessary to have the intertemporal substitution effect greater than the income effect, i.e., $\sigma < 1$.[7]

We now derive a sufficient condition for $\widetilde{k}^* > k^*$. Suppose $R^* < 1 + n$ and $\sigma < 1$ are satisfied. Using (6), we can get

$$1 + \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma} - 1} = \frac{(1 - \alpha) \beta^{\frac{1}{\sigma}} (R^*)^{\frac{1}{\sigma}}}{\alpha (1 + n)}.$$

Substituting this into (12) and rearranging terms gives

$$\underbrace{\left(\frac{R^*}{1 + n} \right)^{\phi(\sigma + \psi) - 1}}_{\Upsilon} > (1 + n)^{1 - \sigma} \left\{ \frac{(1 - \alpha) \beta^{\frac{1}{\sigma}}}{\alpha \left[1 + \beta^{\frac{1}{\sigma}} (1 + n)^{\frac{1}{\sigma} - 1} \right]} \right\}^{\sigma}, \tag{13}$$

where $\phi(\sigma + \psi) - 1 = \frac{\psi + \alpha}{1 - \alpha} - (1 - \sigma)$. Note that the parameter ψ does not affect the value of R^* nor the expression on the right-hand side of (13). Since $R^* < 1 + n$, lowering the value of ψ will raise the value of Υ. Thus, holding other parameters constant, $\widetilde{k}^* > k^*$ is more likely to occur when the value of ψ is low (or equivalently, when the Frisch elasticity of labor supply is large). This, together with $\sigma < 1$, ensures that young consumers will significantly increase their labor supply when interest rate rises. A low value of ψ is not uncommon in macroeconomic studies. In the extreme case when $\psi = 0$, the preferences in (1) become quasi-linear in labor. Hansen (1985) shows that this type of utility function can emerge from a model with indivisible labor. Quasi-linear utility function is now commonly used in business cycle models and monetary-search models.

The main results of this paper are summarized in Proposition 3.

Proposition 3 *(i) Suppose $R^* < 1 + n$. Then a necessary condition for $\widetilde{k}^* > k^*$ is $\sigma < 1$. (ii) Suppose $R^* < 1 + n$ and $\sigma < 1$ are satisfied. Then a sufficient condition for $\widetilde{k}^* > k^*$ is (13).*

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[7]In infinite-horizon models, it is typical to assume that σ is greater than or equal to one. However, in overlapping generations model, it is typical to assume that the intertemporal substitution effect is greater than the income effect. Galor and Ryder (1989) shows that this assumption plays an important role in establishing the existence, uniqueness and stability of both stationary and non-stationary equilibria in an overlapping generations model with exogenous labor supply. Nourry (2001) uses similar conditions to examine the local stability of stationary equilibria in a model with endogenous labor supply. In a well-known study on stochastic bubbles, Weil (1987) focuses on the case when the interest elasticity of savings is positive. Under a constant-relative-risk-aversion utility function, this assumption is equivalent to $\sigma < 1$. There is also some empirical support for $\sigma < 1$. See, for instance, the results in Table III and Table IV of Gourinchas and Parker (2002).

3.3 Numerical Example

We now provide a specific numerical example in which asset bubbles will lead to an expansion in steady-state capital, investment, employment and output. Suppose one model period takes 30 years. Set the annual subjective discount rate to 0.9950 and the annual employment growth rate to 1.6%, which matches the average annual growth rate of U.S. employment over the period 1953-2008. Then we have $\beta = (0.9950)^{30} = 0.8604$ and $n = (1.0160)^{30} - 1 = 0.6099$. We also set $\alpha = 0.30$ and $\psi = 0$. The value of A is calibrated so that l^* is one-third. Under this calibration procedure, $\widetilde{k}^*$ is greater than k^* for any $\sigma \in [0, 0.16]$. In Table 1, we report the results obtained when $\sigma = 0.15$ and $A = 0.5862$. Under these parameter values, the bubbly steady state has a higher level of employment, per-worker capital and per-worker output than the bubbleless steady state.[8]

Table 1: Numerical Results

	Bubbleless Steady State	Bubbly Steady State
R	1.2416	1.6099
k	0.0438	0.0461
a	0	0.0721
l	0.3333	0.5084
w	0.3808	0.3407
y	0.1813	0.2474

Note: The notation y denotes per-worker capital, i.e., $y = k^\alpha l^{1-\alpha}$.

4 Conclusions

In this paper, we show that when labor supply is elastic, deterministic rational bubbles can induce an expansion in aggregate economic activities under certain conditions. In the present study, specific forms of utility function and production function have been used. This allows us to deliver our main results in a clear and concise manner. One direction for future research is to extend our results to general utility functions and production technologies. Another possibility is to extend the model to allow for financial market frictions and agency costs as in Azariadis and Chakraborty (1998).

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[8]Similar results can be obtained for other values of $\{\alpha, \beta, n\}$ and some non-zero values of ψ. In general, one can extend the range of σ under which $\widetilde{k}^* > k^*$ by either raising the value of β or lowering the value of α. On the other hand, changing the value of A has no effect on the relative magnitude between $\widetilde{k}^*$ and k^*.

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