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Profit Efficiency in Wheat Cultivation: An All-India Analysis of Farm Level Data

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Abstract:

In this paper, we use Data Envelopment Analysis (DEA) to measure profit efficiency in wheat farming, drawing on farm-level data from 4,529 farms across eight major wheat-producing states in India for the year 2016–2017. To the best of our knowledge, this is the first study to examine production efficiency in Indian agriculture focused on a single crop rather than the total value of output at a national scale, encompassing all major wheat-growing regions.

We decompose profit efficiency into its technical and allocative components by exploring several alternative approaches to measure technical efficiency, including McFadden's gauge function along with the conventional radial input-oriented distance function and the directional distance function. To explicitly account for agro-climatic heterogeneity across states, we construct state-specific production frontiers using only within-state observations to define the reference technology.

Our empirical findings reveal that while average technical efficiency is not particularly low, extremely low levels of allocative efficiency significantly reduce overall profit efficiency. This result holds consistently across all definitions of technical efficiency considered. We speculate that the measured allocative efficiency could have resulted from market imperfections.

JEL Classification Codes: **C6; Q12**

Key Words: Technical Efficiency; Allocative Efficiency; Nerlovian Efficiency; Gauge Function;

Short Run Variable Profit Function; Data Envelopment Analysis

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October 6, 2025

1. Introduction

Back in the mid-1960s, two successive years of massive crop failure resulted in widespread food shortage in India and posed a real danger of mass starvation. Total foodgrains production in 1965–66 fell to 72.4 million tonnes, a decline of more than 19% from the previous year. This was followed by another year of low output at 74.2 million tonnes. In 1967–68, however, foodgrains output reached 95.1 million tonnes—an increase of 28%—exceeding the previous peak of 89.4 million tonnes in 1964–65 by 6.37%.[1] This marked the arrival of the Green Revolution in India and the beginning of a new era in Indian agriculture.

It may be noted that benefits of modern farming consisting of Mexican High Yielding Variety (HYV) seeds, chemical fertilizers, agricultural machinery, and irrigation in wheat were already evident in 1965–66. Wheat output increased by 10% (from 10.4 million tonnes in 1965–66 to 11.4 million tonnes in 1966–67) while rice production declined slightly from 30.6 to 30.4 million tonnes. In 1967–68 rice production increased by 24% and wheat increased by 45%.

An important point noted in the literature is that growth in food production spurred by the introduction of the modern farming technology was highly concentrated crop-wise, class-wise, and region-wise.[2] The special significance of wheat is evident from its share in total foodgrains production, rising from less than 18% prior to 1966–67 to 36% in 2018–19, nearly catching up with rice, which has generally remained between 40% and 44% throughout the period from 1950–51 to 2018–19.

Although rice production also eventually benefitted from the Green Revolution, it is apparent that wheat served as its principal gateway into Indian agriculture. As of 2024, over five decades after the Green Revolution, India has emerged as the third largest producer of wheat in the world. With a total output of 113.29 million tons metric tons, India trailed only China (140.1 million metric tons) and the European Union (122.12 million metric tons).[3] Its production has significantly surpassed that of the United States, which reported 53.65 million metric tons in output (USDA Foreign Agricultural Service, 2025).

[1] Data Source: Ministry of Agriculture & Farmers Welfare, Government of India.
[2] The main benefits accrued to wheat farmers with relatively large land holding in the North Indian states of Punjab and Haryana.
[3] When considering individual countries, India comes second to China.

In 2016–17, the year analyzed in this study, 15.26% of India's GDP came from agriculture, compared to 31.50% from industry and 53.24% from the service sector. At the same time, 44% of the labor force were employed in agriculture, compared to less than 25% in industry.[4] It is clear that economic wellbeing of a vast majority of India's rural population depends crucially on efficient utilization of resources in agriculture.

In the present paper, we use DEA to measure profit efficiency in wheat farming, using farm-level data from 4,529 farms across eight major wheat producing states of India for the year 2016–17. To the best of our knowledge, this is the first paper to examine production efficiency in Indian agriculture focused on a single crop (wheat, rather than total value of output) at a national scale, covering all major wheat-producing states of the country. In our decomposition of profit efficiency into technical and allocative efficiency components, we introduce McFadden's gauge function to measure technical efficiency as an alternative to the more conventional radial input-oriented distance function or the directional distance function. To explicitly account for agro-climatic heterogeneity across states, we conceptualize and construct state-specific production frontiers, using only observations from each respective state to construct the reference technology for that state.

The main contributions of this paper can be summarized as follows:

- We add to the sparse literature on measuring profit efficiency in Indian agriculture focusing on a single crop (rather than the value of total agricultural production) using farm level data. Ours is an all-India study based on farms from all of the major wheat producing states of the country.
- We use the different approaches described in Ray and Yang (2024) to project a technically inefficient input-output bundle on the production frontier to obtain corresponding measures of technical efficiency of a farm and formulate the appropriate nonparametric DEA (Data Envelopment Analysis) models.
- To the best of our knowledge, this is the first study that employs McFadden's gauge function (even though the concept has been available in the literature since 1978).

[4] Data Source: Reserve Bank of India.

- To avoid agro-climatic heterogeneity, we construct state specific nonparametric frontiers for efficiency evaluation of individual farms.

Our empirical findings show that while the average levels of technical efficiency (no matter how measured) are reasonably high (typically at or over 65%) for all states, levels of profit efficiency are uniformly low in all states. This, in its turn, is due to low levels of allocative efficiency. One possible explanation could be that while farmers make output and input decisions based on expected price (possibly the minimum support price announced by the government), due to market imperfections, they are obliged to sell their crops to the traders (who sell them seeds or fertilizers on credit) at much lower price. We found that for a number of states the average price received for the crop (and even the maximum price per unit received) is below the minimum support price guaranteed by the government. Inadequate access to institutional credit is possibly the main explanation of such apparently low level of allocative efficiency.

The rest of the paper unfolds as follows. Section 2 offers a brief review of the existing literature on measurement of production efficiency in Indian agriculture. Section 3 discusses the theoretical foundations of the neoclassical model, focusing on a profit-maximizing firm operating under perfect competition. Section 4 outlines pure (physical) measures of technical efficiency resulting from various projection directions. Section 5 illustrates these projection directions graphically, along with the corresponding profit levels at each projected point on the production frontier. Sections 6 and 7 present relevant optimization problems used to estimate technical efficiency and formulate the corresponding DEA models. Section 8 provides an in-depth empirical analysis of profit efficiency on Indian wheat farms. Section 9 concludes this paper.

2. Review of Literature

Numerous studies have evaluated production efficiency in general, and technical efficiency in particular, in Indian agriculture during the post-Green Revolution decades. These studies vary widely in respect of the crops analyzed (e.g., rice, jute, or wheat), the level of output aggregation (specific crops or total agricultural production), the aggregation of production units (individual farms versus the district- or state-level), and the geographical scope (farms from a specific state or specific district or states across the country).

Regarding analytical methods, while an overwhelming majority of studies have applied the parametric Stochastic Frontier Analysis (SFA), several have used the nonparametric method of Data Envelopment Analysis (DEA). In fact, some of them have used both approaches.

Jha and Rhodes (1999) used SFA to measure technical efficiency in wheat production, based on 660 farm-level observations from the years 1981–82 and 1982–83. Of these, 282 were from Haryana (HA), where the Green Revolution had already made significant inroads, and the other 378 were from Madhya Pradesh (MP), where the adoption of modern farming lagged considerably behind. Their findings from Haryana (HA) indicated that larger farm size and ownership of land and machinery contributed to higher technical efficiency. In contrast, no such evidence was found for Madhya Pradesh (MP), where traditional farming practices remained more prevalent.

For technical efficiency evaluations of other crops[5] using farm-level data, Tadesse (1997) examined data for 129 HYV Rice (IR-20) cultivators across different ecological regions of Tamil Nadu during the 1992–93 Northeast monsoon cropping season (October–December). Mythili and Shanmugam (2000) used SFA to study the efficiency of 100 rice paddy farmers from Tamil Nadu over the years 1990–91 through 1992–93. Shanmugam (2003) also used SFA to measure technical efficiency in rice cultivation in Tamil Nadu, utilizing 329 farm-level observations from the two major cropping seasons over the years 1990–91 through 1992–93.[6] The study did not find any significant effect of landholding size on technical efficiency in rice production.

Several studies have used the total agricultural production, aggregated over individual crops, as a measure of output. Mithiya et al. (2019) estimated a Cobb Douglas production frontier with district-level aggregated input and output data for 17 districts of West Bengal over a 23-year period, covering 1990–91 through 2013–14. They found a positive and statistically significant effect of an increase in the average size of landholdings on technical efficiency.[7] Pradhan and Mukherjee (2018) used data constructed from two rounds of the Indian Rural Economic and Demographic household Survey (REDS) (both as annual cross sectional data and as a 2-peiord panel data), to estimate a Cobb Douglas production function for the value of aggregate

[5] Kumbhakar (1994) Kumbhakar and Bhattacharyya (1992) analyzed production efficiency in rice production.
[6] The study also covered two cash crops – groundnut and cotton.
[7] Given that their dependent variable is total agricultural production, one cannot infer from this result whether the positive effect is true for any specific crop.

agricultural output using SFA. Their dataset covered 4,487 households from the 1999 round and 4,869 households from the 2006 round of the survey.[8]

In a pioneering study investigating allocative efficiency for profit maximization in Indian agriculture, Hopper (1965) analyzed the allocation of land and other inputs by a sample of 43 farms in the state of Uttar Pradesh (UP) to four crops (barley, wheat, peas, and gram). This study found that, at sample means, the value of the marginal product of the inputs did not differ significantly from their respective market prices. He concluded, therefore, that even within traditional farming systems, farmers' input allocation across alternative crops was consistent with profit maximizing behavior.

In one of the earliest applications of the dual profit function, Lau and Yotopoulos (1971) examined the relative efficiency of small and large farms using state-level average data from six states over a three-year period. Land and capital were treated as fixed inputs, while labor was the only variable input. Output was measured by the gross value of production. The specified dual (variable) profit function was derived from an underlying Cobb Douglas production function. They found significantly lower profit efficiency for large farms, attributing the higher efficiency of small farms to better supervision by owner-managers. Kumbhakar and Bhattacharyya (1992), along with Kumbhakar (1994), estimated a stochastic dual profit function jointly with a system of input-demand and output supply equations. Their models allowed for deviations between shadow prices and market prices of inputs, conditional on farm size and the farmer's education level. Using a dataset of 287 farm-level observations for paddy cultivators in West Bengal from 1980 to 1985, their empirical results indicated that higher levels of farmer education were positively associated with allocative efficiency.

Jha et al. (2000) measured cost efficiency in wheat farming in Punjab for the years 1981-82 and 1982-83. Their data covered 300 observations in 1981-82 and 298 in 1982-83. The authors concluded that, overall, large farms were technically more efficient than small farms.[9] However, the statistical results reported in their Tables 2 and 3 did not fully support this conclusion. For the entire state, their reported rank correlation between technical efficiency and farm size was only

[8] The negative and statistically significant coefficient of acreage in their estimated Cobb Douglas production function (reported in Table 2) across all regressions raises serious questions about the validity of their conclusions about technical efficiency and/or its determinants.

[9] The authors provide no information about the inputs included in their nonparametric DEA models.

0.067 in 1981-82, which was not statistically significant at the 10% level. In 1982-83, the rank correlation statistic increased to 0.135 and was highly significant. When analyzing clusters of 10 farms from different *tehsils* within the state, the rank correlation between technical efficiency and size was significant in only 4 out of 30 cases in 1982-83. By contrast, in the previous year, significant correlations were found in 13 out of 30 cases; however, 3 out of these 13 significant correlations were negative.

Few studies of technical efficiency in Indian agriculture have employed DEA. Apart from Jha et al. (2000) mentioned above, an important nonparametric DEA application is Ray and Ghose (2014), which used state-level aggregate data on inputs and outputs to measure Pareto-Koopmans efficiency in Indian agriculture covering the years 1970–71 through 2000–01. Two categories of aggregate outputs – foodgrains and non-foodgrains (vegetables, jute, cotton, and other cash crops) were considered. Among the inputs, the authors separately analyzed technical efficiency in utilizing modern inputs (chemical fertilizers, agricultural machinery, pump, and power) and traditional inputs (land, labor, and irrigation). Their DEA results showed that, on average, the utilization of modern inputs was highest in the Eastern states and lowest in the Southern states. On average, output technical efficiency was more than 90% in all regions. However, for non-foodgrains outputs, the average efficiency was considerably lower, although the Eastern region still performed the best.

3. The Production Technology

Consider an industry producing *m* outputs using *n* inputs. Denote output bundles by vectors $y \in R_+^m$ and input bundles by $x \in R_+^n$. An input-output combination (x, y) is feasible if the output y can be produced from the input x. The production technology faced by the firms in the industry can be defined by the production possibility or technology set

$$T = \{(x, y): y \text{ can be produced from } x\}. \quad (1)$$

In parametric models, the set T is typically defined, in the single output case, by the production function

$$y = f(x); y \in R_+, x \in R_+^n \quad (2)$$

and, in the multiple output case, by the transformation function

$$F(x,y) = \alpha; x \in R_+^m, y \in R_+^n \tag{3}$$

The corresponding production possibility sets are

$$T = \{(x,y): y \leq f(x); x \in R_+^n; \text{y} \in R_+\} \tag{4}$$

and

$$T = \{(x,y): F(x,y) \leq 0; x \in R_+^n; \text{y} \in R_+^m\} \tag{5}$$

It is assumed that the production set is closed and weakly monotonic in both inputs and outputs. Weak monotonicity implies

$$\frac{\partial f}{\partial x_i} \geq 0, (i = 1,2,\ldots,n) \tag{6}$$

in the single output case, and

$$\frac{\partial F}{\partial x_i} \leq 0, (i = 1,2,\ldots,n) \tag{7}$$

$$\frac{\partial F}{\partial y_j} \geq 0, (j = 1,2,\ldots,m) \tag{8}$$

in the multiple output case.

Weak monotonicity is also described as free disposability of inputs and outputs.

3.1 Profit-maximization and Profit Efficiency

The profit maximization problem of a competitive firm, facing the output price (vector) p in the product market and the input price vector w in the factor market, is formulated as:

$$max\ \Pi = p'y - w'x \quad s.t. \quad F(x,y) \leq 0 \tag{9a}$$

in the multiple-output case and

$$max\ \Pi = py - w'x \quad s.t. \quad y \leq f(x) \tag{9b}$$

in the single output case.

Suppose that the optimal input-output bundle which solves the problem in (9a or 9b) is (x^*, y^*), and the maximum attainable profit is given by $\Pi^* = p'y^* - w'x^*$. Let the observed input output

be (x^0, y^0), with the corresponding actual profit $\Pi_0 = p'y^0 - w'x^0$. The overall performance of the firm[10] can be evaluated by comparing its actual profit Π_0 with the maximum profit Π^*.

3.2 Variable Profit Maximization

In the profit maximization problem (9a-9b), the firm is assumed to be choosing all inputs and outputs subject to their feasibility restriction only. In practice, however, many inputs are treated as fixed in the short run. Similarly, some outputs may also be contractually predetermined. External environmental factors (such as rainfall or the quality of transportation infrastructure) may also influence the amount of output that can be produced from a given set of inputs, even though these factors lie outside the firm's control. Partitioning the input vector $x = (v, z)$ where v is the vector of variable inputs and z is the vector of fixed inputs and/or externally determined factors, McFadden (1978) defined the restricted profit function as

$$\Pi_R = max\, p'\, y - w_v'v \;\; s.t. F(v, z, y) \leq 0 \tag{10}$$

Lau (1978) called this maximum difference between a firm's revenue and its variable costs the short-run profit function. It may be noted that all of the empirical applications cited above — including Kumbhakar and Bhattacharyya (1992), Lau and Yotopoulos (1971), Sidhu (1974), and Sidhu and Baanante (1981) - have estimated and analyzed the variable profit function.

Within the framework of this variable profit function, the profit efficiency of a firm is

$$PE = \frac{\Pi_R^0}{\Pi_R^*} = \frac{p'y^0 - w_v'v^0}{p'y^* - w_v'v^*}. \tag{11}$$

3.3. Ratio and Difference Measures

A firm attains full efficiency when its actual profit $\Pi_0$equals the maximum Π^*. Since $\Pi^* \geq \Pi_0$, by definition, the proximity of a firm's actual profit to the maximum serves an indicator of its efficiency. As argued in Ray and Yang (2024), two alternative measures of this proximity are the profit ratio and the profit difference.

An intuitively appealing measure of profit efficiency is the ratio of actual profit to maximum attainable profit, defined as:

[10] We use 'firm' as a generic production unit and 'farm' for agricultural producers.

$$PE = \frac{\Pi_0}{\Pi^*} = \frac{p'y^0 - w'x^0}{p'y^* - w'x^*}. \quad (12)$$

Two important properties of the ratio measure of profit efficiency should be noted. First, because both the numerator and the denominator are expressed in the same monetary units, this measure is a pure number. Second, since proportional changes in all output and input prices leave the optimal input-output bundle (x^*, y^*) unchanged, this measure is homogeneous of degree zero in prices. Additionally, it is comparable to other efficiency measures, such as cost efficiency, which is defined as the ratio of minimum to actual cost, and revenue efficiency, which is defined as the ratio of actual to maximum revenue.

One potential limitation of the ratio-based measure is that, unlike cost or revenues (whether actual or optimal), profits —both actual and, in some cases, maximum—can be negative. In fact, if both actual and maximum profits are negative, the ratio-based profit efficiency can be greater than 1. Thus, when actual profit is negative, the ratio of actual to maximum profit is not a meaningful indicator of efficiency.

To overcome the difficulties posed by negative actual profit, one can use the difference between actual and maximum profits rather than their ratio. Define:

$$\Delta = \Pi^* - \Pi_0. \quad (13)$$

Note that $\Delta \geq 0$ by definition, even when Π_0 (or even Π^*) is negative. In equation (13), Δ measures the amount of potential profit that remains unrealized due to inefficiency. Clearly, a larger value of this shortfall indicates a higher level of profit inefficiency. However, using the nominal difference between maximum and actual profit as an indicator of inefficiency has its own limitations. First, if all output and input prices increase proportionately, Δ also increase by the same proportion even though neither the actual nor the profit-maximizing input-output bundles would change. Second, this nominal difference does not take account for the scale of operation of a firm.[11]

3.4. Alternative ways to Normalize

[11] For example, consider a relatively small firm *A* with actual profit $\Pi_0^A = 10$ and $\Pi^{A*} = 20$, and a larger firm *B* with actual profit $\Pi_0^B = 985$ and $\Pi^{B*} = 1000$. Thus $\Delta^B = 15 > \Delta^A = 10$ implying that *B* is more inefficient than *A*. However, the shortfall for *B* from the maximum is only 1.5% compared to 50% for *A*.

For meaningful inter-firm comparisons of performance, the profit shortfall Δ must be normalized by a variable that captures the scale of a firm's operations. Chambers et al. (1998), abbreviated as CCF, used the sum of the observed cost ($C_0 = w'x^0$) and observed revenue($R_0 = p'y^0$) for normalization, and measure the inefficiency of the firm[12] as

$$NE = \frac{\Delta}{C_0+R_0} = \frac{\Pi^*-\Pi^0}{C_0+R_0}. \tag{14}$$

Ray (2004) proposed two alternative normalizations: one by the observed cost and the other by the observed revenue, defining as:

$$\Delta_C = \frac{\Pi^*-\Pi^0}{C_0} \tag{15a}$$

and

$$\Delta_R = \frac{\Pi^*-\Pi^0}{R_0}. \tag{15b}$$

The expression in (15a) measures the lost 'return on outlay' while (15b) can be interpreted as the 'lost return on sales'.

Figure 1 graphically illustrates the ratio and alternative difference measures of profit efficiency in the one-input, one-output case. In this diagram, input (x) is measured along the horizontal axis and output (y) along the vertical axis. The curve $y = f(x)$ represents the production function, evaluated at the technological frontier. Points on or below this curve constitute the production possibility set.

Point A denotes the observed input-output bundle (x_0, y_0) of the firm. Given a pair of input and output prices, w and p, the iso-profit line for any target profit level $\tilde{\Pi}$ is defined as:

$$y = \frac{\tilde{\Pi}}{p} + \frac{w}{p}x. \tag{16}$$

These iso-profit lines are parallel, with slope equal to the input-output price ratio $\frac{w}{p}$ and the intercept equal to the normalized profit $\frac{\tilde{\Pi}}{p}$. At this point, we can rescale the output so that the output

[12] CCF designate the normalized difference in (14) above as Nerlovian efficiency (NE) and consider the sum of the observed revenue and cost of the firm as a measure of its size.

price (p) is set to unity. The ray OR through the origin represents the zero-profit line $y = wx$. The parallel line through Point A is the iso-profit line for the actual profit of the firm: $y = \Pi_0 + wx$. The vertical intercept OD measures the actual profit $\Pi_0 = y - wx$.

Point C, where an iso-profit line is tangent to the production function, represents the optimal input-output bundle (x^*, y^*) that yields the maximum profit $\Pi^* = y^* - wx^*$. The intercept of this tangent line is OE, which measures the maximum profit Π^*. The ratio measure of profit efficiency is $PE = \frac{\Pi_0}{\Pi^*} = \frac{OD}{OE}$. The difference $DE = OE - OD = \Pi^* - \Pi_0$ represents the shortfall of actual profit from the maximum profit and serves as a (nominal) measure of profit inefficiency.

As noted above, to be a meaningful criterion of comparative evaluation of performance across firms, the profit difference needs to be normalized. Since the output price is set to unity, the observed revenue becomes $R_0 = py_0 = y_0$, so that revenue coincides with output. Hence, if the actual revenue is used for normalization, we would obtain:

$$\Delta_y = \frac{\Pi^* - \Pi_0}{R_0} = \frac{\Pi^* - \Pi_0}{y_0} = \frac{OE - OD}{AB} = \frac{DE}{OF} \tag{17}$$

An alternative is to use the actual cost $C_0 = wx_0$ for normalization (as in Ray (2004)). To obtain a measure of the actual cost $C_0 = wx_0$, we refer to point G on the zero-profit line, $y = wx$, where $BG = y = wx_0$. The cost-normalized difference measure of profit inefficiency is

$$\Delta_x = \frac{\Pi^* - \Pi_0}{C_0} = \frac{\Pi^* - \Pi_0}{wx_0} = \frac{OE - OD}{BG} = \frac{DE}{OH} \tag{18}$$

Finally, for the Nerlovian efficiency of Chambers et al. (1998), we have:

$$NE = \frac{\Pi^* - \Pi_0}{R_0 + C_0} = \frac{\Pi^* - \Pi_0}{y_0 + wx_0} = \frac{OE - OD}{AB + BG} = \frac{DE}{OF + OH}. \tag{19}$$

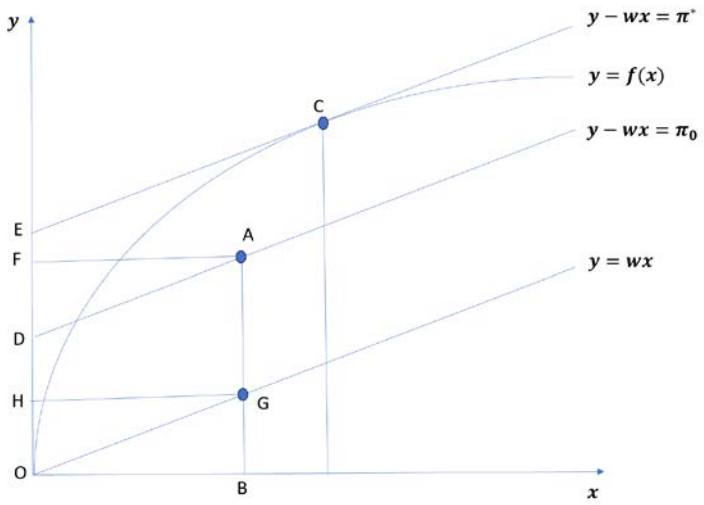

Figure 1 Ratio and Difference Measures of Profit Efficiency

4. Sources of Technical Inefficiency

A firm is said to be technically inefficient if its observed input-output bundle (x^0, y^0) lies in the interior (rather than the frontier) of the production possibility set. In such cases, the firm has opportunities to improve efficiency through one or more of the following adjustments:

1. Input reduction without output loss: The firm can reduce input usage while maintaining the same level of output, thereby increasing profit through cost savings.
2. Output expansion without input increase: The firm can increase output without using additional input, leading to higher revenue without additional cost.
3. Simultaneous input reduction and output expansion: The firm can both reduce input and increase output, resulting in a simultaneous increase in revenue and decrease in cost.

4.1 Alternative Measures of Technical Efficiency

Shephard Distance Function and Farrell's Radial Technical Efficiency

Following the definition of Shephard (1953, 1970), the output-oriented distance function measured at an input-output bundle (x, y) is

$$D^y(x,y) = inf_\mu\{\mu : \left(x, \frac{1}{\mu}y\right) \in T,\ \mu > 0\} \tag{20}$$

Similarly, the input-oriented distance function is

$$D^x(x,y) = sup_\delta\{\delta : \left(\frac{1}{\delta}x, y\right) \in T,\ \delta > 0\} \tag{21}$$

The output distance function is equivalent to the radial output efficiency defined by Farrell (1957):

$$\tau_y(x,y) = \frac{1}{\varphi^*}; \varphi^* = sup_\varphi\{\varphi: (x, \varphi y) \in T, \varphi > 0\} \quad (22)$$

On the other hand, the input-oriented distance function is the inverse of the radial input efficiency of Farrell (1957):

$$\tau_x(x,y) = \theta^* = inf_\theta\{\theta : (\theta x, y) \in T,\ \theta > 0\} \quad (23)$$

Any input-output bundle (x^0, y^0) that is technically inefficient can be projected onto the efficient frontier either by proportionally scaling down the input bundle x^0 by the factor $\theta(=\frac{1}{\delta})$ or by proportionally scaling up the output bundle y^0 by the factor $\varphi(=\frac{1}{\mu})$. These radial projections imply that either all inputs or all outputs are scaled proportionally.

The input-oriented measure of technical efficiency, $\tau_x(x^0, y^0)$, is appropriate for evaluating cost efficiency, where the focus is on proportionally scaling down the input vector x^0 without altering the output vector y^0. Similarly, the output-oriented technical efficiency, $\tau_y(x^0, y^0)$, is suitable for assessing revenue efficiency, as it reflects the potential to proportionally expand output while holding inputs constant.

However, in the context of profit maximization, both input and output vectors are optimally chosen as decision variables. Therefore, a technical efficiency measure that seeks to scale only inputs or only outputs is inadequate as a component of profit efficiency.

Directional Distance Function

Radial input- or output-oriented projections continue to be popular benchmarks for evaluating inefficient input-output bundles located within the interior of the production possibility set. However, the direction of projection can be chosen arbitrarily and need not be limited to proportional scaling of either the input or output bundle.

Chambers et al. (1996), building upon Luenberger (1992)'s benefit function, introduced an efficiency measure that allows projection of any observed input-output bundle onto the production frontier in an arbitrarily chosen direction (g^x, g^y). This measure, known as the directional distance function (DDF), is defined as:

$$\begin{aligned}\vec{D}(x^0, y^0; g^x, g^y) = \beta^* &= sup_\beta\{\beta: (x^0, y^0) + \beta(-g^x, g^y) \in T\} \\ &= sup_\beta\{\beta: (x^0 - \beta g^x, y^0 + \beta g^y) \in T\}. \end{aligned} \quad (24)$$

It is obvious that for $(g^x = 0, g^y = y^0)$, $\beta = \varphi - 1$, while $(g^x = x^0, g^y = 0)$ leads to $\beta = 1 - \theta$.

A commonly used direction in empirical applications, proposed by Briec (1997), is $(g^x = x^0,\ g^y = y^0)$. Under this specification, the DDF becomes

$$\vec{D}(x^0, y^0; x^0, y^0) = sup_\beta\{\beta: \big((1-\beta)x^0, (1+\beta)y^0\big) \in T\}. \quad (25)$$

This formulation of the DDF allows simultaneous contraction of inputs and expansion of outputs in proportion to the observed levels, offering a flexible and interpretable measure of inefficiency.

Graph Hyperbolic Distance Function

Färe et al. (1985,1994) defined the graph hyperbolic distance function as

$$D^{Graph}(x,y) = sup_{\delta}\{\,\delta : \left(\frac{1}{\delta}x, \delta y\right) \in T,\ \delta > 0\}. \tag{26}$$

The corresponding technical efficiency can be measured as

$$\tau_{Graph}(x,y) = \frac{1}{\delta}. \tag{27}$$

By construction, the observed point A and its efficient projection B on the technology set lie on a rectangular hyperbola, as illustrated in Figure 2. This geometric relationship gives rise to what is known as the graph hyperbolic measure of technical efficiency. This measure can be interpreted as the geometric mean of input efficiency and output efficiency, capturing the simultaneous proportional contraction of inputs and expansion of outputs. Fare et al. (1994) treated δ itself as a direct measure of technical efficiency.

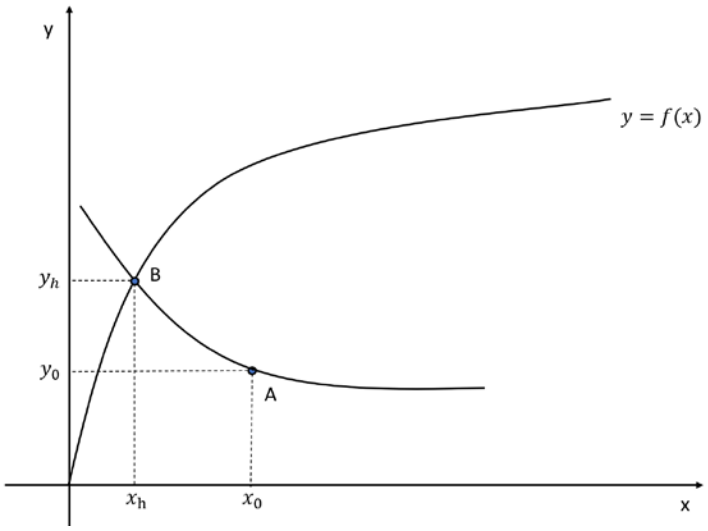

Figure 2. Graph Hyperbolic Projection for a Technically Inefficient Point

The DDF in (25) can be seen to be a linear approximation to D^{Graph} defined in (26) above. Note

$$\frac{1}{\delta} = f(\delta) \approx f(\delta_0) + f'(\delta_0)(\delta - \delta_0) = f(\delta_0) - \frac{1}{\delta_0^2}(\delta - \delta_0). \tag{28}$$

Setting $\delta_0 = 1$, we obtain:

$$\frac{1}{\delta} \approx 2 - \delta \tag{29}$$

Hence, let $\delta = 1 + \beta$. By substitution, one would get (25) from (26). It is important to note, however, that setting $\delta_0 = 1$ implies the input-output bundle lies on the frontier. A linear approximation becomes increasingly inaccurate if the observed input-output bundle is significantly distant from the frontier.

McFadden's Gauge Function

Long before the development of the graph hyperbolic measure or the directional distance function, McFadden (1978) proposed the *gauge function* as a way to measure technical efficiency. This approach involves equi-proportional scaling of both input and output vectors in the *same* direction. To formalize this, McFadden defined the negative image of the production possibility set T in the netput space as

$$T^{-} = \{(-x, y): (x, y) \in T\}. \tag{30}$$

The gauge function is

$$\begin{aligned} H(-x, y) = \gamma^{*} &= inf_{\gamma}\{\gamma : \left(-\tfrac{1}{\gamma}x, \tfrac{1}{\gamma}y\right) \in T^{-},\ \gamma > 0\} \\ &= inf_{\gamma}\{\gamma : \left(\tfrac{1}{\gamma}x, \tfrac{1}{\gamma}y\right) \in T,\ \gamma > 0\}. \end{aligned} \tag{31}$$

For any technically inefficient input-output bundle (x^0, y^0) located in the interior of *T*, we have $\gamma^{*} < 1$. The gauge function projection scales both x^0 and y^0 upwards by the factor $\sigma^{*} = \frac{1}{\gamma^{*}} > 1$ until the bundle reaches the frontier.[13]

Clearly, for McFadden's gauge function, the directional vector is defined as $g^x = -x^0, g^y = y^0$, and the directional distance function is given by:

$$\vec{D}(x^0, y^0; x^0, y^0) = sup_{\beta}: \{\beta: \left((1+\beta)x^0, (1+\beta)y^0\right) \in T\}.[14] \tag{32}$$

McFadden's gauge function stands apart from other widely used measures of technical efficiency due to two key features.

First, when the original input-output bundle yields positive profit, applying the gauge function can lead to an improvement in projected profit even when both inputs and outputs are increased. This is because the function scales both inputs and outputs proportionally in the same direction, potentially increasing both revenue and cost. However, since this scaling preserves the input-output ratio, the overall outcome—typically an increase in profit—reflects a more favorable bundle along the same ray from the origin.

Second, the increase in profit at the gauge projection of an inefficient input-output bundle arises from the homogeneity of degree one in the profit function with respect to input and output prices. That is, profit, defined as the difference between revenue and cost, scale linearly with prices. Importantly, this should not be interpreted as scale efficiency, because the average productivity

[13] An anomaly arises when an input-output bundle is found to be efficient under both input-oriented and output-oriented DEA measures, yet is classified as inefficient according to McFadden's gauge function. This happens when the ray through the origin intersects the production frontier twice – once at the observed input-output bundle and the second time at a point further towards the northeast. The second point is preferable as the technically efficient projection because it would yield a higher profit no matter what the input and output prices. See Ray and Yang (2024) for a detailed discussion of this point.

[14] Thus, $H(x, y) = \gamma = \frac{1}{1+\beta}$.

along the ray remains unchanged when both input and output vectors are scaled by the same factor. The gauge function thus reflects a price-dependent improvement in profitability without implying changes in returns to scale.[15]

Non-Radial Pareto-Koopmans Efficiency

Aparicio et al. (2013) generalized the DDF of Briec (1997) by allowing for different proportional scaling of input and output vectors. However, because both output and input vectors are scaled (up or down) radially, slacks may remain at the resulting projection. As a result, the technically efficient projection may not be a Pareto-Koopmans efficient bundle. To address this limitation, one can consider a non-radial slack-based projection proposed by Tone (2001), which is also known as an enhanced Russell measure (Pastor et al., 1999), or the Pareto-Koopmans efficient projection (Ray, 2004). This approach involves solving the following non-linear optimization problem for a firm with n inputs and m outputs:

$$\Gamma(x,y) = min \frac{\frac{1}{n}\sum_{i=1}^{n}\theta_i}{\frac{1}{m}\sum_{r=1}^{m}\varphi_r} : (\theta_1 x_1, \theta_2 x_2, \dots, \theta_n x_n;\ \varphi_1 y_1, \varphi_2 y_2, \dots, \varphi_m y_m) \in T;$$
$$\theta_i \leq 1, (i = 1, 2, \dots, n);$$
$$\varphi_r \geq 1, (r = 1, 2, \dots, m). \qquad (33)$$

where θ_i and φ_r represent individual scaling factors for each input and output, respectively.

5. Profit at Alternative Technically Efficient Projections

In Figure 3 below, several of the alternative technically efficient projections of an inefficient input-output bundle are shown graphically.

[15] By contrast, (ray) average productivity increases when outputs are radially scaled up or inputs are radially scaled down.

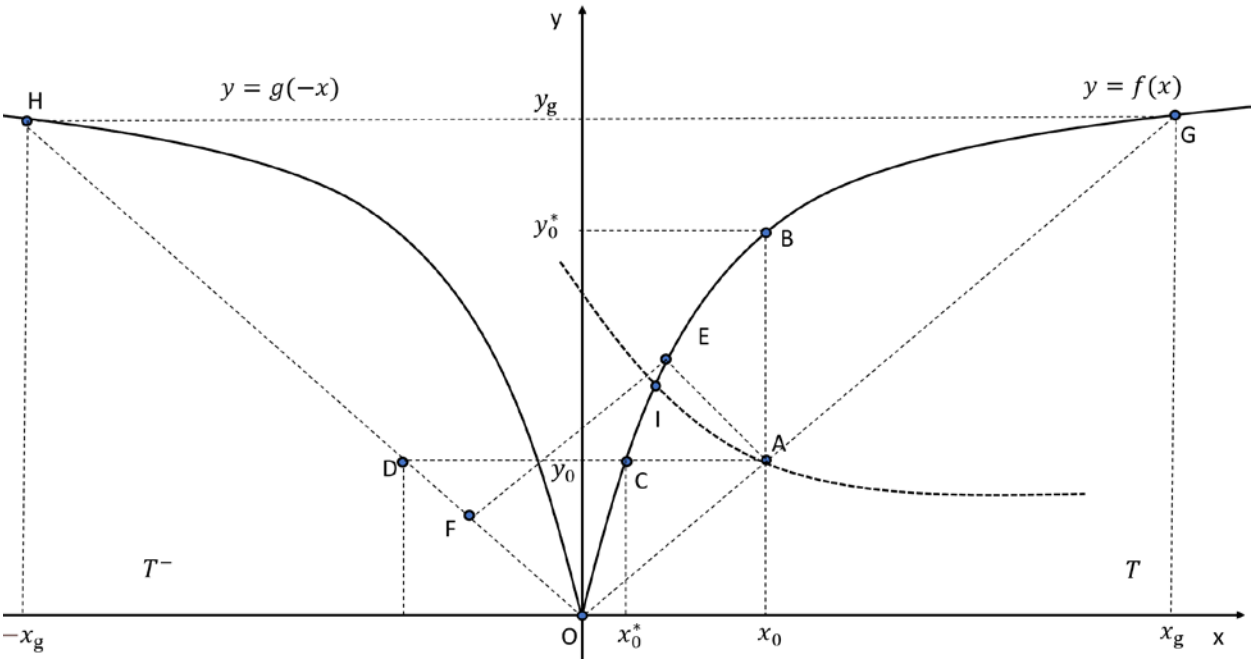

Figure 3. Alternative Technically Efficient Projections on to the Frontier

In Figure 3, point *A* represents the input-output bundle (x^0, y^0), the curve $y = f(x)$ is the production frontier and $y = g(-x)$ is its negative image in the second quadrant. The point *B* vertically above *A* is the output-oriented efficient projection $(x^0, y_0^*) = (x^0, \varphi^* y^0)$. Thus,

$$D^y(x^0, y^0) = \frac{Ax_0}{Bx_0} = \frac{Oy_0}{Oy_0^*}. \tag{34}$$

Point *C* is the input-oriented efficient projection $(x_0^*, y^0) = (\theta^* x^0, y^0)$ and the input-oriented distance function is

$$D^x(x^0, y^0) = \frac{Ay_0}{Cy_0} = \frac{Ox_0}{Ox_0^*} = \frac{1}{\tau_x(x^0, y^0)}. \tag{35}$$

Point D in the northwest quadrant represents the netput $(-x^0, y^0)$ and the point *E* on the production frontier $y = f(x)$ is the efficient projection of (x^0, y^0) in the direction $(g^x = x^0, g^y = y^0)$. The corresponding DDF is

$$\vec{D}(x^0, y^0; g^x = x^0, g^y = y^0) = \beta = \frac{AE}{OD} = \frac{OF}{OD}. \tag{36}$$

Point *G* on the curve $y = f(x)$ is the radial outward projection of the observed input-output bundle at point *A.* Corresponding to point G, point *H* is the projection of the netput bundle *D* on to the curve $y = g(-x)$ leading to the efficient input-output bundle $(x_g, y_g) = (\frac{1}{\gamma}x^0, \frac{1}{\gamma}y^0)$ and the gauge function is

$$H(x^0, y^0) = \gamma = \frac{OA}{OG} = \frac{OD}{OH}. \tag{37}$$

Finally, Point *I* represents the graph hyperbolic projection of inefficient Point A. Note that points *C, I, E, B,* and *G* are all located on the frontier of the production possibility set and are, therefore, technically efficient. As shown in Figure 4 below[16], all of them lead to a higher profit than $\pi_0 = py^0 - wx^0$. For points *C, I, E,* and *B* it is obvious because in all these cases either cost is lower (as at *C)* or revenue is higher (as at *B*) or both (as at *E and I).* At the gauge function projection *G*, both input and output are bigger than at *A* and one may wonder if the cost may increase more than revenue leading to a lower profit than π_0. But, because $(x_g, y_g) = (\sigma x_0, \sigma y_0)$ and $\sigma = \frac{1}{\gamma} > 1$ profit at point *G* is $\sigma\pi_0 > \pi_0$ so long as $\pi_0 > 0$.

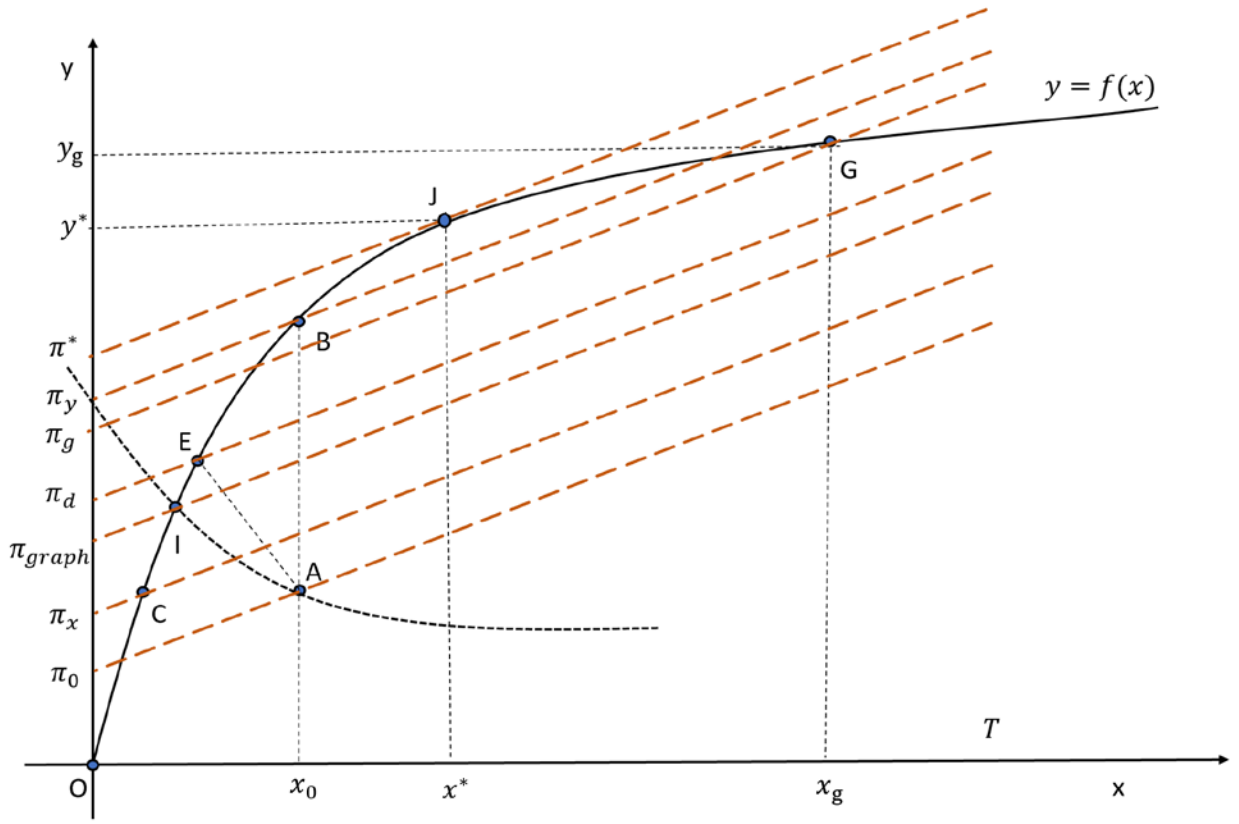

Figure 4. Profit at Alternative Technically Efficient Projections on to the Frontier

However, none of them yields the maximum profit (π^*) attained at Point J. For the different orientations of projection on to the frontier, the corresponding allocative efficiency is obtained as the ratio of π_T and π^*, where π_T is the intercept of the appropriate iso-profit line.

6. Nonparametric Approximation of Technology Set

In the nonparametric Data Envelopment Analysis (DEA) framework—following the seminal work on production analysis of Afriat (1972), Debreu (1951), Farrell (1957), and Shephard

[16] Points in Figure 4 correspond to the different technically efficient projections shown above in Figure 3.ally efficient projections shown above in Figure 3.

(1953, 1970)—the production possibility set is constructed from observed input-output data under the assumptions of convexity and weak monotonicity, without imposing any parametric form on the production or transformation function.
Consider the following input-output data set for N firms in an industry:

$$D = \{(x^j, y^j); x^j \in R_+^n, y^j \in R_+^m, j = 1,2,\dots,N\} \quad (38)$$

Based on the assumptions of convexity and closedness of the production possibility set and free disposability of inputs and outputs, a nonparametric approximation of the technology is the free disposal convex hull of the data set, defined as:

$$S = \{(x,y): x \geq \sum_{j=1}^N x^j; y \leq \sum_{j=1}^N y^j; \sum_{j=1}^N \lambda_j = 1; \lambda_j \geq 0; j = 1,2,\dots,N\} \quad (39)$$

For any observed input-output bundle $(x^k, y^k) \in$ D facing output and input prices (p^k, w^k), the actual profit is $\pi_o^k = p^k y^k - w^k x^k$ and maximum profit is

$$\begin{aligned} &\pi^*(p^k, w^k) = max_{x,y}\, p^k\, y - w^k x \\ &s.t. (x,y) \in S. \end{aligned} \quad (40)$$

For the variety of technical efficiencies and the corresponding technically efficient profit (for any firm *k* in the sample) one needs to solve the relevant optimization problem from below:

Output-oriented Efficiency

$$\begin{aligned} &\varphi_k^* = max\, \varphi \\ &s.t. (x^k, \varphi y^k) \in S. \\ &\tau_y = \frac{1}{\varphi_k^*}. \end{aligned} \quad (41)$$

Input-oriented Efficiency

$$\begin{aligned} &\theta_k^* = min\, \theta \\ &s.t. (\theta x^k, y^k) \in S. \\ &\tau_x = \theta_k^*. \end{aligned} \quad (42)$$

Directional Distance Function

$$\begin{aligned} &\beta_k^* = max\, \beta \\ &s.t. ((1-\beta)x^k, (1+\beta)y^k) \in S. \\ &\tau_d = \frac{1-\beta_k^*}{1+\beta_k^*}. \end{aligned} \quad (43)$$

Graph Hyperbolic Efficiency

$$\begin{aligned} &\delta_k^* = \max \delta \\ &s.t. (\tfrac{1}{\delta} x^k, \delta y^k) \in S. \\ &\tau_{Graph} = \frac{1}{\delta_k^*}. \end{aligned} \quad (44)$$

McFadden's Gauge Function

$$\gamma_k^* = min\, \gamma$$

$$s.t.\left(\frac{1}{\gamma}x^k, \frac{1}{\gamma}y^k\right) \in S.$$
$$\tau_g = \gamma_k^*. \quad (45)$$

Non-radial Pareto-Koopmans Efficiency

$$\Gamma_k(x,y) = min\frac{\frac{1}{n}\sum_{i=1}^{n}\theta_i^*}{\frac{1}{m}\sum_{r=1}^{m}\varphi_k^*} : \left(\theta_1 x_1^k, \theta_2 x_2^k, \dots, \theta_n x_n^k;\ \varphi_1 y_1^k, \varphi_2 y_2^k, \dots, \varphi_m y_m^k\right) \in S;$$
$$\theta_i \leq 1, (i = 1, 2, \dots, n);\ \varphi_r \geq 1, (r = 1, 2, \dots, m).$$
$$\tau_{PK} = \Gamma_k{}^*. \quad (46)$$

Once the profit maximization problem in (40) has been solved, profit at the technically efficient projection for each unit can be obtained from the optimal solution to problems (41) – (46).

7. Empirical Measures of Technical Efficiency

To estimate physical and profit-oriented technical efficiencies, the following DEA model variations are considered.

7.1.1. Input-oriented Projection:

$$min_{\lambda,\theta}\ \theta$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x^j \leq \theta x^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j y^j \geq y^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

$$\lambda_j \geq 0, j = 1,2,\dots,N \quad (47)$$

Thus, the physical technical efficiency is defined as $\tau_x(x^0, y^0) = \theta *.$ (48)

The corresponding profit is $\pi_x = p'y^0 - \theta^* w'x^0.$ (49)

7.1.2. Output-oriented Projection:

Banker et al. (1984) formulated[17] the following linear programming (LP) problem for measuring the output-oriented radial technical efficiency of (x^0, y^0) :

$$max_{\lambda,\varphi}\ \varphi$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x^j \leq x^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j y^j \geq \varphi y^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

$$\lambda_j \geq 0, j = 1,2,\ldots,N \tag{50}$$

Thus, the physical technical efficiency is defined as $\tau_y(x^0, y^0) = \frac{1}{\varphi^*}$. (51)

The corresponding profit is $\pi_y = \varphi^* p' y^0 - w' x^0$. (52)

7.1.3. Directional Distance Function Projection:

$$max_{\lambda,\beta}\ \beta$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x^j + \beta x^0 \leq x^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j y^j - \beta y^0 \geq y^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

[17] The initial formulation of DEA by Charnes et al.(1978) assumed constant returns to scale, while this formulation allows variable returns. It should be noted that although BCC are credited with this generalization, this formulation was already there in Afriat (1972) for the single output case.

$$\lambda_j \geq 0, j = 1,2,\ldots,N \quad (53)$$

Thus, the physical technical efficiency is defined as $DDF(x^0, y^0) = \beta^*$. (54)

The corresponding profit is $\pi_d = (1+\beta^*)p'y^0 - (1-\beta^*)w'x^0 = p'y^0 - w'x^0 + \beta^*(p'y^0 + w'x^0)$ (55)

7.1.4. Graph Hyperbolic Projection:

$$max_{\lambda,\delta}\ \delta$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x^j + \delta x^0 \leq 2x^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j y^j \geq \delta y^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

$$\lambda_j \geq 0, j = 1,2,\ldots,N \quad (56)$$

Thus, the physical technical efficiency is defined as $\tau_{Graph}(x^0, y^0) = \frac{1}{\delta^*}$. (57)

The corresponding profit is $\pi_{Graph} = \delta^* p'y^0 - \frac{1}{\delta^*} w'x^0$. (58)

7.1.5. McFadden's Gauge Projection:

$$max_{\lambda,\sigma}\ \sigma$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x^j \leq \sigma x^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j y^j \geq \sigma y^0\ ;$$

$$\sum_{j=1}^{N} \lambda_j = 1$$

$$\lambda_j \geq 0, j = 1,2,\ldots,N \tag{59}$$

Thus, the physical technical efficiency is defined as $\tau_{Gauge}(x^0, y^0) = \gamma = \frac{1}{\sigma^*}$. (60)

The corresponding profit is $\pi_{Gauge} = \frac{1}{\gamma^*}(p'y^0 - w'x^0) = \frac{1}{\gamma^*}\pi_{0.}$ (61)

7.1.6. Pareto-Koopmans Projection:

As shown in Ray (2004), the nonlinear objective function can be linearized as:

$$\Gamma = f(\theta, \varphi) \approx f(\theta^0, \varphi^0) + \sum_i (\theta_i - \theta_i^0)\left(\frac{\partial f}{\partial \theta_i}\right)\Big|_0 + \sum_r (\varphi_r - \varphi_r^0)\left(\frac{\partial f}{\partial \varphi_r}\right)\Big|_0 \tag{62}$$

Note that $\frac{\partial f}{\partial \theta_i} = \frac{\frac{1}{n}}{\frac{1}{m}\sum_r \varphi_r}$ and $\frac{\partial f}{\partial \varphi_r} = -\frac{\frac{1}{n}\sum \theta_i}{\frac{1}{m}(\sum_r \varphi_r)^2}$. (63)

Thus, using $\theta_i^0 = 1$ for all i and $\varphi_r^0 = 1$ for all r, $\Gamma \approx 1 + \frac{1}{n}\sum_i \theta_i - \frac{1}{m}\sum_r \varphi_r$. (64)

Then we can iteratively solve the following minimization problem:

$$min_{\lambda,\theta,\varphi}\, \tilde{\Gamma} = \frac{1}{n}\sum_i \theta_i - \frac{1}{m}\sum_r \varphi_r$$

$$s.t.$$

$$\sum_{j=1}^{N} \lambda_j x_{ij} \leq \theta_i x_{i0}, (i = 1,2,,n);$$

$$\sum_{j=1}^{N} \lambda_j y_{rj} \geq \varphi_r y_{ro}, (r = 1,2,\ldots,m);$$

$$\sum_{j=1}^{N} \lambda_j = 1;$$

$$\lambda_j \geq 0, (j = 1,2,\ldots,N)$$

$$0 \leq \theta_i \leq 1, (i = 1,2,\ldots,n);$$

$$\varphi_r \geq 1, (r = 1,2,\ldots,m); \tag{65}$$

Thus, the physical technical efficiency is defined as $\tau_{PK}(x^0, y^0) = \frac{\frac{1}{n}\sum_{i=1}^{n}\theta_i^*}{\frac{1}{m}\sum_{i=1}^{m}\varphi_r^*}$. (66)

The corresponding profit is $\pi_{pk} = \sum_{r=1}^{m}\varphi_r^* p_r y_r - \sum_{i=1}^{n}\theta_i^* w_i x_i$. (67)

8. The Empirical Application on Indian Farms

8.1 The Data and Construction of Variables

Our empirical analysis uses the nonparametric method of Data Envelopment Analysis (DEA) to measure the variable profit efficiency for 4,529 wheat-producing farms across eight major wheat-growing states in India: Bihar (BI), Gujarat (GU), Haryana (HA), Madhya Pradesh (MP), Maharashtra (MH), Punjab (PU), Rajasthan (RA), and Uttar Pradesh (UP). Collectively, these states account for over 80% of the nation's total wheat production.

The data was constructed from the holding level input and output information obtained from the *Cost of Cultivation Report* published by the Government of India for the year 2016-2017. The quantity of the single output, wheat, is measured in quintals[18] and the output price per unit is derived from the reported value of the main produce. The inputs included in this analysis are:

1. Land (LAND)
2. Labor (LAB)
3. Seed (SEED)
4. Fertilizers (FRTL)
5. Farm machinery (MACH)
6. Irrigation (IRR)

The land input (LAND) is directly reported in hectares as the size of the holding.

The labor input (LAB) is measured as the total number of hours worked by hired, family, and casual workers. A single wage rate per hour was constructed by dividing the total expenditure on all types of labor by the total labor hours.

The quantity of the seed (SEED) is measured in kilograms, and its price is calculated by the expenditure on seeds per kilogram.

The quantity of fertilizers (FRTL) is reported as the total kilograms of nitrogen, phosphates, and potassium (NPK) applied. The price per kilogram was derived by dividing the reported fertilizers expenditure by the total quantity applied.

The Cost of Cultivation data separately reports two types of machinery: farm machines (e.g., power tillers) and irrigation machines (e.g., pumps). For the capital or machinery input, we use

[18] One quintal is 100 kilograms.

only the reported number of farm machinery (both owned and hired). The price per hour of farm machinery is calculated from the corresponding reported expenditure.

Measuring the irrigation input (IRR) posed a greater challenge, as the expenditure on irrigation (presumably for canal irrigation) was reported in rupees, but no information was available on the actual quantity of irrigation water used. However, both hours and expenditure were reported for irrigation machinery (owned and hired). To convert irrigation expenditure into equivalent machine hours, we divided the total irrigation cost by the hourly operating cost of the irrigation equipment.[19]

Table 1 shows the distribution of farms across states and the average holding size (in hectares) for each. Bihar (BI) and Uttar Pradesh (UP) together account for nearly half of the sample, representing over 46% of the total observations. These two states also have the smallest average landholding sizes. In contrast, Haryana (HA) and Punjab (PU) show significantly larger average holdings. Historically, Haryana (HA) and Punjab (PU) played a pioneering role in the adoption of modern agricultural practices and were central to India's Green Revolution. The compatibility of mechanized farming with high-yielding wheat varieties in these states has encouraged the consolidation of fragmented landholdings. On the other hand, both Bihar (BI) and Pradesh (UP), known as part of the *BIMARU* (or ailing) group of economically lagging states[20], were slower to adopt modern techniques and continued to exhibit economically unviable landholding patterns.

There were 4,697 farms with relevant input-output price and quantity information available. However, as shown in Table 1, each state includes some observations where the variable cost exceeded the value of output, resulting in negative variable profits. In a perfectly competitive market with free entry and exit, a fundamental principle is that firms will shut down if they are unable to cover their variable costs, even in the short run. The presence of so many farms with negative profits suggests that they have subsequently exited the market, or more likely, experienced unexpected crop failures during that year. These cases are mostly found in states of Gujarat (GU), Maharashtra (MH) and Rajasthan (RA), all of which are drought-prone regions. Farms in these areas likely faced additional constraints that are not applicable to others in the sample, making their inclusion in broader analysis inappropriate.

Table 2 presents the summary statistics of input-output quantities for the 4,529 farms included in our analysis. The all-India average area under wheat cultivation is 0.62 hectares (1.53 acres). The average yield is 31.98 quintals per hectare (4,862 kg per acre).

7.2 Results

Table 3 reports the mean levels of short-run variable profit efficiency across different states. As evident from the data, the average efficiency is low in all states. Maharashtra (MH) records the lowest mean efficiency at 7.4%, followed by Gujarat (GU), Rajasthan (RA), and Uttar Pradesh (UP), where the mean ranges between 10.5% and 11.6%. Even Punjab (PU), often regarded as

[19] Given the small share of this input in the total cost, such admittedly crude approximation is unlikely to distort the results in any significant way.

[20] The other two states in this group are Madhya Pradesh (MP) and Rajasthan (RA).

the breadbasket of India, shows a modest mean efficiency of 18%. Somewhat surprisingly, Madhya Pradesh (MP) is the only state with a mean efficiency exceeding 25%. These figures highlight the extremely low producer surplus for the average wheat farmer in India. When accounting for land rent, whether actually paid on leased land or imputed on owned land, the net return falls even further below the maximum attainable level.

Table 4 summarizes the mean levels of Nerlovian efficiency, an alternative (normalized) difference-based measure. Unlike the ratio measures reported in Table 3, which show the proportion of the maximum achievable profit that is actually realized, these Nerlovian efficiency measures capture the shortfall from the maximum profit as a proportion of the sum of a farm's revenue and (variable) cost. As argued by Ray and Yang (2024), this form of normalization may lack intuitive sense and likely comes from the linear programming dual formulation of the optimization problem in the directional distance function (DDF) framework. In any case, the mean Nerlovian (in)efficiencies are broadly consistent with the ratio-based profit efficiency measured reported in Table 3, with one notable exception: Punjab (PU). While Punjab (PU) shows a profit efficiency of 18%, its Nerlovian inefficiency is unusually high 775.75%, highlighting a potential discrepancy in interpretation between the ratio-based and difference-based profit efficiencies.

As discussed in detail in the preceding sections, there are two sources of any shortfall from the maximum achievable profit for a farm. The first is *technical inefficiency*, which occurs when the chosen input-output bundle lies below the frontier or the production possibility set. The second one is the *allocative inefficiency*, which arises when the technically efficient projection of the observed bundle deviates from the profit-maximizing bundle on the frontier.

Unlike profit efficiency, technical efficiency depends on how the inefficient input-output bundle is projected onto the frontier. Moreover, while physical measures of technical efficiency rely solely on input-output quantities, the technical efficiency components of profit efficiency (defined as the ratio of actual profit to the profit at the technically efficient projection) also depends on input and output prices. As a result, the technical efficiency component of the profit efficiency typically differs from their corresponding physical efficiency. The only exception is McFadden's gauge function, which aligns the physical and price-dependent technical efficiency.

Table 5 presents the mean levels of various alternative measures of (physical) technical efficiency across different states. For each state, the average of these different measures is reported as the overall mean. The biggest overall mean of physical technical efficiency is observed in Haryana (HA) at 83%, followed by Punjab (PU) at 81.7%. On the other hand, Gujarat (GU) exhibits the lowest overall mean technical efficiency of 60.8%. These results suggest that, regardless of the projection direction, Indian farmers are generally reasonably efficient in terms of physical technical efficiency.

Across all alternative measures and states, the highest level of average technical efficiency recorded is in Haryana (HA) at 92% using the graph hyperbolic approach. The lowest record is recorded at 46.6% in Gujarat (GU) using McFadden's gauge function. The range of variation

between alternative measures of average technical efficiency is the widest (between 46.6% and 80.7%) in Gujarat (GU). For all other states, the range is quite similar (about 24%).

Table 6 presents an alternative measure of technical efficiency, defined as the ratio of actual profit to the profit achievable under various technically efficient projections. This score evaluates the extent to which farms are able to realize their potential profitability, given their current resource usage and allocation. Several noteworthy observations emerge from this analysis.

First, under McFadden's gauge function, the physical and profit-based efficiency scores are identical. This outcome arises because the technically efficient projection in this case involves a uniform proportional increase in both inputs and outputs. Consequently, costs and revenues change by the same proportion, resulting in an unchanged profit ratio at the technically efficient projection.

Second, the average levels of profit-based technical efficiency are consistently lower than the corresponding physical efficiency measures. This discrepancy can be primarily attributed to price sensitivity: even when a farm is physically efficient in converting inputs into outputs, it may rely on high-cost inputs or face unfavorable output prices, thereby reducing overall profitability. Additionally, a suboptimal input mix, reflected in low allocative efficiency, can further diminish profit levels despite high output.

State-level mean allocative efficiency values, corresponding to various profit-based technical efficiency measures, are presented in Table 7. The data clearly indicate that the low levels of profit efficiency reported in Table 3 are primarily attributable to poor allocative performance. With the exception of Madhya Pradesh (MP), the overall mean allocative efficiency across all states remains below 25%. This pattern persists even in agriculturally advanced states such as Haryana (HA) and Punjab (PU), underscoring the widespread inefficiencies in input allocation despite relatively high physical efficiency.

The overall conclusion from Tables 5-7 is that while Indian wheat farmers demonstrate reasonable (physical) technical efficiency, implying that they are relatively effective in converting inputs into maximum possible output, they fall short in terms of profit and allocative efficiency. This suggests that although inputs are not grossly wasted, the choice of input combinations and output levels is far from optimal.

The generally low levels of profit efficiency, and particularly allocative efficiency, are concerning. Unlike the 1950s, when Indian agriculture was largely subsistence-based farming with limited commercialization, the post-Green Revolution era has seen farmers increasingly reliant on market-purchased inputs and producing primarily for commercial sale. These findings challenge Hopper (1965)'s optimistic assertion that farmer's input choices were broadly consistent with the principle of equating input prices to the respective values of their marginal products.

Ray and Bhadra (1993) applied Varian's (1984) Weak Axiom of Cost Minimization (WACM) to test the cost-minimizing behavior of farmers cultivating Monsoon (Aman) paddy, using 149 farm-level observations collected from various districts in West Bengal between 1980 and 1985.

WACM offers a simple nonparametric test of cost-minimizing behavior that relies entirely on free disposability of output. To test whether farm *k* is cost minimizing, one first weakly orders the observations such that $j \geq k$ only if $y_j \geq y_k$. Suppose that the input price vector paid by farm *k* is w^k, Weak axiom of Cost Minimization implies that firm k's choice of inputs is not consistent with cost minimization if there exists any observed input bundle x^j such that $w^k x^j < w^k x^k$ while $y_j \geq y_k$. In their empirical analysis, Ray and Bhadra found that only 16 out of the 149 observations satisfied the conditions of WACM, suggesting limited adherence to cost-minimizing behavior among the sampled farms.

Ray and Bhadra (1993) further examined variable cost minimization by treating land and capital as fixed inputs, while labor, seed, and fertilizer were considered variable. In this framework, the input vector is partitioned as $x = (x_v, z)$, where x_v represents variable inputs and z denotes fixed inputs. Correspondingly, the input price vector is partitioned as $w = (w_v, r)$. To test for variable cost minimization, the variable cost for farm *k* is calculated as $VC_k = w_v^{k\prime} x_v^k$, and compared with $w_v^{k\prime} x_v^j$ for all farms $y_j \geq y_k$ and $Z_j \geq Z_k$. Farm *k*'s choice of its variable inputs is consistent with minimization of variable cost so long as $VC_k = w_v^{k\prime} x_v^k \leq w_v^{k\prime} x_v^j$ for all *j*. When tested for variable cost minimization, 60 of the 149 observations proved to be consistent with the Weak Axiom of Variable Cost Minimization. The authors concluded that, in general, these farmers were making rational choices regarding their variable inputs in response to relative input prices.

In 2016–17, the all-India Minimum Support Price (MSP) was set at ₹1,625 per quinta (Department of Food and Public Distribution, 2025). However, the average output price received by farmers in several states within our sample fell below this benchmark. As shown in Table 8, farmers in Bihar (BI) received an average price of ₹1,527.27 per quintal, with the highest recorded price at ₹1,617.97, still below the MSP. Similar trends were observed in Madhya Pradesh (MP) (₹1,572.89) and Uttar Pradesh (UP) (₹1,574.10), where the average prices also failed to meet the guaranteed support level.

This disparity is not coincidental. These three states are among the most economically disadvantaged regions in India, characterized by a high concentration of poor, marginal farmers with limited access to formal credit. As a result, many rely on village moneylenders and local traders, who often provide cash or agricultural inputs on credit under the condition that farmers sell their produce to them at below-market prices. Additionally, farmers are frequently compelled to sell immediately after harvest—when market prices are lowest—in order to repay loans.

It is well known that farmers often need to make decisions on the choice of inputs and output based on anticipated rather than actual prices. To the extent that farmers rely on the minimum support price from the government, any discrepancy between the price expected and price actually realized will show up as allocative inefficiency.[21]

Underdeveloped infrastructure in these regions further exacerbates the issue, limiting farmers' ability to transport their products to larger wholesale markets (mandis) where prices are typically

[21] DeCanio (1980) conducted a study on Kansas farmers to examine the economic losses resulting from forecasting errors.

higher. Therefore, what appears to be allocative inefficiency may, in fact, stem from market imperfections rather than irrational decision-making by these vulnerable farming communities.

9. Conclusion

The extremely low levels of profit efficiency observed in the sample, covering farms across all major wheat-producing states in India, reflect a concerning reality. Despite India's emergence as the world's second-largest producer of wheat, many farmers are barely recovering their variable costs. In contrast, average technical efficiency consistently exceeds 65%, regardless of the measurement approach. This disparity suggests that the primary source of low profit efficiency lies in poor allocative efficiency. These findings challenge Hopper (1965)'s reassuring conclusion that Indian farmers make rational input and output decisions. Consequently, any analysis of production technology, including input demand and output supply elasticities derived from a dual profit function based on competitive profit maximization (like those in Sidhu and Baanante (1981)) may be fundamentally flawed.

A compelling question arises: why do such low levels of allocative efficiency persist in Indian wheat cultivation, a crop for which purchased inputs constitute the majority of production costs, even decades after the onset of the Green Revolution? The long-standing narrative of the uninformed village farmer with limited access to market towns no longer suffices as a plausible explanation. A more reasonable explanation is that the prices recorded (or even those actually paid) diverge significantly from the shadow prices that guide farmers' input-output decisions. This discrepancy may reflect market imperfections and constraints in access to reliable price information. Enhancing farmers' access to formal credit markets could reduce their dependence on exploitative input suppliers and informal lenders. Coupled with guaranteed prices support for outputs, such interventions may enable farmers to make more economically rational decisions, thereby improving allocative and profit efficiency.

Table 1: Number of Farms and Average Holding Size by State

	BI	GU	HA	MP	MH	PU	RA	UP
Total Number of Farms	1050	528	343	567	288	534	354	1033
Number of Farms with Negative Variable Profit	4	70	0	0	63	1	24	6
Number of Farms with Positive Variable Profit	1046	458	343	567	225	533	330	1027
Average Holding Size (in Hectares)	0.622	0.658	2.171	1.432	0.599	1.633	0.943	0.589

Table 2: Summary Statistics of Output and Input Data (All India)

	Output	Land	Labor	Machine	Seed	Fertilizer	Irrigation
Measurement Units	Quintals	Hectares	Hours	Hours	Kilograms	Kilograms	Machine Hours
Count	4529	4529	4529	4529	4529	4529	4529
Mean	19.83	0.62	215.43	5.77	70.90	85.19	27.01
Std	13.66	0.43	142.77	4.35	47.96	62.73	19.81
Min	1.10	0.03	18.67	0.30	3.00	2.66	1.00
Max	90	3	1157	30.75	360	532	127

Table 3: Profit Efficiency ($\Pi_0/\ \Pi^*$) by State

	BI	GU	HA	MP	MH	PU	RA	UP
Count	1046	458	343	567	225	533	330	1027
Mean	0.152	0.105	0.155	0.263	0.074	0.180	0.106	0.116
Std	0.125	0.112	0.155	0.175	0.106	0.174	0.126	0.110
Min	0.002	0.000	0.000	0.020	0.000	0.005	0.000	0.001
Max	1	1	1	1	1	1	1	1

Table 4: Nerlovian (In)Efficiency by State

	BI	GU	HA	MP	MH	PU	RA	UP
Count	1046	458	343	567	225	533	330	1027
Mean	2.741	5.284	4.652	1.604	5.180	7.757	3.227	4.939
Std	2.749	4.569	3.965	1.471	4.226	9.897	2.514	5.093
Min	0	0	0	0	0	0	0	0
Max	30.597	41.744	29.216	9.797	29.479	92.615	17.074	58.045

Table 5: Mean (Physical) Technical Efficiency by State

	BI	GU	HA	MP	MH	PU	RA	UP
Input	0.730	0.625	0.810	0.763	0.728	0.834	0.746	0.699
Output	0.776	0.635	0.866	0.769	0.730	0.846	0.795	0.744
DDF	0.733	0.613	0.868	0.759	0.721	0.834	0.759	0.698
Graph	0.866	0.807	0.920	0.879	0.861	0.917	0.880	0.849
Gauge	0.701	0.466	0.830	0.697	0.614	0.797	0.729	0.690
Pareto-Koopmans	0.611	0.500	0.686	0.621	0.622	0.671	0.657	0.622
Min	0.611	0.466	0.686	0.621	0.614	0.671	0.657	0.622
Max	0.866	0.807	0.920	0.879	0.861	0.917	0.880	0.849

Table 6: Mean (Price-dependent) Technical Efficiency by State

	BI	GU	HA	MP	MH	PU	RA	UP
Input	0.643	0.570	0.806	0.730	0.559	0.858	0.602	0.644
Output	0.605	0.472	0.778	0.654	0.507	0.798	0.587	0.591
DDF	0.643	0.570	0.806	0.730	0.559	0.858	0.602	0.644
Graph	0.655	0.585	0.811	0.738	0.570	0.861	0.612	0.656
Gauge	0.701	0.466	0.830	0.697	0.614	0.797	0.729	0.690
Pareto-Koopmans	0.611	0.448	0.691	0.612	0.468	0.797	0.534	0.570
Min	0.605	0.448	0.691	0.612	0.468	0.797	0.534	0.570
Max	0.701	0.585	0.830	0.738	0.614	0.861	0.729	0.690

Table 7: Mean Allocative Efficiency by State

	BI	GU	HA	MP	MH	PU	RA	UP
Input	0.199	0.139	0.172	0.309	0.103	0.188	0.147	0.149
Output	0.246	0.205	0.192	0.399	0.138	0.214	0.173	0.191
DDF	0.228	0.169	0.183	0.349	0.118	0.202	0.162	0.173
Graph	0.223	0.164	0.182	0.344	0.114	0.208	0.158	0.171
Gauge	0.211	0.195	0.181	0.361	0.109	0.216	0.138	0.163
Pareto-Koopmans	0.287	0.217	0.211	0.431	0.154	0.215	0.192	0.199
Min	0.199	0.139	0.172	0.309	0.103	0.188	0.138	0.149
Max	0.287	0.217	0.211	0.431	0.154	0.216	0.192	0.199

Table 8: Summary Statistics of Output Price (Rs/Quintal)

	BI	GU	HA	MP	MH	PU	RA	UP
Mean	1527.280	1723.352	1625.792	1572.899	1799.503	1625.000	1657.283	1574.11
Std	55.545	7.055	10.623	169.602	180.908	0.003	120.697	51.269
Min	1400	1350	1624.987	1300	1500	1624.960	1200	1155.556
Max	1617.978	2500	1794.731	3000	2500	1625.014	2100	1800

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