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A Time-Varying Approach of the US Welfare Cost of Inflation

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Abstract. Money demand specifications exhibits instability, especially for long spans of data. This paper reconsiders the welfare cost of inflation for the US economy using a flexible time-varying cointegration methodology to estimate the money demand function. We find evidence that the time-varying cointegration estimation provides a better fit of the actual data than a time-invariant estimation and that the throughout unitary income elasticity only exists for the log-log form over the entire sample period. Our estimate of the welfare cost of inflation for a 10-percent inflation rate lies in the range of 0.025 to 0.75 percent of GDP and averages 0.27 percent. When we plug in the actual inflation rate series over the sample period, we find that the welfare cost of inflation lies in the range of 0.009 to 0.33 percent of GDP. In sum, our findings fall well within the ranges of existing studies of the welfare cost of inflation. Finally, the interest elasticity of money demand shows substantial variability over our sample period.

Keywords: Money Demand Function, Welfare cost of inflation, Time-varying cointegration

JEL classification: C32, E52, G10

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1. Introduction

Macroeconomists borrow ideas from microeconomics to consider the welfare cost of inflation, which refers to the changes in social welfare caused by inflation. Bailey (1956) and Friedman (1969) develop the now traditional approach that treats real money balances as a consumption good and inflation as a tax on real balances. This approach measures the welfare cost as the appropriate area under the money demand curve.

Ireland (2009) re-examines the welfare cost estimates reported in Lucas (2000), noting that the extension of the Lucas's sample of annual data from 1900 to 1994 to 1900 to 2004 adds another period of extremely low interest rates with which to estimate the money demand function. Ireland (2009, p. 1043, Fig. 2) plots the data, showing that the semi-log specification may fit the more recent data (i.e., post-1979) better than the log-log specification that Lucas (2000) uses. Ireland (2009) considers only the post-1979 period, using quarterly rather than annual observations. He concludes that the semi-log specification dominates the log-log specification, reporting a welfare cost for a 10-percent inflation rate of less than 0.25 percent of income (2009, p. 1048). Lucas finds a welfare cost for 10-percent inflation of just over 1.8 percent of income for the log-log specification and just less than 1.2 percent for the semi-log specification.

Our paper addresses this possibility of structural change by using a time-varying parameter cointegration approach for quarterly data from 1959 to 2010. In addition, the more recent data adds another period of very low interest rates with which to estimate the money demand function. We find strong evidence of time-varying parameters in the cointegration relationship and the log-log specification, once again, dominates the semi-log specification. Finally, our time varying estimates of the welfare cost of inflation generally fall closer to the

findings of Ireland, implying smaller effects than in Lucas (2000).

The welfare cost of inflation considers the long-run effects of inflation and abstracts from the effects of inflation on redistribution because of any difference between expected and actual inflation. That is, the calculation of the welfare cost assumes that the private sector expects the current inflation rate, which could describe an economy with a well-functioning inflation-targeting regime. Moreover, all contracts reflect the actual and expected inflation rate so that no distortions exist in real decisions.

Since the welfare cost of inflation captures long-run effects, the first step in calculating this welfare cost searches for the long-run money demand relationship. Typically, that means determining if a cointegrating relationship exists amongst the variables in the money demand function. The rejection of traditional cointegration, however, does not necessarily mean that cointegration does not exist. Rather, the cointegrating relationship may reflect either instability or nonlinearity, or both. Papers that address the instability issue include, for example, Khan (1974), Duprey (1980), Tesfatsion and Veitch (1990), Hafer and Jansen (1991), Miller (1991), Lütkepohl (1993), Ireland (2009), Rao and Kumar (2011), Wang (2011), Nakashima and Saito (2012) and Lucas and Nicolini (2013) and papers that address the nonlinearity issue include Vinod (1998), Serletis and Shahmoradi (2005), Bae and DeJong (2007), Jawadi and Sousa (2013), and Gupta and Majumdar (forthcoming). A linear time-varying function can summarize nonlinearity of any form (Granger, 2008) and also capture structural change by considering in the limit each point in time as a different regime, as it seems empirically true for the case of the money demand function.

This paper reconsiders the welfare cost of inflation for the US economy using the system-based time-varying cointegration method of Bierens and Martins (2010) to estimate the long-run

relationship between money, income, and the interest rate according to the Meltzer (1963) log-log and the Cagan (1956) semi-log specifications. We find significant, but not uniform, evidence of time-varying cointegration against the standard cointegration approach of time-invariant coefficients.

The unitary income elasticity only exists for the log-log form over the entire sample period. This specification produces estimates of the welfare cost of inflation for a 10-percent inflation rate that lie in the range of 0.025 to 0.75 percent of GDP over time and averaging 0.27 percent in sample. This compares favorably to the values of about 0.2 to 0.3 percent of income that Fisher (1981), Serletis and Yavari (2004) and Ireland (2009) report but differ from those that Lucas (2000) reports of closer to one.

Our model with time-varying coefficients fits the data better and is more general than the standard time-invariant specification adopted by the authors cited above. Therefore, our results probably indicate that the single-valued welfare cost of inflation obtained from standard cointegration methods capture the sample average of the estimated welfare costs at each point of time. We can relate the periods when the welfare cost falls below or above average to the position of the US business cycle. That is, we find that the welfare cost averages from 12.0-, 10.3-, and 7.4-percent higher during expansions than recessions for 0-, 2-, and 10-percent inflation rates. To the best of our knowledge, this is the first paper to estimate time-varying, long-run money demand functions for the US economy, and more importantly, also the first to provide a time-varying measure of the associated welfare costs of inflation, using quarterly data on the measure of real money balances, real income, and nominal interest rate over the period of 1959:Q1 to 2010:Q4.

Zuo and Park (2011) and Barigozzi and Conti (2014) provide estimates of time-varying

long-run money demand functions for China and Europe, based on the single-equation Park and Hahn (1999) and Bierens and Martins (2010) approaches, respectively. Still, neither of them do (time-varying) welfare analysis. On the other hand, Kumar (2014) estimates the (time-varying) welfare cost of inflation in India over the period of 1996:Q2 to 2013:Q1 using a different methodology: the Kalman filter. The measurement money demand equation is an AR(1) model for money over income with nominal interest rate as a regressor. The state equation for the time-varying (semi) elasticity of the money demand is a standard random walk process. He concludes that the welfare cost increased in recent years (about 0.04 percent in 2012).

The paper conforms to the following outline. Section 2 briefly discusses the existing literature in this area. Section 3 lays out the theoretical issues involved in calculating the welfare cost of inflation. Section 4 describes the econometric methodology, discusses the data, implements the method, and analyzes the findings. Section 5 concludes.

2. Existing Empirical Estimates

We can primarily categorize the voluminous literature on the welfare costs of inflation in the U.S., and internationally,¹ under three alternative approaches. First, the simplest analysis computes the deadweight loss by evaluating the area under the money demand curve (Fischer, 1981; Lucas, 1981; Gillman, 1995; Lucas, 2000; Serletis and Yavari, 2004; Ireland, 2009; Lim et al., 2012; Gupta and Majumdar, forthcoming). Second, another method computes the welfare cost from general equilibrium models (Cooley and Hansen, 1989; Gillman, 1993; Gomme, 1993; Lucas, 1994; Dotsey and Ireland, 1996; Aiyagari et al., 1998; Pakko, 1998; Wu and Zhang, 1998; Lagos and Wright, 2005; Burstein and Hellwig, 2008; Craig and Rocheteau, 2008;

¹ For a detailed review of the international literature on the welfare costs of inflation, refer to Gupta and Uwilingiye (2010).

Henriksen and Kydland, 2010; Silva 2012; Adão and Silva, 2013). Third, other authors use partial equilibrium models that capture the interaction between capital income taxation and inflation (Feldstein, 1997, 1999). Understandably, these three approaches reach different conclusions regarding the sizes of the welfare cost of inflation. In general, welfare costs obtained from calculating the deadweight loss under the long-run money demand function produces estimates substantially lower than those obtained from general equilibrium models and partial equilibrium models that account for the interaction between capital income taxation and inflation.² This is expected, since the former approach accounts only for the money demand distortion brought about by positive nominal interest rates, while, in general equilibrium, increases in inflation can distort other marginal decisions, affecting on both the level and growth rate of aggregate output. Further, the interactions between inflation and a not-completely-indexed tax code can add substantially to the welfare cost of inflation as well.

Fischer (1981) and Lucas (1981) calculate relatively low welfare costs of inflation. Fischer (1981) computes the deadweight loss generated by increasing the inflation rate from zero to 10 percent at just 0.3 percent of GDP, using the monetary base (government money) as the definition of money. Lucas (1981) calculates the welfare cost of the same change in the inflation rate from zero to 10 percent inflation at 0.45 percent of GDP, using M1 as the measure of money. Lucas (2000) revises his estimate of the welfare cost upward, to slightly less than 1 percent of GDP.

Ireland (2009) more recently investigates the welfare cost of money, using quarterly US

² Gupta and Majumdar (forthcoming) provide the exception. They estimate a nonparametric long-run money demand function and find welfare costs comparable to general equilibrium estimates, since the data coverage by the nonparametric function far exceeds the coverage in a linear money demand specification.

data covering the period of 1980:Q1–2006:Q4.³ He cannot reject the null of unitary long-run income elasticity of money demand. As a consequence, he expresses the money demand function as the relationships between the nominal money–income ratio and the nominal interest rate. He chooses the (cointegrated) semi-log formulation of money demand over the competing (spurious) log-log specification over his sample period of 1980:Q1 to 2006:Q4. Finally, he finds that the welfare cost of inflation lies between 0.014 and 0.232 percent of GDP for inflation rates between 0 and 10 percent, which compares closely to the welfare estimates of Fischer (1981) and Lucas (1981) but not so much to Lucas (2000) for the US. Essentially, the larger value obtained by Lucas is explained by a different sample period (1900-1994) and model specification (log-log).

Structural models provide a recent alternative to econometric estimates of the triangle under an estimated money demand curve. Cooley and Hansen (1989) calibrate a cash-in-advance version of a business cycle model. They find that the welfare cost of 10 percent inflation is about 0.4 percent of GNP. In a follow-up paper, Cooley and Hansen (1991) consider the effects of distortionary taxes on the welfare cost measure, finding that the welfare cost rises to nearly one percent when the cash good proves more important in the utility function than the credit good in their cash-in-advance model. Silva (2012) extends the cash-in-advance model to give agents the flexibility to choose when they convert bonds into cash and shows that the welfare cost rises by 10-fold from 0.1 percent in the benchmark model to 1 percent in the model with agent flexibility. Other recent general-equilibrium models that estimate the welfare cost of inflation include Dotsey and Ireland (1996), Aiyagari, et al. (1998), Burstein and Hellwig (2008), and Henriksen and Kydland (2010).

³ As noted in the introduction, Ireland (2009) begins his sample in 1980, since he sees possible structural change in the quarterly sample between 1979 and 1980. He suspects that the semi-log specification will perform better in the post-1979 period rather than the log-log specification that Lucas (2000) uses. Ireland confirms that the semi-log specification dominates the log-log specification in his quarterly sample.

Pakko (1998) develops a shopping-time model of money demand. He estimates that the welfare cost of raising the inflation rate from zero to 10 percent equals 1.3 percent. Craig and Rocheteau (2008) argue that a search-theoretic framework is necessary for appropriately measuring the welfare cost of inflation. Lagos and Wright (2005) model monetary exchange and provide estimates for the annual cost of 10 percent inflation of between 3 and 5 percent of consumption, which translates into 2 to 3.5 percent of GDP in the US.

In sum, various methods and specifications to estimate the welfare cost of inflation exist in the literature. Their conclusions do not differ too much with welfare costs as a fraction of GDP below 5 percent. To summarize, welfare cost estimates are found to range between 0.3 percent of GDP (Fischer, 1981) to 5.98 percent of GDP (Wu and Zhang, 1998) for a 10-percent inflation rate. In this paper, we consider the size of the time-varying welfare costs of inflation based on the distortion of inflation to the money demand only.

3. Welfare Costs over Time

Ireland (2009) suggests that structural change may affect the welfare cost calculations and confines his sample to quarterly data from 1980 to 2006. We address the possibility of structural change by implementing the time-varying cointegration approach of Bierens and Martins (2010). As such, we calculate a time-varying welfare cost.

Calculating the welfare cost of inflation depends critically on the specification of the money demand function. Lucas (2000) employs two money-demand specifications, which come from Meltzer (1963) and Cagan (1956). The first specification due to Meltzer (1963) relates the natural logarithms of real money balances (M/P), real income (Y/P), and nominal interest rate (r), with M equals the nominal money supply and Y equals nominal income. That is,

$$\ln(M/P)_t = \ln A_t + \alpha_t \cdot \ln(Y/P)_t - \eta_t \cdot \ln r_t, \quad (1)$$

where $A_t > 0$ is a time-varying intercept, α_t is the time-varying income elasticity of money demand, and $\eta_t > 0$ measures the absolute value of the time-varying interest elasticity of money demand.

The second specification due to Cagan (1956) links the natural logarithms of real money balances and real income, and the level of the nominal interest rate as follows:

$$\ln(M / P)_t = \ln B_t + \beta_t \cdot \ln(Y / P)_t - \xi_t \cdot r_t, \quad (2)$$

where $B_t > 0$ is a time-varying intercept, β_t is the time-varying income elasticity of money demand, and $\xi_t > 0$ measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

Then, following the literature, we analyze the specifications for which we cannot reject the null hypothesis that the income elasticity equals unity for all t . Thus, we can write the corresponding time-varying relationships in equations (1) and (2), respectively, in terms of money-income ratio (m), as follows:

$$\ln m_t = \ln A_t - \eta_t \ln r_t, \text{ and} \quad (3)$$

$$\ln m_t = \ln B_t - \xi_t r_t. \quad (4)$$

Lucas (2000) applied the methods outlined in Bailey (1956) to transform the evidence on money demand into a welfare cost estimate. Bailey (1956) identified the welfare cost of inflation as the area under the inverse money-demand function (the “consumer surplus”) gained by reducing the interest rate to zero from its existing value. Thus, for an estimated money demand function given by $m(r, t)$ and the implied inverse demand function represented by $\psi(m, t)$, then we can calculate the welfare cost as follows:

$$w(r, t) = \int_{m(r)}^{m(0)} \psi(x, t) dx = \int_0^r m(x, t) dx - r \cdot m(r, t) \quad (5)$$

The second integral in equation (5) shows an alternative way to calculate the consumer surplus. Here, we integrate under the money-demand curve as the interest rate rises from zero to a positive value, giving the lost consumer surplus. Then, we deduct the associated seigniorage revenue (i.e., $r \cdot m(r, t)$) to deduce the deadweight loss.

Remember that the function m measures money as a fraction of income. Thus, the function w also measures values as a fraction of income. In this case, the value of $w(r, t)$ measures the fraction of income that people need, as compensation, to become indifferent between living in a steady-state with an interest rate constant at r or in a steady-state with an interest equal to zero. Lucas (2000) shows that when the money-demand function conforms to the log-log specification, then $m(r) = A \cdot r^{-\eta}$, so that in equation (3) the level $m(r, t) = A_t \cdot r^{-\eta_t}$. Thus, the welfare cost of inflation as a fraction of GDP equals the following expression:

$$w(r, t) = A_t \left(\frac{\eta_t}{1 - \eta_t} \right) r^{1 - \eta_t} \quad (6)$$

When the money demand function corresponds to the semi-log specification in equation (4), then the level of $m(r, t) = B_t \cdot e^{-\xi_t \cdot r}$. Now, the welfare cost of inflation conforms to the following expression:

$$w(r, t) = \frac{B_t}{\xi_t} \left[1 - (1 + \xi_t r) e^{-\xi_t r} \right] \quad (7)$$

Equations (6) and (7) imply that the time-varying interest elasticity and semi-elasticity of money demand play crucial roles in evaluating the welfare cost of inflation. Thus, in our empirical reassessment of the welfare cost analysis, we first test for the unit income elasticity throughout the sample period and then determine the long-run (cointegrating) relationship between the ratio of money to income and the nominal interest rate in the two specifications –

log-log and semi-log models.

4. Methodology, Data, and Results

4.1 Econometric Method

In his analysis using models with time-invariant coefficients, Ireland (2009) tested the log-log and semi-log specifications for cointegration, finding that the semi-log form exhibited cointegration while the log-log form did not. We revisit the issue of the welfare cost of inflation, using the time-varying-parameter method of cointegration developed by Park and Hahn (1999) and Bierens and Martins (2010).⁴

Park and Hahn (1999) consider a single equation cointegrating regression in the Engle-Granger (1987) tradition with time-varying parameters. They model the parameters of the cointegrating vector to follow smooth functions of time, using Fourier series expansions. They employ semi-nonparametric sieve estimators, deriving their asymptotic properties, and develop residual-based specification tests. Bierens and Martins (2010) permit the cointegrating vectors in a vector error-correction (VEC) model in the Johansen (1988, 1991) tradition to follow smooth functions of time, similar to Park and Hahn (1999). They model the time-varying cointegrating vectors, however, using Chebyshev time polynomials. They estimate the extended VEC model following Johansen's maximum likelihood (ML) approach and develop a likelihood ratio test for time-varying cointegration, where time-invariant cointegration is the null hypothesis.

Using the time-varying parameter cointegration methods admits the possibility of a

⁴ Other approaches to modifying the original linear specification of cointegration include sudden deterministic structural breaks and Markov-switching approaches. Regarding time-varying error-correction models, Hansen (2003) generalizes reduced-rank methods to cointegration under sudden regime shifts with a known number of break points, while Andrade, et al. (2005) develop tests on the cointegration rank and on the cointegration space under known and unknown break points. The Markov-switching approach of Hall, Psaradakis, and Sola (1997) and the smooth transition model of Saikkonen and Choi (2004) provide interesting approaches to modeling shifts in cointegrating vectors.

nonlinear long-run money demand function and thus putting aside the discussion of whether the model is of a semi-log or log-log form (Bae and deJong, 2007). See also Granger (2008): “Any non-linear model can be approximated by a time-varying parameter linear model”⁵. By specifying a model with time-varying coefficients we are able to stick to the literature and obtain results for the semi-log and log-log forms. On the other hand, we adopt a money demand system-based specification as it is more general and robust to endogeneity than the single equation strategy. It allows us to accommodate for the possibility that more than one cointegrating relationship may exist between the real measure of money, real income, and the nominal interest rate (Wolters and Lutkepohl, 1998).

Following the notation in Bierens and Martins (2010), consider the time-varying $VEC(p)$ model with a drift and Gaussian errors:

$$\Delta Z_t = \mu + \Pi_t' Z_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Z_{t-j} + \varepsilon_t \quad (8)$$

where $Z_t \in \mathbb{R}^k$, μ is a $k \times 1$ vector of intercepts, $\varepsilon_t \sim N_k[0, \Omega]$, and T is the number of observations. Our first objective is to test the null-hypothesis of time-invariant (TI) cointegration, $\Pi_t' = \Pi' = \alpha\beta'$, where α and β are fixed $k \times r$ matrices with rank r , against time-varying (TV) cointegration of the type: $\Pi_t' = \alpha\beta_t'$, where α is the same as before but now the β_t 's are time-varying $k \times r$ matrices with rank r . In both cases, Ω and the Γ_j 's are fixed $k \times k$ matrices, and $1 \leq r \leq k$. In the case of our model, $k=3$ or $k=2$ and $r=1$ and the first equation gives the money demand function. If we find evidence for TV cointegration, we compute the TV welfare costs out of the estimated TV cointegrating vector.

⁵ The literature that examines nonlinear long-run relationships is not new and it includes, for example, Blake and Fomby 1997, de Jong 2001, Granger and Yoon 2002, Harris, et al. 2002, and Juhl and Xiao 2005.

Assuming that the function of discrete time β_t is smooth (see Bierens and Martins, 2010, for details), and defining $\xi_{i,T} = \frac{1}{T} \sum_{t=1}^T \beta_t P_{i,T}(t)$, $i = 0, \dots, T-1$, as unknown $k \times r$ matrices, we can write β_t as follows:

$$\beta_t = \beta_m(t/T) = \sum_{i=0}^m \xi_{i,T} P_{i,T}(t) \quad (9)$$

for some fixed $m < T-1$, where the orthonormal Chebyshev time polynomials $P_{i,T}(t)$ are defined by: $P_{0,T}(t)=1$, $P_{i,T}(t) = \sqrt{2} \cos(i\pi(t-0.5)/T)$, $t = 1, 2, \dots, T$, $i=1, 2, 3, \dots, m$. Here, we choose m (and also p) according to the standard model selection procedures. We can then specify the error-correction model more conveniently with TI coefficients as follows:

$$\Delta Z_t = \mu + \alpha \xi' Z_{t-1}^{(m)} + \Gamma X_t + \varepsilon_t, \quad (10)$$

where $\xi' = (\xi_0', \xi_1', \dots, \xi_m')$ is an $r \times (m+1)k$ matrix of rank r , $Z_{t-1}^{(m)}$ is defined by

$$Z_{t-1}^{(m)} = (Z_{t-1}', P_{1,T}(t)Z_{t-1}', P_{2,T}(t)Z_{t-1}', \dots, P_{m,T}(t)Z_{t-1}')' \quad (11)$$

and $X_t = (\Delta Z_{t-1}', \dots, \Delta Z_{t-p+1}')'$. To test for the null hypothesis of standard time-invariant (TI) cointegration, Bierens and Martins (2010) propose a Likelihood Ratio (LR) test statistic where the restricted model takes $\xi' = (\beta', O_{r,k,m})$ and is asymptotically distributed as χ_{mkr}^2 (see Bierens and Martins (2010) for further details).

4.2 Data

In this study, we use quarterly time-series data from the first quarter of 1959 (1959:Q1) to the fourth quarter of 2010 (2010:Q4). Data come from the Federal Reserve Bank of St. Louis FRED database, except that we adjust the series for the measure of money supply (M1) by adding back the funds removed by retail deposit sweep programs using estimates stock based on

the MIRS aggregate defined by Cynamon et al., (2006).⁶ We measure nominal income and the nominal interest rates by nominal GDP (Y) and the three-month US Treasury bill rate (r), respectively. We seasonally adjust all series, except for the Treasury bill rate. When we use real money balances (M/P) and real GDP (Y/P) independently in the regressions, we divide the corresponding nominal series for MIRS and GDP by the GDP deflator (P), but when we use the money income ratio (m), we just divide MIRS by GDP (i.e., $m = MIRS/Y$).

Before conducting the cointegration analysis, we consider the time-series properties of the variables -- the natural logarithms of money to GDP, real money, real GDP, the interest rate, as well as the level of the interest rate -- using the augmented Dickey-Fuller (ADF, 1981), the Phillips-Perron (PP, 1988), the Dickey-Fuller generalized least squares (DF-GLS, Elliott, Rothenberg, and Stock 1996), and the Ng-Perron (2001) unit-root tests with an intercept and with an intercept and trend. Table 1 reports the findings. We conclude that all series conform to I(1) processes.

4.3 *Empirical Results*

We begin by testing for a long-run relationship between money, income, and the interest rate according to the Meltzer (1963) log-log and the Cagan (1956) semi-log specifications. Since the money demand function provides an important component of many macroeconomic models, economic theory suggests that a long-run relationship should exist between money, income, and the interest rate.

Since we evaluate welfare costs as a percentage of the GDP, we need to test for the assumption of unitary income elasticity and, when evidence in its favor is found, impose it and

⁶ We also performed the analysis using M1 and not correcting for sweep programs. We find better performance for the measure of M1 that adjusts for sweep programs.

estimate long-run money demand equations, where the natural logarithm of the money-income ratio depends on the natural logarithm of the nominal interest rate (log-log) or the nominal interest rate (semi-log). We analyze the confidence sets of the estimated time-varying income elasticity parameter in the cointegrated equations that involves real money balances, real income, and the interest rate, and choose the functional form (log-log or semi-log) for which the estimate falls within the confidence set of an income elasticity of unitary.

In Table 2 we present the results for testing standard TI cointegration of Johansen (1988, 1991) versus TV cointegration of Bierens and Martins (2010). In all cases, we reject the null hypothesis of time-invariant cointegration in favor of time-varying cointegration. This table reports the results with and without imposing the constraint that the income elasticity of real money demand equals one.

Although we initially estimate the model without imposing the restriction that the income elasticity of money demand equals one, the calculation of the welfare cost of inflation requires a unitary income elasticity for all t . Figure 1 reports the time-varying coefficients for the long-run relationship with $k=3$, where b_1 , b_2 , and b_3 are the coefficients of the natural logarithms of real money, real GDP, and the interest rate (levels, in Figure 1a), respectively.⁷ The coefficient of the interest rate variable follows its own path, but appears much more stable for the log-log specification of the model. We note that for both the log-log and semi-log specifications, the movement in the coefficients of the natural logarithm of real money and real GDP mirror each other such that the ratio tends to remain relatively constant and, possibly, equal to one in absolute value, as also suggested from Figure 2 (plots the income elasticity of real money demand relative to one). Further, we test whether in fact the real income elasticity of money demand equals one

⁷ Comparing these coefficients to equations (1) and (2), α and β both equal $(-b_2/b_1)$ and η and ξ both equal b_3 .

throughout. Figure 3 plots the upper and lower 90% confidence bands for $-b_2/b_1$ relative to one as well as the median points, all based on the wild bootstrap procedure. We conclude that only the log-log specification exhibits an income elasticity that does not differ significantly from one during the whole sample.

Ireland (2009) provides graphs that show the actual values of the nominal interest rate and the money to income ratio as well as the semi-log and log-log money demand specifications (see Figure 2, p. 1043). He also identifies actual values from 1900-1979 and from 1980 to 2006, the end of his sample period. The latter, more-recent data did not associate with large increases in money demand as the interest rate went to low levels, around 1 percent in his sample period. As such, Ireland (2009) suggested "... the semi-log specification ... may now provide a more accurate description of money demand. ... the new data points appear to trace out a demand curve that is far less interest-elastic ... (than) the earlier data ..." (p. 1043). Our time-varying cointegration relationship sees parameters changing over time. Thus, in Figure 4, we also plot the actual and fitted values for the money to income ratio ($MIRS/Y$). The estimated time-varying cointegration model in log-log form fits the actual data closely, which clearly did not occur in Ireland (2009) where he found spurious regression.

Further, Ireland suggested that the interest elasticity of money demand changed in the post-1980 period. We plot the time varying elasticity in Figure 5. We see that the elasticity varies between 0.013 and 0.250 and although the elasticity varies over the sample period, we find an average elasticity of 0.123. Moreover, the average elasticity over the same sample period considered by Ireland (2009) equals 0.113, whereas he finds an elasticity of 0.0873. Thus, our time-varying cointegration does not differ dramatically from his. Moreover, we do not find a big change in the interest elasticity of money demand between pre-1980, where it equals 0.129 and

post-1979, where it equals 0.119.

Since we use the log-log specification, we calculate the welfare cost of inflation from equation (6) for three different values of inflation -- 0-, 2-, and 10-percent. We plot the three different measures of welfare cost in Figure 6 measured as a percent, along with the interest rate also measured as a percent. Finally, the chart also includes the National Bureau of Economic Research recession dates in grey bars.

Examining the average welfare costs, recessions experience, on average, lower welfare costs than expansions. More specifically, expansions average 12, 10, and 7 percent higher welfare costs than recessions for the 0-, 2-, and 10-percent inflation rates, respectively. The maximum and minimum values of the welfare cost across the three values of inflation occur in 1962:Q4 and 1998:Q2, respectively. In addition, the Treasury-bill rate averages 48 percent higher (i.e., 7.21 versus 4.87 percent) during recessions relative to expansions.

Table 3 reports the empirical distribution of welfare costs for 0-, 2-, and 10-percent inflation rates.⁸ The distributions tend to concentrate at the lower end of the welfare cost distribution. The mean and median welfare costs rise as we move from 0-percent to 2-percent to 10-percent inflation. Ireland (2009) reports welfare costs as a percent of income for the static OLS model of 0.0131, 0.0356, and 0.219 for the 0, 2, and 10 percent inflation rates, respectively. Our welfare cost results exceed Ireland's, equaling 0.08, 0.123, and 0.277 as a percent of income, respectively.

5. Conclusion

This paper revisits the estimation of the welfare costs of inflation, using time-varying

⁸ Similar Figures and Tables on the 0- and 2-percent inflation rates are available from the authors.

cointegration to estimate the long-run money demand in the log-log and semi-log specifications of Meltzer (1963) and Cagan (1956), respectively. In preliminary tests, we find strong evidence against the standard time-invariant specification of Johansen (1988, 1991) and in favor of the vector error-correction model of Bierens and Martins (2009) where the cointegration vector is time-varying according to a flexible Fourier function of time, thus providing a much better fit of the actual data. This means that Fisher (1981), Serletis and Yavari (2004), and Ireland's (2009) estimates of the welfare cost of inflation for the US economy probably, in fact, measure the average of welfare cost of an actual changing welfare cost over time.

We conclude that the semi-log specification does not exhibit unit income elasticity. Instead, the log-log model does present a unitary elasticity for the whole sample and our estimate of the welfare cost of inflation for a 10-percent inflation rate lies in the range of 0.025 to 0.75 percent of GDP. In sum, our findings fall well within the ranges of existing studies of the welfare cost of inflation.. The interest elasticity of money demand shows substantial variability over our sample period.

Two natural extensions of the analysis of time-varying welfare costs of inflation include the alternative specifications of general equilibrium models (Cooley and Hansen, 1989) and partial equilibrium models (Feldstein 1997, 1999) as well as studying the implications of assuming an interest rate that equals zero in the limit.

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Table 1: Unit root test results

Variable	Test Type	Intercept	Conclusion	Intercept and Trend	Conclusion
<i>ln(m)</i>	ADF	-2.503	I(1)	-0.581	I(1)
	PP	-2.712*	I(0)	-0.621	I(1)
	DF-GLS	-0.324	I(1)	-0.204	I(1)
	Ng-Perron	-0.279	I(1)	-0.676	I(1)
<i>ln(M/P)</i>	ADF	1.557	I(1)	-1.301	I(1)
	PP	1.861	I(1)	-1.072	I(1)
	DF-GLS	2.891	I(1)	-0.694	I(1)
	Ng-Perron	2.508	I(1)	-2.097	I(1)
<i>ln(Y/P)</i>	ADF	-1.933	I(1)	-2.131	I(1)
	PP	-2.168	I(1)	--1.671	I(1)
	DF-GLS	2.699	I(1)	-1.263	I(1)
	Ng-Perron	1.317	I(1)	-4.914	I(1)
<i>ln(r)</i>	ADF	0.629	I(1)	-0.080	I(1)
	PP	-1.156	I(1)	-1.008	I(1)
	DF-GLS	-0.227	I(1)	-0.815	I(1)
	Ng-Perron	2.035	I(1)	-0.187	I(1)
<i>r</i>	ADF	-1.715	I(1)	-2.194	I(1)
	PP	-2.034	I(1)	-2.363	I(1)
	DF-GLS	-1.456	I(1)	-1.540	I(1)
	Ng-Perron	-5.192	I(1)	-6.190	I(1)

Note: The critical values are:

- ADF and PP with intercept (intercept and trend): -3.461, -2.875, and -2.574 (-4.002, -3.431, and -3.139) at the 1%, 5%, and 10% level of significance, respectively;
- DF-GLS with intercept (intercept and trend): -2.576, -1.942, and -1.615 (-3.461, -2.928, and -2.636) at the 1%, 5%, and 10% level of significance, respectively;
- Ng-Perron with intercept (intercept and trend): -13.800, -8.100, and -5.700 (-23.800, -17.300, and -14.200) at the 1%, 5%, and 10% level of significance, respectively;
- *, **, ***: Significant at 10%, 5% and 1% levels.

Table 2: TV Cointegration analysis of Money Demand

Money Variable	Interest Rate Variable	Lag-Length Information Criterion	p^*	m^*	TVC (LR)	WB	SB
$k=3$							
$\ln(M/P)$	r	SBC	2	15	(0.000)	(0.000)	(0.000)
		HQ	4	22	(0.000)	na	na
	$\ln(r)$	SBC	2	19	(0.000)	(0.000)	(0.000)
		HQ	5	23	(0.000)	na	na
$k=2$							
$\ln(m)$	r	SBC	4	1	(0.006)	(0.030)	(0.033)
		HQ	4	22	(0.000)	na	na
	$\ln(r)$	SBC	2	24	(0.000)	(0.000)	(0.000)
		HQ	7	29	(0.000)	na	na

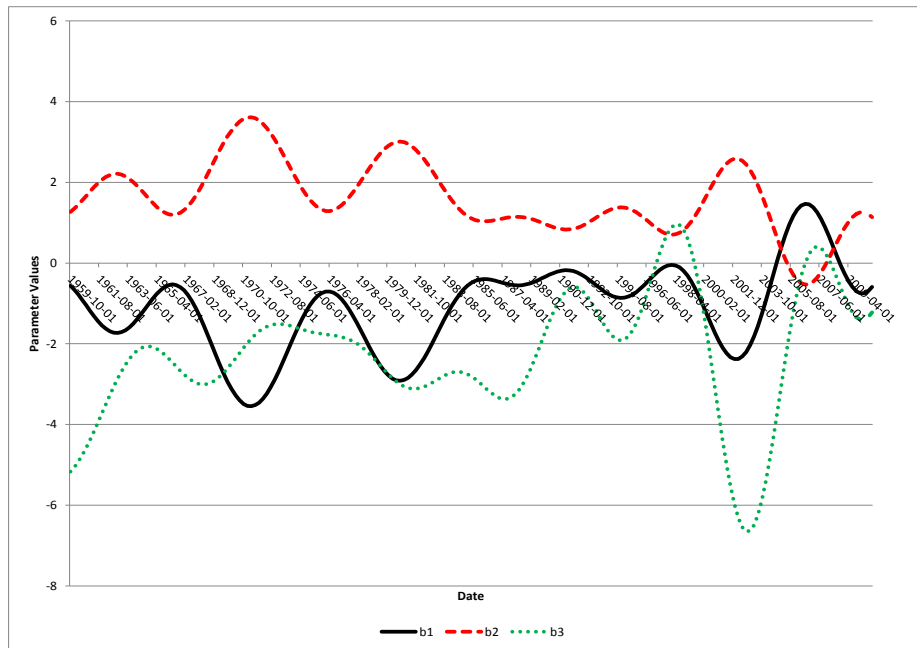
Notes: Numbers in parentheses are p-values for null hypothesis of standard TI cointegration (Johansen) against the alternative hypothesis of TV cointegration (Bierens and Martins): TVC is the original (chi-squared) statistic; WB is the Wild Bootstrap TVC statistic; and SB is the Sieve Bootstrap TVC statistic. k is the number of variables in the cointegration system. That is, when k equals 3, the additional variable in the cointegration equations is the natural logarithm of real GDP (Y/P). p^* is the lag order chosen according to the SBC or HQ criteria. m^* is the chosen number of Chebishev polynomials, given p^* . "na" means that the estimation of the reduced rank regression is not possible.

Table 3: Descriptive Statistics: Welfare Cost (1959:Q4-2010:Q4)

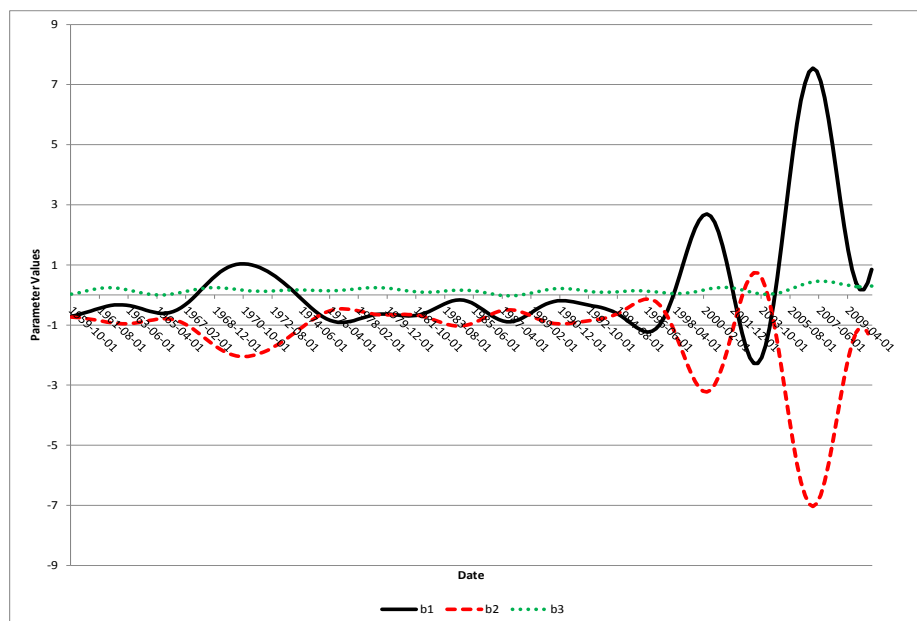
Inflation Rate	Mean	Median	Maximum	Minimum	Standard Deviation	Jarque-Bera
0 percent	0.080	0.067	0.243	0.006	0.047	69.005
			[1962:Q4]	[1998:Q2]		(0.00)
2 percent	0.123	0.106	0.357	0.010	0.069	56.371
			[1962:Q4]	[1998:Q2]		(0.00)
10 percent	0.277	0.2444	0.731	0.026	0.142	36.999
			[1962:Q4]	[1998:Q2]		(0.00)

Notes: Numbers in brackets correspond to the specific quarter for which maximum and minimum welfare costs are attained. Numbers in parentheses indicates p-value of the Jarque-Bera test.

Figure 1: Time-Varying Cointegration Coefficients

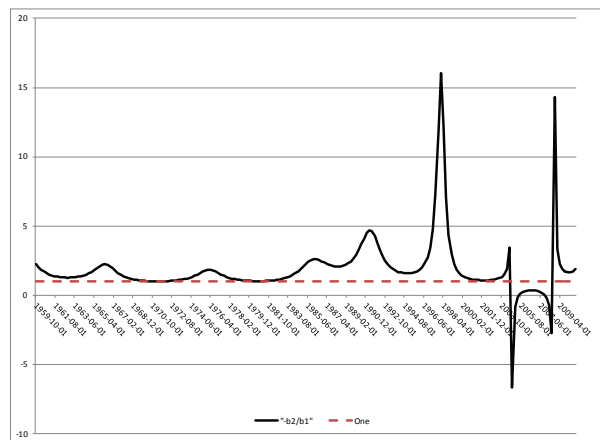


a. Semi-Log Specification

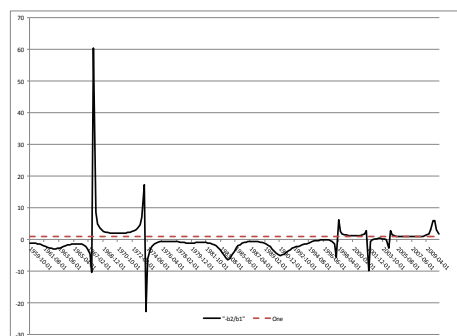


b. Log-Log Specification

Figure 2: Time-Varying Income Elasticities

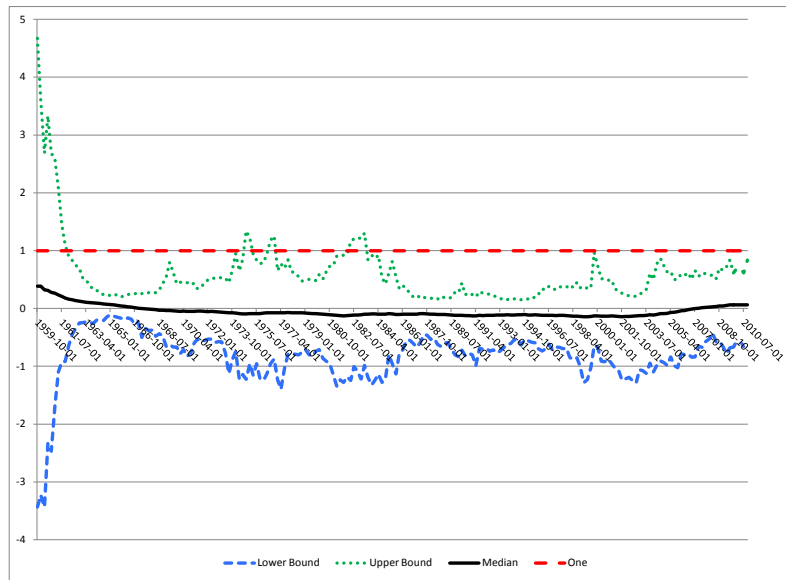


a. Semi-Log Specification

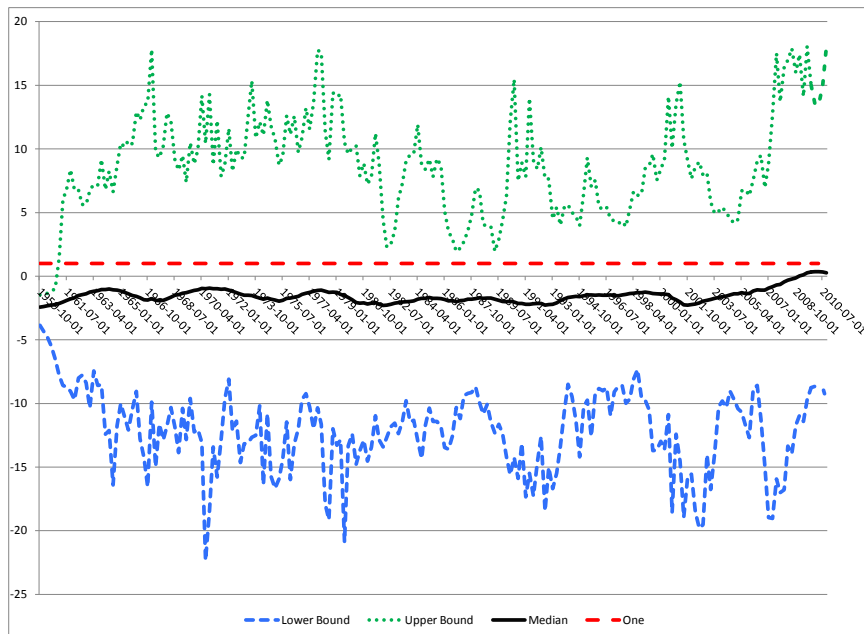


b. Log-Log Specification

Figure 3: Income Elasticities: Upper and Lower Bounds



a. Semi-Log Specification



b. Log-Log Specification

Figure 4: Fitted and Actual Money/Income versus the Nominal Interest Rate

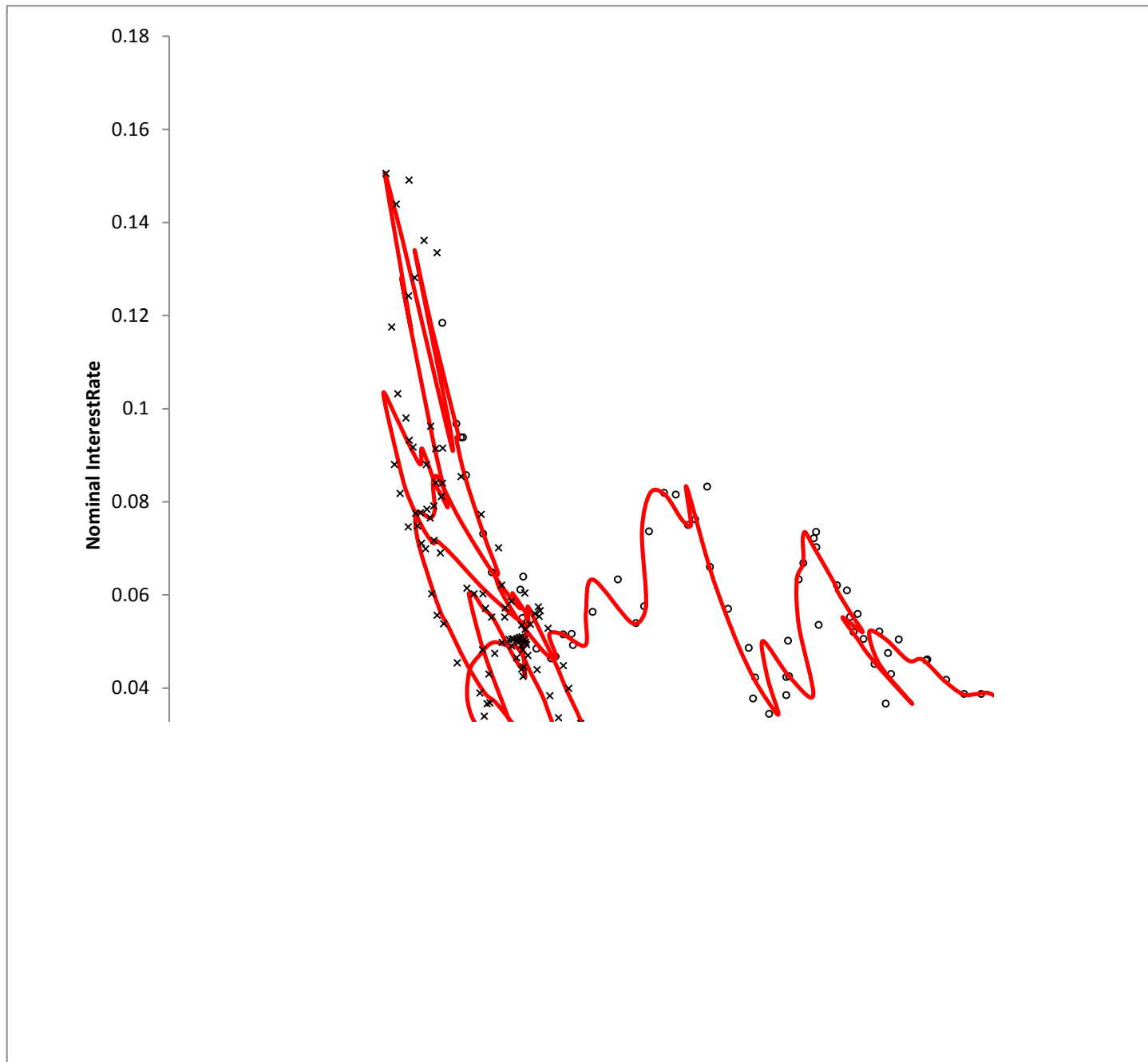


Figure 5: Time-Varying Interest Elasticity of Money Demand

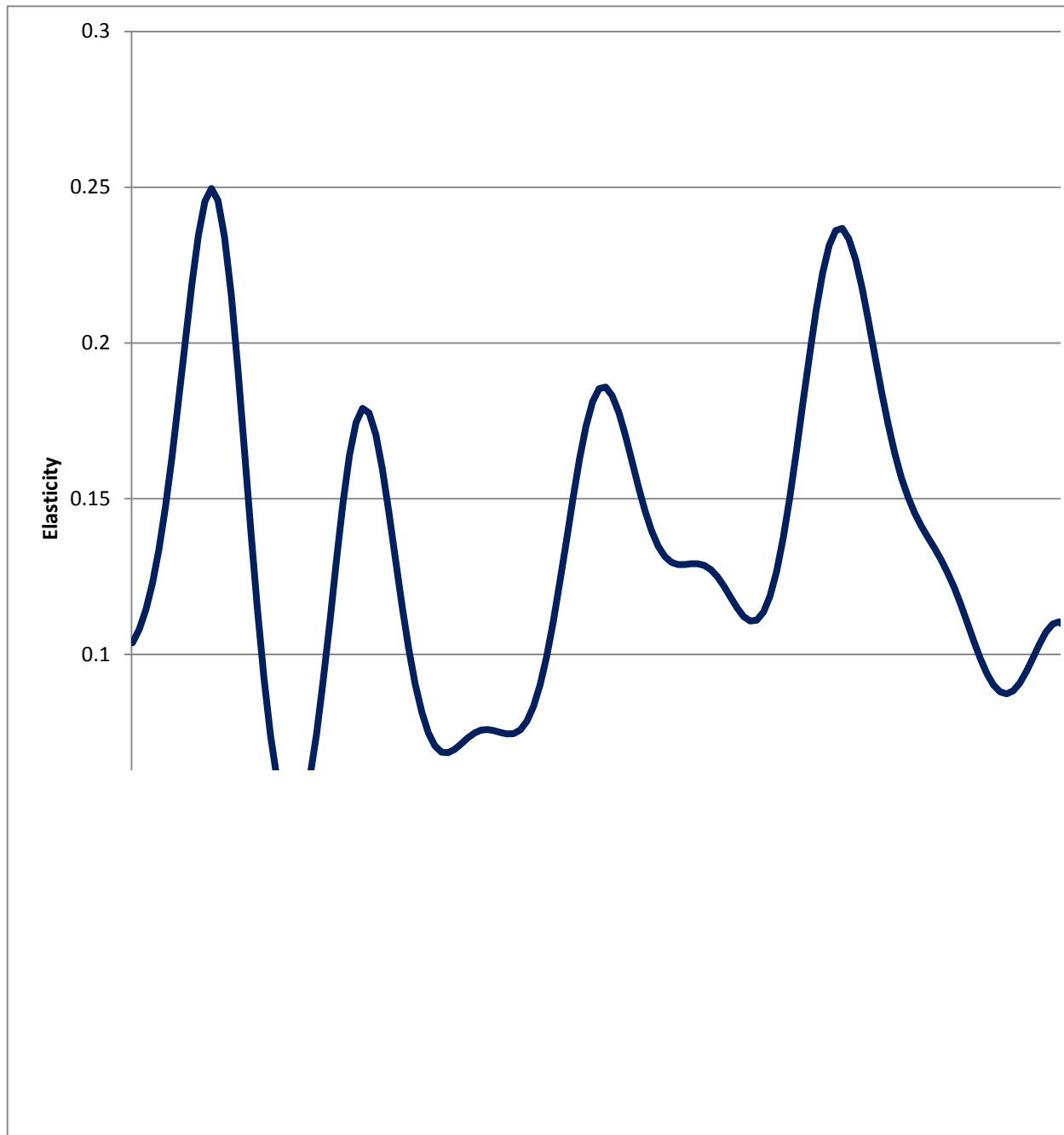


Figure 6: Time-Varying Welfare Cost of Inflation, 0-, 2-, and 10-Percent Inflation

